User Guide for the `Mathematica` Package `TwoMassTadpoles.m`

J. Grigo, J. Hoff, P. Marquard and M. Steinhauser

version: June 2012

The package `TwoMassTadpoles.m` contains a collection of tadpole integrals with two different mass scales. For convenience also all related single mass scale integrals are included. For further details see [1]. If you use any of the contents of this file, please refer to [1] in the corresponding publication.

The integrals with two different mass scales are expressed in terms of HPLs, the `Mathematica` package `HPL.m` [2, 3] is needed. Note that depending on the `Mathematica` version the application of `Series[]` to HPLs might lead to wrong results.

Functions

`TwoMassTadpoles.m` provides two functions:

- `TwoMassTadpoles[id, ep, x [,options]]`
  for integrals with two mass scales (Figs. 1, 2 and 3) and
- `OneMassTadpoles[id, ep [,options]]`
  for integrals with one mass scale (Fig. 4).

Parameters

- `id` Identifier of the integral (see figures), e.g. `"m-3l-1"`;
- `ep` Symbol of the variable ϵ in $d = 4 - 2\epsilon$ dimensions and
- `x` (for `TwoMassTadpoles[...]` only) mass ratio $\mathtt{x} = \frac{m_2}{m_1}$.

Return value

The functions `TwoMassTadpoles` and `OneMassTadpoles` provide the integrals corresponding to the identifier `id`. The form of the result is adjustable via the options.

Options

- `epOrder -> no`

  By default the integral is given as a series expansion in ϵ or (if known) with exact ϵ-dependence. Via setting `epOrder -> n` (integer) the `n`-th coefficient of the ϵ-expansion is returned instead. If the requested coefficient is not known,

  ```
  unknown[integral:<id>,epOrder: <n>,<x>]
  ```

  will be returned. In Figs. 1 – 4 we indicate below each individual diagram the identifier and the highest known ϵ-coefficient, in case only a limited number of expansion terms are available.

- `massM1 -> 1`

  This option allows to adjust the mass m_1. This implies that for the integrals with two masses the second mass is given by $m_2 = xm_1$. Note that in the logarithmic contributions we set $\mu = 1$.

- `loopConstant -> I*Pi^(2-ep)*Gamma[1+ep]`

  The result is divided by this constant per loop order.

- `switchPropagatorSign -> False`

  The global sign corresponds to propagators in Minkowski space as follows:

  $$\frac{1}{-p^2+m^2} \qquad \text{and} \qquad \frac{1}{-p^2}\,.$$

  Setting this option to `True` switches the propagator sign, i.e. for each integral an additional factor

  $$(-1)^{\text{number of propagators}}$$

  is multiplied.

- `solutionType -> "numeric"` (integrals in Fig. 3 only)
  For the integrals in Fig. 3, which are not known analytically, one can choose from different approximations:

  - `"numeric"`
    Interpolation based on series expansions and numerical evaluation of the integrals.
    - for the integrals in Fig. 3 with four lines:
      asymptotic expansion ($0 \le x \le 0.2$), the numerical evaluation ($0.2 < x < 0.5$) and the Taylor expansion ($0.5 \le x \le 1$);
    - for the integrals in Fig. 3 with six lines: asymptotic expansion ($0 \le x \le 0.3$) and the Taylor expansion ($0.3 < x \le 1$);
  - `"expansion0"`
    asymptotic expansion around $x = 0$, numerical coefficients
  - `"expansion0-symbolic"`
    asymptotic expansion around $x = 0$, exact coefficients expressed by the symbols `B4`, `D3`, `S2`, `OepS2` and `T1ep` (see below)
  - `"expansion0-exact"`
    asymptotic expansion around $x = 0$, exact coefficients expressed by `Mathematica`-known functions and constants (e.g. `Im[PolyLog[2, Exp[I Pi/3]]]`)
  - `"expansion1"`
    Taylor expansion around $x = 1$, exact coefficients
  - `"interpolation"` (integrals with four lines only)
    interpolation, based on the numerical evaluation for $x \in [0.1, 1]$, step size 0.005

- `warn -> True`
  If set to `False`, warnings are suppressed.

