

Readme for `kernel3.m`

The file `kernel3.m` contains the results for the three-loop kernel functions $K^{(3a)}(s)$, $K^{(3b)}(s)$, $K^{(3b,lbl)}(s)$ and $K^{(3c)}(s, s')$ of Ref. [1]. $K^{(3a)}$, $K^{(3b)}$ and $K^{(3b,lbl)}$ are expanded in M_μ^2/s and M_e/M_μ whereas $K^{(3c)}$ also depends on s' and thus a further assumption on the hierarchy between s and s' is necessary. We provide results for $s \approx s' \gg M_\mu^2$ and $s' \gg s \gg M_\mu^2$ from which approximations of $K^{(3c)}(s, s')$ valid for all s and s' can be constructed. In the case of $K^{(3c)}(s, s')$ only the leading non-vanishing term in M_e/M_μ is computed.

The following table contains the information about the expansion depth of the expressions in `kernel3.m`:

kernel	symbol in <code>kernel3.m</code>	highest available expansion term
$K^{(3a)}$	K3a	$(M_\mu^2/s)^4$
$K^{(3b)}$	K3b	$(M_\mu^2/s)^4, (M_e^2/s)^1(M_\mu^2/s)^3$
$K^{(3b,lbl)}$	K3bLBL	$(M_\mu^2/s)^4, (M_e^2/s)^1(M_\mu^2/s)^3$
$K^{(3c)}$	K3cH1	$(M_\mu^2/s)^5, (M_\mu^2/s)^3[(\sqrt{s'} - \sqrt{s})/\sqrt{s}]^3$
	K3cH2	$(M_\mu^2/s')^5, (M_\mu^2/s')^1(s/s')^4$

The symbols used in `kernel3.m` have the following meaning:

symbol	Mmu	Mel	Ms	Msp	deltaMspMs
meaning	M_μ	M_e	\sqrt{s}	$\sqrt{s'}$	$\sqrt{s'} - \sqrt{s}$

[1] Alexander Kurz, Tao Liu, Peter Marquard, Matthias Steinhauser, “*Hadronic contribution to the muon anomalous magnetic moment to next-to-next-to-leading order*”, SFB/CPP-14-19, TTP14-009.