User Guide for the `Mathematica` Package `coefhl2.m`

J. Grigo, J. Hoff, P. Marquard and M. Steinhauser

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This package contains the moments of non-diagonal current correlators presented in [1]. If you use any of the contents of this file, please refer to [1] in the corresponding publication. `coefhl2.m` contains improved results compared to the original package `coefhl.m` introduced in [2]. For further details see [1].

In order to use the (partly) known exact analytic results, the `Mathematica` package `HPL.m` [3, 4] is needed. Note that depending on the `Mathematica` version the application of `Series[]` to HPLs might lead to wrong results. The numerical results and series expansions contained in `coefhl2.m` do not need any additional package. `coefhl2.m` provides the results in terms of two functions:

- `Cbar2[case, aOrd, zOrd, x [,options]]`
  for the moments of the (pseudo-)scalar and the transverse part of the (axial-)vector correlator and
- `CbarL2[case, aOrd, zOrd, x [,options]]`
  for the longitudinal part of the (axial-)vector correlator.

Parameters

- `case`: `"s"`, `"p"`, `"v"`, `"a"`

  scalar, **p**seudo-scalar, **v**ector or **a**xial-vector current
- `aOrd`: `0,1,2`

  order of the perturbative expansion in $\frac{\alpha_s}{\pi}$
- `zOrd`: if `aOrd` < 2: `zOrd` $\in \{\texttt{-1}, \ldots, \texttt{9}\}$, if `aOrd` $= 2$: `zOrd` $\in \{\texttt{-1}, \ldots, \texttt{4}\}$
  order of the momentum expansion in $z = q^2/m_1^2$
- `x`: symbol or number $\in [0, 1]$
  mass ratio $\mathtt{x} = m_2/m_1$

Return value

Following the notation introduced in [1] we have:

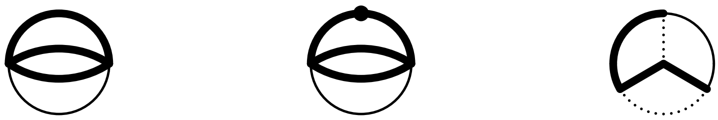
`Cbar2[case, aOrd, zOrd, x ]`	returns	$\bar{C}^{(\mathtt{aOrd}),\mathtt{case}}_{\mathtt{zOrd}}(\mathtt{x})$,
`CbarL2[case, aOrd, zOrd, x ]`	returns	$\bar{C}^{(\mathtt{aOrd}),\mathtt{case}}_{L,\mathtt{zOrd}}(\mathtt{x})$.

At one- and two-loop level (`aOrd` $= 0, 1$) the exact analytic result in terms of HPLs is returned. At three-loop level (`aOrd` $= 2$) by default a numerical solution (using `InterpolatingFunction`) valid for $x \in [0,1]$ is returned, see `"numeric"` below.

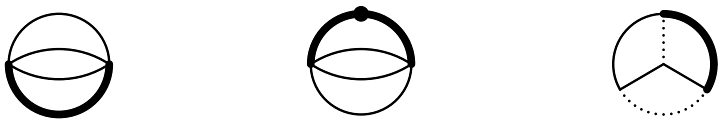
By default all results are given for the gauge group SU(3) with two massive (m_1 and m_2, $m_1 \geq m_2$) and three massless quarks at the renormalization scale $\mu = m_1$. There are several options to obtain the result in a different form.

Options

- `"solutionType" -> "numeric"` (for `aOrd` $= 2$ only)
  This option concerns the treatment of the master integrals for which only series expansions and numerical results are known.
  - `"numeric"`
    Interpolation based on
    * (integrals $J^{(3)}_{8a,b}$ and $J^{(3)}_{9a,b}$) the asymptotic expansion ($0 \leq x \leq 0.2$), the numerical evaluation ($0.2 \leq x \leq 0.5$) and the Taylor expansion ($0.5 \leq x \leq 1$);
    * (integrals $J^{(3)}_{10a,b}$) the asymptotic expansion ($0 \leq x \leq 0.3$) and the Taylor expansion ($0.3 \leq x \leq 1$);
  - `"expansion0"`
    Expansion around $x = 0$;
  - `"expansion1"`
    Expansion around $x = 1$;
  - `"exact"`
    Exact analytic results where the (so far unknown) finite parts of the following integrals remain as symbols (the poles have already been plugged in):



$J^{(3)}_{8a}$, `master3l8a[x]` $J^{(3)}_{9a}$, `master3l9a[x]` $J^{(3)}_{10a}$, `master3l10a[x]`



$J^{(3)}_{8b}$, `master3l8b[x]` $J^{(3)}_{9b}$, `master3l9b[x]` $J^{(3)}_{10b}$, `master3l10b[x]`

- `SU3 -> True`
  The moments are given by default for the SU(3) case. If set to `False`, the result is given in terms of `ca` $=$ `nc` and `cf` $= \frac{\texttt{nc}^2-1}{2\texttt{nc}}$ corresponding to SU(`nc`);

- `nh -> 1`
  Number of heavy quarks (mass m_1);

- `nm -> 1`
  Number of medium heavy quarks (mass m_2);

- `nl -> 3`
  Number of massless quarks;
- `imu -> 1`
  Ratio of the renormalization scale μ and the heavy mass m_1:
  $$\text{imu} = \frac{\mu}{m_1}$$
- `subtractPoles -> True`
  If set to `False` the remaining poles in ϵ of the moments with `zOrd` $= -1$ or 0 are not subtracted in the $\overline{\text{MS}}$ scheme but left untouched.

References

[1] J. Grigo, J. Hoff, P. Marquard, and M. Steinhauser, "Moments of heavy quark correlators with two masses: Exact mass dependence to three loops," *Nuclear Physics B* (2012) , `arXiv:1206.3418 [hep-ph]`. `http://www.sciencedirect.com/science/article/pii/S0550321312003902`.

[2] J. Hoff and M. Steinhauser, "Moments of heavy-light current correlators up to three loops," *Nucl.Phys.* **B849** (2011) 610–627, `arXiv:1103.1481 [hep-ph]`.

[3] D. Maitre, "HPL, a mathematica implementation of the harmonic polylogarithms," *Comput.Phys.Commun.* **174** (2006) 222–240, `arXiv:hep-ph/0507152 [hep-ph]`.

[4] D. Maitre, "Extension of HPL to complex arguments," *Comput.Phys.Commun.* **183** (2012) 846, `arXiv:hep-ph/0703052 [HEP-PH]`.