`MATAD` symbols

The following constants `B4`, `D3`, `S2`, `OepS2` and `T1ep` are introduced in [4]:

$$
\begin{aligned}
\texttt{D3} &= 6\zeta_3 - \frac{15}{4}\zeta_4 - 6\left[\mathrm{Cl}_2\left(\frac{\pi}{3}\right)\right]^2 , \\
\texttt{B4} &= -4\zeta_2 \ln^2 2 + \frac{2}{3}\ln^4 2 - \frac{13}{2}\zeta_4 + 16\mathrm{Li}_4\left(\frac{1}{2}\right) , \\
\texttt{S2} &= \frac{4}{9\sqrt{3}}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) , \\
\texttt{OepS2} &= -\frac{763}{32} - \frac{9\pi\sqrt{3}\ln^2 3}{16} - \frac{35\pi^3\sqrt{3}}{48} + \frac{195}{16}\zeta_2 - \frac{15}{4}\zeta_3 + \frac{57}{16}\zeta_4 \\
&\quad + \frac{45\sqrt{3}}{2}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) - 27\sqrt{3}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{e^{-i\pi/6}}{\sqrt{3}}\right)\right] , \\
\texttt{T1ep} &= -\frac{45}{2} - \frac{\pi\sqrt{3}\ln^2 3}{8} \\
&\quad - \frac{35\pi^3\sqrt{3}}{216} - \frac{9}{2}\zeta_2 + \zeta_3 + 6\sqrt{3}\mathrm{Cl}_2\left(\frac{\pi}{3}\right) - 6\sqrt{3}\mathrm{Im}\left[\mathrm{Li}_3\left(\frac{e^{-i\pi/6}}{\sqrt{3}}\right)\right] .
\end{aligned}
$$

References

[1] J. Grigo, J. Hoff, P. Marquard, and M. Steinhauser, "Moments of heavy quark correlators with two masses: Exact mass dependence to three loops," *Nuclear Physics B* (2012) , `arXiv:1206.3418 [hep-ph]`. `http://www.sciencedirect.com/science/article/pii/S0550321312003902`.

[2] D. Maitre, "HPL, a mathematica implementation of the harmonic polylogarithms," *Comput.Phys.Commun.* **174** (2006) 222–240, `arXiv:hep-ph/0507152 [hep-ph]`.

[3] D. Maitre, "Extension of HPL to complex arguments," *Comput.Phys.Commun.* **183** (2012) 846, `arXiv:hep-ph/0703052 [HEP-PH]`.

[4] M. Steinhauser, "MATAD: A program package for the computation of massive tadpoles," *Comput. Phys. Commun.* **134** (2001) 335–364, `arXiv:hep-ph/0009029`.

