Classical Theoretical Physics II

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Exercise sheet 0

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Exercise 1: Basic techniques

(a) Solve the differential equation $\ddot{x}(t) = -\omega^2 x(t)$ with boundary conditions $x(t_1) = x_1$ and $x(t_2) = x_2$.

(b) Calculate the eigenvalues and eigenvectors of the matrix $R = \left(\begin{smallmatrix} 3 & 1 \\ -2 & 0 \end{smallmatrix}\right)$.

(c) Express the vector $\vec{A} = A_x\,\vec{e}_x + A_y\,\vec{e}_y$ in terms of polar coordinates. That is, determine A_r and A_θ in $\vec{A} = A_r\vec{e}_r + A_\theta\vec{e}_\theta$.

(d) Show that the divergence $\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}$ is given in polar coordinates by

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_r}{\partial r} + \frac{A_r}{r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta}\ . \tag{1}$$

(e) The Levi-Civita symbol ϵ_{ijk} is a three-index object, defined as

$$\epsilon_{ijk} = \begin{cases} 0, & \text{if any two indices are equal} \\ 1, & \text{if } i,j,k \text{ is an even permutation of 1,2,3} \\ -1, & \text{if } i,j,k \text{ is an odd permutation of 1,2,3} \end{cases} \tag{2}$$

Show that

$$(\vec{A}\times\vec{B}\,)_i = \epsilon_{ijk}A_jB_k\ , \tag{3}$$

where A_j is the jth component of the vector $\vec{A}$, etc.

(f) Show that

$$\sum_{i=1}^{3}\epsilon_{ijk}\epsilon_{i\ell m} = \delta_{j\ell}\delta_{km} - \delta_{jm}\delta_{k\ell}\ . \tag{4}$$

(g) Equations (3) and (4) are very useful for deriving all kinds of vector identities. For instance, prove the following relations:

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})\ , \tag{5}$$

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A} + \vec{B}\times(\vec{\nabla}\times\vec{A}) + \vec{A}\times(\vec{\nabla}\times\vec{B})\ . \tag{6}$$

(h) Calculate the following integrals:

$$\int \frac{dx}{x+a}\ , \quad \int \frac{dx}{\sqrt{1-x^2}}, \quad \int \frac{dx}{1+e^x}, \qquad \int \frac{dx}{x^2-5x+6}, \tag{7}$$

$$\int_0^{2\pi} \mathrm{d}\varphi \cos^2(\varphi), \quad \int_a^b \frac{\mathrm{d}x}{\sqrt{(x-a)(b-x)}}. \tag{8}$$