User Guide for the `Mathematica` File `rho.m`

J. Grigo, J. Hoff, P. Marquard and M. Steinhauser

version: June 2012

The file `rho.m` contains the results related to the ρ parameter presented in [1] in a `Mathematica`-readable form. If you use any of the contents of this file, please refer to [1] in the corresponding publication.

Z boson self energy

$$\frac{\Sigma_Z(0)}{M_Z^2} = \frac{3G_F m_{t'}^2}{16\pi^2\sqrt{2}} \left(\bar{C}^a_{-1,\mathrm{diag}} + \left(\frac{\alpha_s}{\pi}\right)^2 \delta\bar{C}^{a,(2)}_{-1,\mathrm{db}} + \left(\frac{\alpha_s}{\pi}\right)^2 \delta\bar{C}^{a,(2)}_{-1,\mathrm{sing}} \right)$$

$$\begin{aligned} \bar{C}^a_{-1,\mathrm{diag}} &= \texttt{CbarDiagMsBar} \\ \delta\bar{C}^{a,(2)}_{-1,\mathrm{db}} &= \texttt{CbarDbMsBar} \\ \delta\bar{C}^{a,(2)}_{-1,\mathrm{sing}} &= \texttt{CbarSingMsBar} \end{aligned}$$

ρ parameter

in the $\overline{\mathbf{MS}}$ scheme

$$\delta\rho(x) = \frac{3G_F m_{t'}^2}{16\pi^2\sqrt{2}} \left(\delta^{(0)}(x) + \frac{\alpha_s}{\pi}\delta^{(1)}(x) + \left(\frac{\alpha_s}{\pi}\right)^2 \delta^{(2)}(x) + \mathcal{O}(\alpha_s^3) \right)$$

$$\begin{aligned} \delta^{(0)}(x) &= \texttt{rhoMsBar0} \\ \delta^{(1)}(x) &= \texttt{rhoMsBar1} \\ \delta^{(2)}(x) &= \texttt{rhoMsBar2Exp0} + \mathcal{O}(x^{31}) \\ \delta^{(2)}(x) &= \texttt{rhoMsBar2Exp1} + \mathcal{O}((1-x)^{21}) \end{aligned}$$

and in the on-shell scheme

$$\Delta\rho(X) = \frac{3G_F M_{t'}^2}{16\pi^2\sqrt{2}} \left(\Delta^{(0)}(X) + \frac{\alpha_s}{\pi}\Delta^{(1)}(X) + \left(\frac{\alpha_s}{\pi}\right)^2 \Delta^{(2)}(X) + \mathcal{O}(\alpha_s^3) \right)$$

$$\Delta^{(0)}(X) = \texttt{rhoOnShell0}$$
$$\Delta^{(1)}(X) = \texttt{rhoOnShell1}$$
$$\Delta^{(2)}(X) = \texttt{rhoOnShell2Exp0} + \mathcal{O}(X^{31})$$
$$\Delta^{(2)}(X) = \texttt{rhoOnSehll2Exp1} + \mathcal{O}((1-X)^{21})$$

Used symbols and functions

Strong coupling constant and gauge group

Instead of the strong coupling constant α_s we use

$$\texttt{api} = \frac{\alpha_s}{\pi} \ .$$

The results are expressed in terms of the SU(N_c) invariants $C_A = N_c$ and $C_F = (N_c^2-1)/(2N_c)$ where $T_F = 1/2$ is chosen. We use the symbols `nc`, `ca` and `cf`.

Renormalization scale and masses

The renormalization scale dependence is expressed by

$$\texttt{lm} = \log\frac{\mu^2}{m_{t'}^2}\ (\overline{\text{MS}}\text{ scheme}) \qquad \texttt{lM} = \log\frac{\mu^2}{M_{t'}^2}\ (\text{on-shell scheme})$$

where μ stands for the renormalization scale and $m_{t'}(M_{t'})$ for the heavier $\overline{\text{MS}}$ (on-shell) quark mass. The other quark mass $m_{b'}(M_{b'})$ is given by

$$m_{b'} = \texttt{x}\ m_{t'} \qquad (M_{b'} = \texttt{X}\ M_{t'}) \qquad \Rightarrow \qquad \texttt{x} = \frac{m_{b'}}{m_{t'}} \qquad (\texttt{X} = \frac{M_{b'}}{M_{t'}})\ .$$

Massless Quarks

The number of massless quarks occurring in closed loops is given by

`nl`.

`MATAD` symbols

Some results depend on the constants `B4`, `D3`, `S2`, `OepS2` and `T1ep`, introduced in [2]:

$$\begin{aligned}
\texttt{D3} &= 6\zeta_3 - \frac{15}{4}\zeta_4 - 6\left[\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right]^2 , \\
\texttt{B4} &= -4\zeta_2 \ln^2 2 + \frac{2}{3}\ln^4 2 - \frac{13}{2}\zeta_4 + 16\mathrm{Li}_4\left(\frac{1}{2}\right) , \\
\texttt{S2} &= \frac{4}{9\sqrt{3}}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) , \\
\texttt{OepS2} &= -\frac{763}{32} - \frac{9\pi\sqrt{3}\ln^2 3}{16} - \frac{35\pi^3\sqrt{3}}{48} + \frac{195}{16}\zeta_2 - \frac{15}{4}\zeta_3 + \frac{57}{16}\zeta_4 \\
&\quad + \frac{45\sqrt{3}}{2}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) - 27\sqrt{3}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{e^{-i\pi/6}}{\sqrt{3}}\right)\right] , \\
\texttt{T1ep} &= -\frac{45}{2} - \frac{\pi\sqrt{3}\ln^2 3}{8} \\
&\quad - \frac{35\pi^3\sqrt{3}}{216} - \frac{9}{2}\zeta_2 + \zeta_3 + 6\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) - 6\sqrt{3}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{e^{-i\pi/6}}{\sqrt{3}}\right)\right] .
\end{aligned}$$

They can be replaced by `Mathematica`-known symbols and functions via the replacement rules in the list `matadSymbols`.

HPLs

The one- and two- loop results are expressed by harmonic polylogarithms (`HPL[...]`). To work with those functions we recommend the package `HPL.m` [3, 4]. Note that depending on the `Mathematica` version the application of `Series[]` to HPLs might lead to wrong results. Alternatively it is possible to use the replacement rules in `hplsToKnownFunctions` to express the HPLs through known functions (`Log[...]` and `PolyLog[...]`).

References

[1] J. Grigo, J. Hoff, P. Marquard, and M. Steinhauser, "Moments of heavy quark correlators with two masses: Exact mass dependence to three loops," *Nuclear Physics B* (2012) , `arXiv:1206.3418 [hep-ph]`. `http://www.sciencedirect.com/science/article/pii/S0550321312003902`.

[2] M. Steinhauser, "MATAD: A program package for the computation of massive tadpoles," *Comput. Phys. Commun.* **134** (2001) 335–364, `arXiv:hep-ph/0009029`.

[3] D. Maitre, "HPL, a mathematica implementation of the harmonic polylogarithms," *Comput.Phys.Commun.* **174** (2006) 222–240, `arXiv:hep-ph/0507152 [hep-ph]`.

[4] D. Maitre, "Extension of HPL to complex arguments," *Comput.Phys.Commun.* **183** (2012) 846, `arXiv:hep-ph/0703052 [HEP-PH]`.