Institut für Theoretische Teilchenphysik

Einführung in die Flavourphysik
WiSe 2017/18

Prof. U. Nierste
Dr. I. Nisandzic

Exercise Sheet 1

Problem 1: CKM matrix and unitarity triangle

a) Starting from the standard convention for the CKM matrix find a new phase convention in which 5 CKM elements are exactly real and positive. Keep V_{ud}, V_{us}, V_{cb}, V_{tb} as they are and choose one of the remaining elements to be real. For which CKM elements is this possible?

b) In the Wolfenstein approximation one has $V_{ub} = |V_{ub}| \exp(-i\gamma)$ and $V_{td} = |V_{td}| \exp(-i\beta)$, while all other CKM elements are real. Rephase V_{ij} in order to express V such that the only non-zero phases are

   i) α and β

   ii) α and γ.

c) Show explicitly that the three angles α, β and γ of the unitarity triangle are independent of the phase convention of V.

d) Two sides of the unitarity triangle are

$$R_u = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \quad , \quad R_t = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2} \quad ,$$

and the angles γ and β are given by

$$\gamma = \arctan \frac{\bar{\eta}}{\bar{\rho}} \quad , \quad \beta = \arctan \frac{\bar{\eta}}{1-\bar{\rho}} \quad .$$

Express

   i) R_t and β in terms of R_u and γ,

   ii) R_u and R_t in terms of β and γ.

Problem 2: Jarlskog invariant

For $i \neq k$ and $j \neq l$ consider

$$J = \sigma_{ik}\sigma_{jl}\ \mathrm{Im}\big[V_{ij}V_{il}^{*}V_{kl}V_{kj}^{*}\big] \quad \text{with} \quad \sigma_{ik} = \sum_{m=1}^{3} \varepsilon_{ikm} \quad .$$

a) Show that J is independent of the phase convention of V.

b) Show that J is independent of i, k, j, l.

c) Express J in terms of the (improved) Wolfenstein parameters A, λ, $\bar{\rho}$ and $\bar{\eta}$.