INSTITUT FÜR THEORETISCHE TEILCHENPHYSIK

Einführung in die Flavourphysik
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Sheet 12

Problem 18: The $\boldsymbol{\beta}$-function for $\boldsymbol{\alpha_s}$

The value of the strong coupling α_s depends on therenormalization scale μ. The dependence is determined through differential equation

$$\frac{d\alpha_s(\mu)}{d(\ln \mu)} = \beta(\alpha_s(\mu)) = -\frac{\beta_0}{2\pi}\alpha_s(\mu)^2 - \frac{\beta_1}{4\pi^2}\alpha_s(\mu)^3 - \dots \quad .$$

The coefficients β_0, β_1 etc. can be calculated:

$$\beta_0 = 11 - \frac{2}{3}n_f \quad , \quad \beta_1 = 51 - \frac{19}{3}n_f \quad ,$$

where n_f is the number of flavors.

a) Assume that the α_s is known at the scale $\mu = Q$. Calculate $\alpha_s(\mu)$ for $\mu \neq Q$ to first order (i.e. neglect the term with β_1).

b) Show that one can write $\alpha_s(\mu)$ as

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \quad .$$

Express the Λ through $\alpha_s(Q)$. How does Λ depend on n_f? What happens for $\mu \approx \Lambda$?

c) Now take into account the term with β_1. Show that, to a leading order in $\ln(\mu^2/\Lambda^2)^{-1}$ the following holds:

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}\left(1 - \frac{2\beta_1 \ln(\ln(\mu^2/\Lambda^2))}{\beta_0^2 \ln(\mu^2/\Lambda^2)}\right) + \mathcal{O}(1/\ln^3(\mu^2/\Lambda^2))$$

with

$$\Lambda = Q e^{-2\pi/\beta_0\alpha_s(Q)}\left(\frac{4\pi + 2\beta_1\alpha_s(Q)/\beta_0}{\beta_0\alpha_s(Q)}\right)^{\beta_1/\beta_0^2} \quad .$$