Prof. U. Nierste, Dr. M. Blanke
Dr. I. Nišandžić

Exercise Sheet 8

Problem 13: Leptonic B-decays

a) The decay process $B^+ \to \tau^+\nu_\tau$ is induced in the standard model by diagram in Fig. 1(a). Show that the decay can be described at the parton level with effective four-fermion interaction in the form:

$$\mathcal{L}_W^{\rm eff}(x) = -C_W[\bar{b}(x)\gamma_\mu P_L u(x)][\bar{\nu}_\tau(x)\gamma^\mu P_L \tau(x)] + \text{h.c.}$$

by expanding the W propagator to first non-vanishing order in p_B^2/M_W^2, where p_B is the momentum of the B^+ meson. Express the coefficient C_W with Fermi constant G_F and the CKM-matrix element V_{ub}.

b) The decay amplitude is:

$$\langle \tau^+(p,\sigma)\nu_\tau(k,\sigma')|i\mathcal{L}_W^{\rm eff}(0)|B^+(p_B)\rangle = i(2\pi)^4\delta^{(4)}(p_B - p - k)\mathcal{M}_{\sigma\sigma'}(p,k)$$

with

$$i\mathcal{M}_{\sigma\sigma'}(p,k) = -iC_W\bar{u}(k,\sigma')\gamma^\mu P_L v(p,\sigma)\langle 0|\bar{b}(0)\gamma_\mu P_L u(0)|B^+(p+k)\rangle \quad ,$$

where σ, σ' are spin indices and p, k are the four-momenta of the τ^+ and ν_τ. Parametrize the hadronic matrix element $\langle 0|\bar{b}(0)\gamma_\mu P_L u(0)|B^+(p+k)\rangle$ with the decay constant f_{B^+} and calculate the partial decay width $\Gamma(B^+ \to \tau^+\nu_\tau)$. Set $k^2 = 0$, $p^2 = m_\tau^2$ and $(p+k)^2 = m_B^2$.

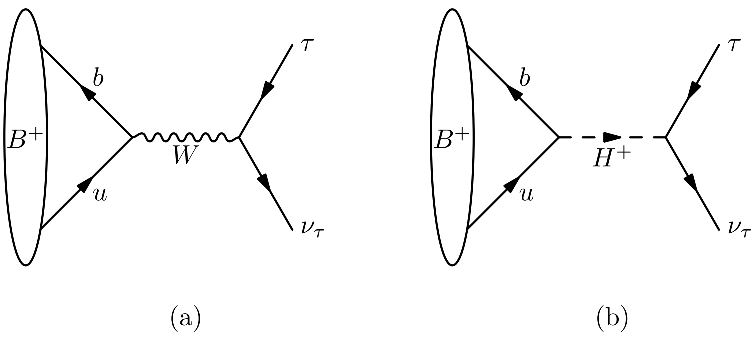

Abbildung 1: Tree level diagrams for $B^+ \to \tau^+\nu_\tau$ in standard model and two-Higgs-doublet model.

c) The diagram in two-Higgs-doublet models is given in Fig. 1(b). The relevant couplings in two-Higgs-doublet type-II (2HDM-II) (after neglecting up-quark and neutrino masses) are given by

$$\mathcal{L}_{H^\pm f\bar{f}} = \tan\beta \; m_b V_{ub}^* \frac{\sqrt{2}}{v} \bar{b} P_L u H^- + \tan\beta \; m_\tau \frac{\sqrt{2}}{v} \bar{\nu}_\tau P_R \tau H^+ + \text{h.c.} \quad .$$

d) In analogy to (a) express the interaction in Fig. 1(b) with

$$\mathcal{L}_H^{\text{eff}}(x) = -C_H [\bar{b}(x) P_L u(x)][\bar{\nu}_\tau(x) P_R \tau(x)] + \text{h.c.}$$

and determine the coefficient C_H.

e) Show

$$\langle 0 | \bar{b}(x) \gamma_5 u(x) | B^+(p_B) \rangle = -i f_{B^+} \frac{m_B^2}{m_b} e^{-ip_B x}.$$

For this you can use the equations of motion for the quark fields:

$$i \not{D} b(x) = m_b b(x) \quad , \quad i \not{D} u(x) = 0,$$

where $D_\mu = \partial_\mu - ig A_\mu^a T^a$ and A_μ^a denotes the gluon field.

f) Calculate, in analogy to (b), the partial decay rate $\Gamma(B^+ \to \tau^+ \nu_\tau)$ in 2HDM-II.

g) The present average for the several measurements of the branching ratio $\text{Br}(B^+ \to \tau^+ \nu_\tau)$ is $(1.15 \pm 0.23) \cdot 10^{-4}$. Compare this value to your standard model prediction. Sketch the region in the parameter space of $m_{H^\pm}, \tan\beta$ presently allowed by the experiment. For the following values of the remaining parameters you can use:

$$m_{B^+} = 5.27925\,\text{GeV} \quad , \quad f_{B^+} = 0.192\,\text{GeV} \quad , \quad \Gamma_{B^+} = 4.011 \cdot 10^{-13}\,\text{GeV} \quad ,$$
$$m_b = 4.8\,\text{GeV} \quad , \quad m_\tau = 1.77682\,\text{GeV} \quad , \quad |V_{ub}| = 0.00409 \quad ,$$
$$G_F = 1.1663787 \cdot 10^{-5}\,\text{GeV}^{-2} \quad , \quad v^2 = \frac{1}{\sqrt{2} G_F} \quad .$$