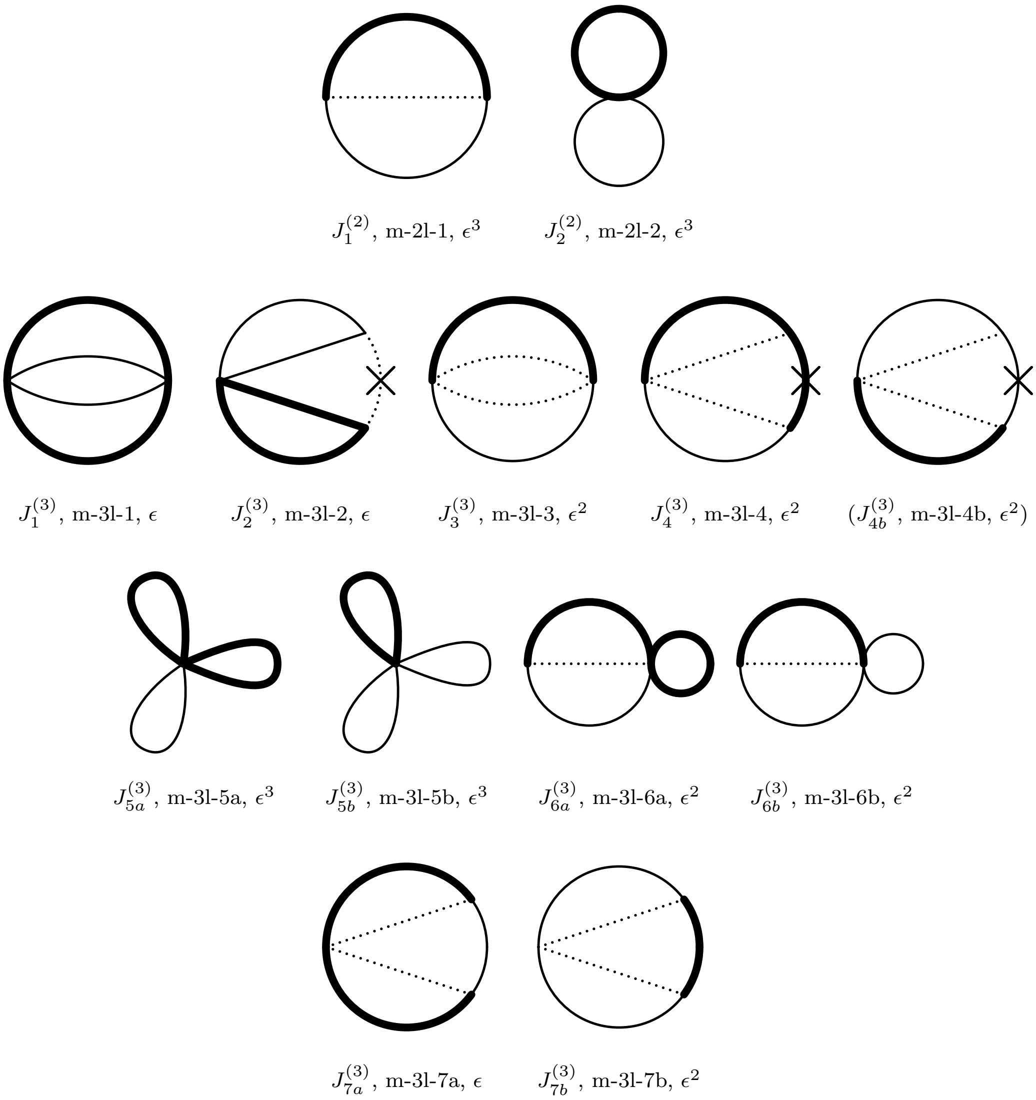

Figure 1: Analytically known master integrals. Thick, thin and dotted lines denote massive (m_1, m_2) and massless propagators. A cross on a line indicates that the corresponding propagator is raised to power minus one. m-3l-4b is no master integral since it can be expressed by m-3l-3 and m-3l-4. It is listed here for the sake of convenience.

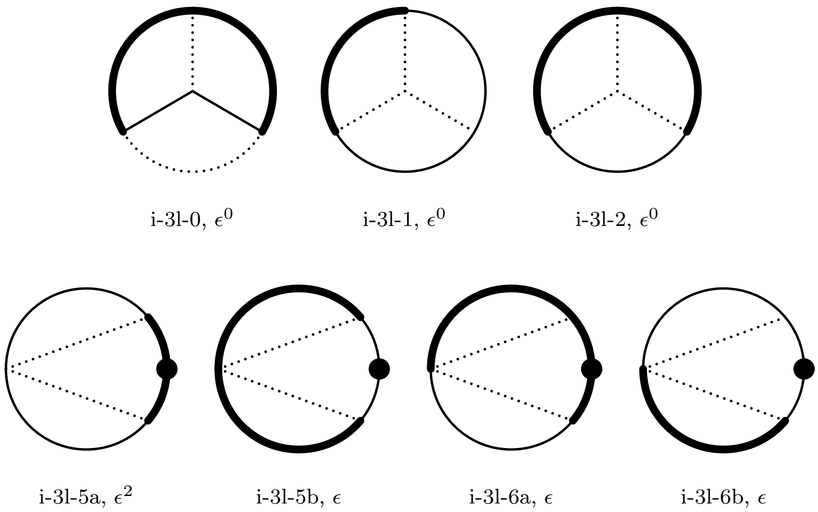

Figure 2: Analytically known integrals which are reducible to the master integrals in Fig. 1. A dot on a line indicates that the corresponding propagator is squared.

