Institut für Theoretische Teilchenphysik

Einführung in die Flavourphysik
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Exercise Sheet 3

Problem 4: Elements of the gamma matrix algebra

To compute physical observables involving fermions like cross-sections and decay rates, one often needs to manipulate Dirac matrices. Many identities can make this work easy and fast. To solve this exercise, only the defining anticommutation relation for the four Dirac matrices, $\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$, is needed. Also, the Fermi theory is a chiral theory, its vertices involve the γ_5 matrices. It is defined as $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, so it anticommutes with all the others, $\{\gamma^\mu, \gamma_5\} = 0$, but not with itself $\{\gamma_5, \gamma_5\} = 2$.

a) Prove that

$$
\begin{aligned}
i &: tr\left(\gamma^\mu\gamma^\nu\right) = 4g^{\mu\nu} && (1)\\
ii &: tr\left(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) = 4\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right) && (2)\\
iii &: tr(\text{odd number of } \gamma^\mu\text{'s}) = 0 && (3)\\
iv &: tr\left(\gamma_5\right) = tr\left(\gamma_5\gamma^\mu\gamma^\nu\right) = 0 && (4)\\
v &: tr\left(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\right) = -4i\epsilon^{\mu\nu\rho\sigma}\,. && (5)
\end{aligned}
$$

b) Prove the following identities:

$$
\begin{aligned}
i &: \gamma^\mu\gamma_\mu = 4 && (6)\\
ii &: \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu && (7)\\
iii &: \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho} && (8)\\
iv &: \gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_\mu = -2\gamma^\sigma\gamma^\rho\gamma^\nu\,. && (9)
\end{aligned}
$$

c) Prove that $P_L = (1-\gamma_5)/2$ and $P_R = (1+\gamma_5)/2$ are orthogonal projectors, i.e. such that $P_{L,R}^2 = P_{L,R}$, $P_L P_R = P_R P_L = 0$.

Problem 5: Semileptonic *B*-decays

The semileptonic decays $\bar{B}^0 \to X_c \ell \nu_\ell$ (with $\ell = e, \mu$) are induced by the partonic process $b \to c\ell\nu_\ell$. The X_c stands for all final states with flavor quantum number $C = 1$. Partonic processes can be described with an effective 4-fermion interaction:

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{cb} [\bar{c}\gamma^\mu(1-\gamma_5)b][\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell] + \text{h.c.},$$

where $G_F = 1.16637 \cdot 10^{-5}\,\text{GeV}^{-2}$ is the Fermi constant.

a) Calculate the squared, spin-averaged matrix element $|\mathcal{M}|^2$ for the process $b \to c\ell\nu_\ell$. Neglect the lepton and neutrino masses. You can use the formulas (1-9) and

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m, \qquad \sum_s v^s(p)\bar{v}^s(p) = \not{p} - m.$$