Classical Theoretical Physics II

Lecture: Prof. Dr. K. Melnikov – Exercises: Dr. H. Frellesvig, Dr. R. Rietkerk

Exercise Sheet 13

Issue: 13.07.18 – Submission: 20.07.18 before 09:30

Exercise 1: Plate — 8 points

We borrow from the TTP-kitchen a thin circular plate of radius R, and with a homogeneous (areal) mass density such that the mass of the plate is m. We will position the plate in the x, y-plane, with its center in the origin.

(a) What is the (scalar) moment of inertia of the plate around the z-axis?

(b) What is the (scalar) moment of inertia of the plate around the x-axis? And around the y axis?

(c) Consider an axis parallel with the z-axis but touching the edge of the plate. Calculate the (scalar) moment of inertia of the plate around that axis using the parallel axis theorem.

(d) Repeat the previous question by performing the integral directly, and get agreement.

(e) Consider now drilling a hole (with radius q) with its center at $(r, 0, 0)$, such that $q < r$ and $q + r < R$. What are now the (scalar) moments of inertia of the plate around the x, y, and z axes?

(f) Consider now the plate (without the hole) rotating around the x-axis, with angular velocity ω. What is the kinetic energy of the plate?

(g) Consider now the plate (without the hole) rolling along a floor, with velocity v. What is the kinetic energy of the plate?

Exercise 2: Polygon — 4 points

Consider a homogeneous thin regular polygon with mass m, area A and N sides.

(a) Calculate the (scalar) moment of inertia I_N of the polygon with respect to the axis perpendicular to the polygon passing through its center.

(b) Show that the general result for the previous question reproduces the moments of inertia for the square and the circle:

$$I_{\text{square}} = \frac{mA}{6}\,, \qquad I_{\text{circle}} = \frac{mA}{2\pi}\,. \tag{1}$$

Hint: use the fact that $\lim_{N\to\infty} N \tan(\pi/N) = \pi$.

Exercise 3: Rocking Chair

8 points

After having worked all day with plates and polygons we take some rest in a rocking chair. It has a mass m and moment of inertia I_{cm} around its center of mass. The legs of the chair are wooden arcs with radius of curvature R. When the chair stands up straight, the center of mass is at a height $h < R$ straight above the point of contact with the floor. The aim of this problem is to determine the 'rocking frequency' of the chair.

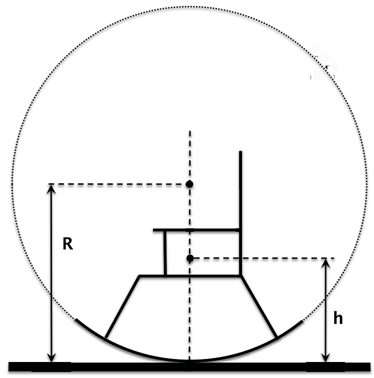

Figure 1: Rocking chair.

(a) Find the position of the center of mass $(x_{\text{cm}}(\theta), y_{\text{cm}}(\theta))$ as a function of the angle θ between the rocking chair and the vertical. Define the angle in such a way, that $\theta = 0$ when the chair stands up straight and that $\theta > 0$ when the chair leans backward, see fig. 2. Choose the origin such that $(x_{\text{cm}}(0), y_{\text{cm}}(0)) = (0, h)$.

(b) Determine the potential energy of the rocking chair (due to gravity) as a function of θ. Perform a Taylor expansion of the potential energy around the equilibrium point. Why are small oscillations of the rocking chair around the equilibrium point stable?

(c) Determine the kinetic energy of the rocking chair as a function of θ. Taylor expand it around the equilibrium point.

(d) Show that the frequency of small oscillations of the rocking chair is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{mg(R-h)}{I_{\text{cm}} + mh^2}}\ . \tag{1}$$

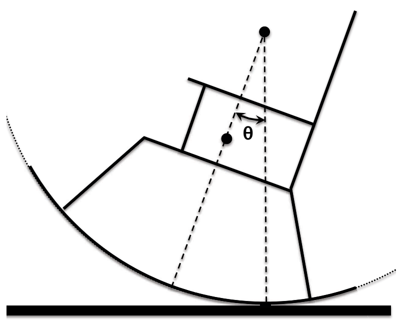

Figure 2: The backwards leaning rocking chair makes an angle θ with the vertical axis.