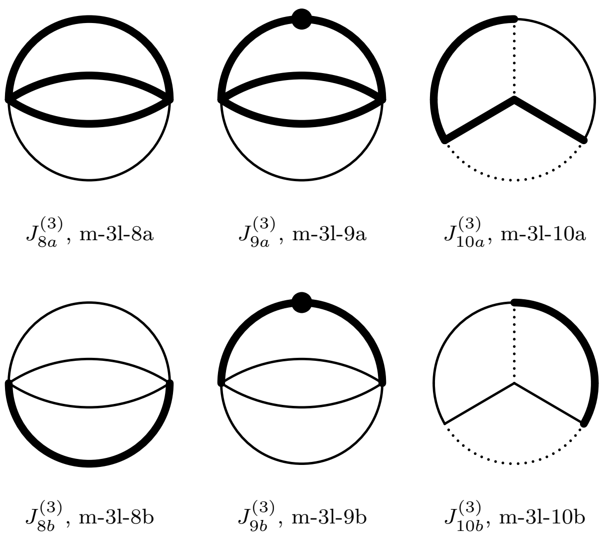

Figure 3: Integrals for which the finite part is only known numerically.

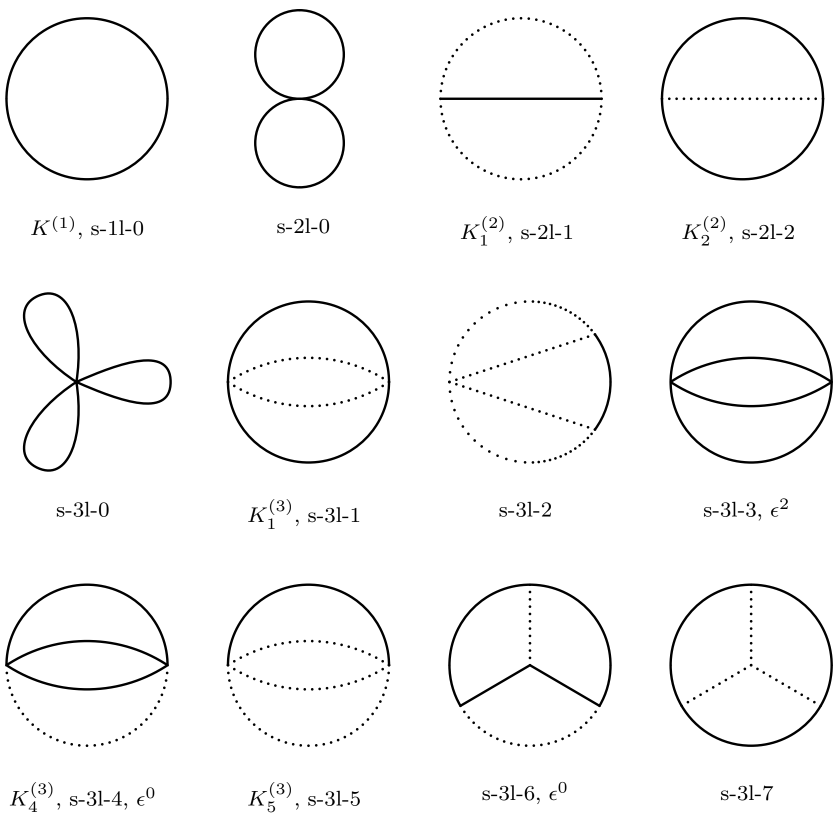

Figure 4: Integrals with one mass scale.