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Exercise Sheet 6

Problem 9: Two-Higgs doublet model

In two-Higgs doublet model one extends the Higgs sector of the standard model with one additional Higgs doublet. Consider two Higgs doublets:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad , \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$$

with hypercharge $+1$.

a) The vacuum expectation values of Φ_1 and Φ_2 break the $SU(2) \times U(1)$ symmetry to $U(1)_{\text{em}}$. Through suitable global phase transformations of Φ_1 and Φ_2 one can bring them to the form

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad , \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

where v_1 and v_2 are real and positive. Express the W- and Z-boson masses through v_1, v_2 and the gauge couplings of the standard model.
Hint: Consider the so called *Higgs basis*, that is defined through $\Phi = \Phi_1 \cos\beta + \Phi_2 \sin\beta$ and $\Phi' = -\Phi_1 \sin\beta + \Phi_2 \cos\beta$ with the definition $\tan\beta = v_2/v_1$.

b) The quark Yukawa sector of the two-Higgs doublet model has the general form

$$\mathcal{L}_Y = -Y_{ij}^{1u} \bar{Q}_{Li} \tilde{\Phi}_1 u_{Rj} - Y_{ij}^{1d} \bar{Q}_{Li} \Phi_1 d_{Rj} - Y_{ij}^{2u} \bar{Q}_{Li} \tilde{\Phi}_2 u_{Rj} - Y_{ij}^{2d} \bar{Q}_{Li} \Phi_2 d_{Rj} + \text{h.c.}$$

with $\tilde{\Phi}_{1,2} \equiv \varepsilon \Phi_{1,2}^* \equiv i\sigma_2 \Phi_{1,2}^*$. The i and j are generation indices and Y^{1u}, Y^{1d}, Y^{2u} and Y^{2d} are Yukawa matrices. The mass eigenstates for the Higgs bosons are $H^\pm$, $G^\pm$, G^0, A^0, H^0 and h^0. For the CP-conserving Higgs sector these fields are defined through:

$$\Phi_1 = \begin{pmatrix} c_\beta G^+ - s_\beta H^+ \\ \frac{1}{\sqrt{2}}(v_1 + c_\alpha H^0 - s_\alpha h^0 + i c_\beta G^0 - i s_\beta A^0) \end{pmatrix} \quad ,$$
$$\Phi_2 = \begin{pmatrix} s_\beta G^+ + c_\beta H^+ \\ \frac{1}{\sqrt{2}}(v_2 + s_\alpha H^0 + c_\alpha h^0 + i s_\beta G^0 + i c_\beta A^0) \end{pmatrix}$$

with $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $s_\alpha = \sin\alpha$ and $c_\alpha = \cos\alpha$. The angle α is determined by the couplings in the Higgs potential.

Diagonalize the quark mass matrices through unitary mixings of the quark fields

$$u_{Li} \to V_{ij}^{u_L} u_{Lj} \quad , \quad u_{Ri} \to V_{ij}^{u_R} u_{Rj} \quad , \quad d_{Li} \to V_{ij}^{d_L} d_{Lj} \quad , \quad d_{Ri} \to V_{ij}^{d_R} d_{Rj}$$

and determine the couplings of the Higgs fields A^0, H^0 and h^0 to the quark mass eigenstates. Which terms in $\mathcal{L}_Y$ lead to flavor changing neutral currents (FCNCs)?

c) To forbid the FCNCs one should add an additional symmetry. The simplest choice is a $\mathbb{Z}_2$-symmetry Z in the form

$$(\Phi_1, \Phi_2, u_{Ri}, d_{Ri}) \underset{Z}{\to} (\pm\Phi_1, \pm\Phi_2, \pm u_{Ri}, \pm d_{Ri}) \quad .$$

For which combinations of the signs does the Z forbid the FCNCs?

Problem 10: Dirac basis and Fierz identities

One can construct the basis for the space of complex 4×4 matrices from the Dirac matrices γ^μ. The basis matrices are:

$$\mathcal{B} = \{P_L \ , \ P_R \ , \ \gamma^\mu P_L \ , \ \gamma^\mu P_R \ , \ \sigma^{\mu\nu} \mid \mu, \nu = 0, \ldots, 3\}$$

with

$$\sigma^{\mu\nu} = \tfrac{i}{2}[\gamma^\mu, \gamma^\nu] \quad , \quad P_L = \tfrac{1}{2}(\mathbb{1} - \gamma_5) \quad , \quad P_R = \tfrac{1}{2}(\mathbb{1} + \gamma_5) \quad .$$

a) Calculate the traces $\mathrm{Tr}(\Gamma_1\Gamma_2)$ with $\Gamma_1, \Gamma_2 \in \mathcal{B}$.

b) Let the Γ be arbitrary complex 4×4 matrix. In the Dirac basis one can expand Γ as

$$\Gamma = s^L P_L + s^R P_R + v_\mu^L \gamma^\mu P_L + v_\mu^R \gamma^\mu P_R + c_{\mu\nu}\sigma^{\mu\nu} \quad .$$

Express the coefficients $s^{L,R}$, $v_\mu^{L,R}$ and $c_{\mu\nu}$ through the traces $\mathrm{Tr}(\Gamma'\Gamma)$ with $\Gamma' \in \mathcal{B}$.

c) Expand the matrices $\sigma^{\mu\nu} P_L$ and $\sigma^{\mu\nu} P_R$ in the Dirac basis $\mathcal{B}$. Calculate the outer product $(\sigma_{\mu\nu} P_L)_{ab} (\sigma^{\mu\nu} P_R)_{cd}$.

d) One can expand the tensors T_{abcd} with four Dirac indices in the basis of outer products of Dirac matrices:

$$T_{abcd} = \sum_{\Gamma,\Gamma' \in \mathcal{B}} C_{\Gamma,\Gamma'} \Gamma_{ab} \Gamma'_{cd} \quad .$$

Calculate the coefficients $C_{\Gamma,\Gamma'}$ of these expansions for

i) $T_{abcd} = (P_L)_{ad}(P_L)_{cb}$ and $T_{abcd} = (P_R)_{ad}(P_R)_{cb}$,

ii) $T_{abcd} = (\gamma^\mu P_L)_{ad}(\gamma_\mu P_L)_{cb}$ and $T_{abcd} = (\gamma^\mu P_R)_{ad}(\gamma_\mu P_R)_{cb}$,

iii) $T_{abcd} = (\gamma^\mu P_L)_{ad}(\gamma_\mu P_R)_{cb}$ and $T_{abcd} = (\gamma^\mu P_R)_{ad}(\gamma_\mu P_L)_{cb}$.

These identities are called *Fierz identities.*