Problem 15: Leptonic decay $B_s \rightarrow \ell^+\ell^-$

The relevant electroweak effective Lagrangian for the process $B_s \rightarrow \ell^+\ell^-$ is given by \[1\]

$$
\mathcal{L} = N C_A(\mu_b) \left( \bar{b} \gamma_\alpha \gamma_5 s \right) \left( \bar{\mu} \gamma^\alpha \gamma_5 \mu \right) + \text{h.c.},
$$

with the normalization $N = V_{tb}^* V_{ts} G_F^2 M_{B_s}^2 / \pi^2$, the scale $\mu_b \sim m_b$ and Wilson coefficients $C_A(\mu_b)$. Latter are given at NLO EW and NNLO QCD as \[1\]

$$
C_A(\mu_b = 5 \text{ GeV}) = 0.4690 R_{1.53}^{-0.09},
$$

with $R_\alpha = \frac{\alpha_s(M_Z)}{0.1184}$ and $R_t = M_t / (173 \text{ GeV})$. The decay $B_s \rightarrow \mu^+\mu^-$ has been observed in the experiments CMS and LHCb at CERN\[2\].

a) Draw the leading order diagrams (in the unitary gauge) for $B_s \rightarrow \ell^+\ell^-$.  

b) Evaluate the numerical value for $B(\mu^+\mu^-)$. Use:

$$
\langle 0| \bar{b} \gamma^\alpha \gamma_5 s |B_s(p)\rangle = ip^\alpha f_{B_s}.
$$

The dependence of $B(\mu^+\mu^-)$ from $|\mathcal{M}|^2$ is given through:

$$
B(\mu^+\mu^-) = \frac{1}{16\pi M_{B_s} \Gamma_{H}^s} \sqrt{1 - \frac{4m^2_{\ell}}{M_{B_s}^2}} |\mathcal{M}|^2.
$$

You can find numerical values for the needed inputs in Ref. [3]. Test the standard model by comparing your result with the experimental value from Ref. [2].

Problem 16: Leptonic kaon decays

Consider leptonic $P^\pm \rightarrow \ell^\pm \nu$ decays ($P_{l2}$), with \[4\]

$$
\Gamma^{\text{SM}}(P^\pm \rightarrow \ell^\pm \nu) = \frac{G_F^2 M_P M^2_{\ell}}{8\pi} \left( 1 - \frac{M^2_{\ell}}{M^2_P} \right)^2 f^2_P |V_{q\nu}|^2.
$$

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Let us introduce the ratio:

\[ R_K = \frac{\Gamma(K_{e2})}{\Gamma(K_{\mu2})}. \] (6)

What is the leading standard model expectation for this observable? Which special property does \( R_K \) have? The current values of inputs can be found in Ref. [3]. Now also use the result of the 2-loop calculation in chiral perturbation theory in Ref. [5], that evaluates QED-corrections, and then test the standard model by comparing to experimental value [4]

\[ R_K^{\text{exp}} = (2.488 \pm 0.010) \times 10^{-5}. \] (7)

References


