

Problem 15: Leptonic decay $B_s \rightarrow \ell^+ \ell^-$

The relevant electroweak effective Lagrangian for the process $B_s \rightarrow \ell^+ \ell^-$ is given by [1]

$$\mathcal{L} = NC_A(\mu_b) (\bar{b}\gamma_\alpha \gamma_5 s) (\bar{\mu}\gamma^\alpha \gamma_5 \mu) + h.c., \quad (1)$$

with the normalization $N = V_{tb}^* V_{ts} G_F^2 M_W^2 / \pi^2$, the scale $\mu_b \sim m_b$ and Wilson coefficients $C_A(\mu_b)$. Latter are given at NLO EW and NNLO QCD as [1]

$$C_A(\mu_b = 5 \text{ GeV}) = 0.4690 R_t^{1.53} R_\alpha^{-0.09}, \quad (2)$$

with $R_\alpha = \alpha_s(M_Z)/0.1184$ and $R_t = M_t/(173.1 \text{ GeV})$. The decay $B_s \rightarrow \mu^+ \mu^-$ has been observed in the experiments CMS and LHCb at CERN [2].

- Draw the leading order diagrams (in the unitary gauge) for $B_s \rightarrow \ell^+ \ell^-$.
- Evaluate the numerical value for $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$. Use:

$$\langle 0 | \bar{b}\gamma^\alpha \gamma_5 s | B_s(p) \rangle = ip^\alpha f_{B_s}. \quad (3)$$

The dependence of $\mathcal{B}(B_s \rightarrow \ell^+ \ell^-)$ from $|\overline{\mathcal{M}}|^2$ is given through:

$$\mathcal{B}(B_s \rightarrow \ell^+ \ell^-) = \frac{1}{16\pi M_{B_s} \Gamma_H^s} \sqrt{1 - \frac{4m_\ell^2}{M_{B_s}^2}} |\overline{\mathcal{M}}|^2. \quad (4)$$

You can find numerical values for the needed inputs in Ref. [3]. Test the standard model by comparing your result with the experimental value from Ref. [2].

Problem 16: Leptonic kaon decays

Consider leptonic $P^\pm \rightarrow \ell^\pm \nu$ decays (P_{l2}), with [4]

$$\Gamma^{\text{SM}}(P^\pm \rightarrow \ell^\pm \nu) = \frac{G_F^2 M_P M_\ell^2}{8\pi} \left(1 - \frac{M_\ell^2}{M_P^2}\right)^2 f_P^2 |V_{qq'}|^2. \quad (5)$$

Let us introduce the ratio:

$$R_K \equiv \frac{\Gamma(K_{e2})}{\Gamma(K_{\mu2})}. \quad (6)$$

What is the leading standard model expectation for this observable? Which special property does R_K have? The current values of inputs can be found in Ref. [3]. Now also use the result of the 2-loop calculation in chiral perturbation theory in Ref. [5], that evaluates QED-corrections, and then test the standard model by comparing to experimental value [4]

$$R_K^{\text{exp}} = (2.488 \pm 0.010) \times 10^{-5}. \quad (7)$$

References

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