Institut für Theoretische Teilchenphysik

Einführung in die Flavourphysik
WiSe 2017/18

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Exercise Sheet 7

Problem 11: Discreete symmetries and quark currents

The transformation property of a Dirac spinor field $\psi(t,\mathbf{x})$ under the discreete transformations C, P and T is

$$C\psi(t,\mathbf{x})C^{-1} = \mathcal{C}\bar{\psi}(t,\mathbf{x})^{\mathsf{T}} \quad , \quad P\psi(t,\mathbf{x})P^{-1} = \mathcal{P}\psi(t,-\mathbf{x}) \quad ,$$
$$T\psi(t,\mathbf{x})T^{-1} = \mathcal{T}\psi(-t,\mathbf{x}) \quad ,$$

with $\mathcal{C} = i\gamma^0\gamma^2$, $\mathcal{P} = \gamma^0$ and $\mathcal{T} = \gamma^1\gamma^3$.

a) Consider two Dirac fields $\psi(t,\mathbf{x})$ and $\psi'(t,\mathbf{x})$. How do the products $\bar{\psi}(t,\mathbf{x})P_L\psi'(t,\mathbf{x})$ and $\bar{\psi}(t,\mathbf{x})P_R\psi'(t,\mathbf{x})$ (with $P_L = (\mathbb{1}-\gamma_5)/2$ and $P_R = (\mathbb{1}+\gamma_5)/2$) transform under C, P, T, CP and CPT?

b) How do the products $\bar{\psi}(t,\mathbf{x})\gamma^\mu P_L\psi'(t,\mathbf{x})$ and $\bar{\psi}(t,\mathbf{x})\gamma^\mu P_R\psi'(t,\mathbf{x})$ transform under C, P, T, CP and CPT?

c) How does the product $\bar{\psi}(t,\mathbf{x})\sigma^{\mu\nu}\psi'(t,\mathbf{x})$ (with $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$) transform under C, P, T, CP and CPT?

Problem 12: Hadronic hyperon decay

The decay of a Λ-hyperon into a π^- and a proton can be described through the effective interaction Lagrangian

$$\mathcal{L}_{\Lambda p\pi} = -\bar{\Lambda}(g_S + g_P\gamma_5)p\pi^- + \text{h.c.} \quad ,$$

where Λ and p are the spinor fields of a Λ-hyperon and a proton,respectively, $\pi^\pm \equiv (\pi^\mp)^\dagger$ denotes the scalar field of the pions and $g_{S,P}$ are complex coupling constants. The transformation of the pion field under the discrete transformations is

$$C\pi^\pm(t,\mathbf{x})C^{-1} = -\pi^\mp(t,\mathbf{x}) \quad , \quad P\pi^\pm(t,\mathbf{x})P^{-1} = -\pi^\pm(t,-\mathbf{x}) \quad ,$$
$$T\pi^\pm(t,\mathbf{x})T^{-1} = \pi^\pm(-t,\mathbf{x}) \quad ,$$

a) How does $\mathcal{L}_{\Lambda p\pi}$ transform under C, P, T, CP and CPT? Show that the physical observables are invariant under CPT-transformation.

b) Which relations have to be fulfilled by the couplings g_S and g_P, so that the theory is invariant under

   i) P-transformations?

   ii) CP-transformations?

   iii) T-transformations?