Three-loop β-functions for top-Yukawa and the Higgs self-interaction in the Standard Model

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Here we give the results of TTP12-012, SFB/CPP-12-23 in Mathematica code. We consider a model described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{y_t} + \mathcal{L}_{\Phi}.$$

with

$$\begin{aligned}
\mathcal{L}_{QCD} =& -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} - \frac{1}{2(1-\xi)} \left(\partial_\mu A^{a\,\mu}\right)^2 + \partial_\mu \bar{c}^a \partial^\mu c^a + g_s f^{abc}\, \partial_\mu \bar{c}^a A^{b\,\mu} c^c \\
&+ \sum_q \left\{ \frac{i}{2} \bar{q} \overleftrightarrow{\partial\!\!\!/}\, q + g_s \bar{q} A\!\!\!/^a T^a q \right\}, \\
\mathcal{L}_{y_t} =& -y_t \left\{ \left(\bar{t} P_R t\right) \Phi_2^* + \left(\bar{t} P_L t\right) \Phi_2 - \left(\bar{b} P_R t\right) \Phi_1^* - \left(\bar{t} P_L b\right) \Phi_1 \right\}, \\
\mathcal{L}_{\Phi} =& \partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2
\end{aligned}$$

and the counterterms

$$\begin{aligned}
\delta\mathcal{L}_{QCD} =& -\frac{1}{4} \delta Z_3^{(2g)} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 - \frac{1}{2} \delta Z_1^{(3g)} g_s f^{abc} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) A^b_\mu A^c_\nu \\
&- \frac{1}{4} \delta Z_1^{(4g)} g_s^2 \left(f^{abc} A^b_\mu A^c_\nu\right)^2 + \delta Z_3^{(2c)} \partial_\mu \bar{c}^a \partial^\mu c^a + \delta Z_1^{(ccg)} g_s f^{abc}\, \partial_\mu \bar{c}^a A^{b\,\mu} c^c \\
&+ \sum_q \left\{ \frac{i}{2} \bar{q} \overleftrightarrow{\partial\!\!\!/} \left[\delta Z_{2,L}^{(2q)} P_L + \delta Z_{2,R}^{(2q)} P_R\right] q + g_s \bar{q} A\!\!\!/^a T^a \left[\delta Z_{1,L}^{(qqg)} P_L + \delta Z_{1,R}^{(qqg)} P_R\right] q \right\}, \\
\delta\mathcal{L}_{Yukawa} =& -\delta Z_1^{(tb\Phi)} y_t \left\{ \left(\bar{t} P_R t\right) \Phi_2^* + \left(\bar{t} P_L t\right) \Phi_2 - \left(\bar{b} P_R t\right) \Phi_1^* - \left(\bar{t} P_L b\right) \Phi_1 \right\}, \\
\delta\mathcal{L}_{\Phi} =& \delta Z_2^{(2\Phi)} \partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \delta Z_{\Phi^2} \Phi^\dagger \Phi + \delta Z_1^{(4\Phi)} \left(\Phi^\dagger \Phi\right)^2.
\end{aligned}$$

All renormalization constants are defined as $Z = 1 + \delta Z$. The file `"beta_SM_gs_yt_lambda"` contains the β-functions, `"anomalous_dim_SM_gs_yt_lambda"` contains the anomalous dimensions and `"renconst_SM_gs_yt_lambda"` contains the renormalization constants of our model. All results are given in the $\overline{\text{MS}}$-scheme. β-functions for a coupling X and anomalous dimensions of a field f are defined as

$$\beta_X = \mu^2 \frac{\partial X}{\partial \mu^2} \qquad = \texttt{beta`X'l3}$$

$$\gamma_2^f = -\mu^2 \frac{\partial \ln Z_f^{-1}}{\partial \mu^2} \qquad = \texttt{gamma`ff'} \text{ or } \texttt{gamma`ff'L}, \texttt{gamma`ff'R}$$

where `L, R` marks the left-, right-handed part and μ = `mu` is the $\overline{\text{MS}}$-renormalization scale. The fields are `t` (top), `b` (bottom), `q` (other quarks), `ph=(ph1,ph2)` (scalar), `g` (gluon). We use the following notation:

$$\begin{aligned}
\lambda &= \texttt{lambda}\\
y_t &= \texttt{yt}\\
g_s &= \texttt{gs}\\
m^2 &= \texttt{m\^{}2=m2}\\
C_A &= \texttt{ca}\\
C_F &= \texttt{cf}\\
d_R &= \texttt{dR}\\
T_F &= \texttt{tr}\\
\zeta(3) &= \texttt{z3=Zeta[3]}\\
n_f &= \texttt{nf}\\
\frac{1}{16\pi^2} &= \texttt{h} \qquad \text{(loop counter)}
\end{aligned}$$

The operator $O_{2\Phi} = \Phi^\dagger\Phi$ is named `Ophi` and its renormalization constant $Z_{_{\Phi^2}} = \texttt{ZOphi}$ is related to the renormalization of m^2 via

$$Z_{m^2} = \left(Z_2^{(2\Phi)}\right)^{-1} Z_{\Phi^2}.$$