Low- and High-Energy Expansion of Heavy-Quark Correlators at Next-To-Next-To-Leading Order

Master Integrals

The master integrals appearing in the calculation of the correlators have the form

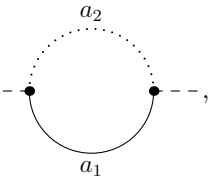
$$\mathcal{M} = \int [\mathrm{d}l_1] \ldots \int [\mathrm{d}l_L] \; \frac{1}{(p_1^2 - m_1^2)^{a_1}} \cdots \frac{1}{(p_N^2 - m_N^2)^{a_N}} \tag{1}$$

with $L = 1, 2, 3$ loops and $1 \leq N \leq 8$ different propagators. Note that the master integrals can be defined in such a way that no irreducible scalar products remain. The p_i are linear combinations of the loop momenta and the external momentum q. It is convenient to set the mass of the heavy quark to one, i.e. $m_i \in \{0, 1\}$. The expression for an arbitrary heavy-quark mass m can be retrieved by simply multiplying $\mathcal{M}$ with a factor of $(m^2)^{(2-\epsilon)L - \sum a_i}$. The integration measure is

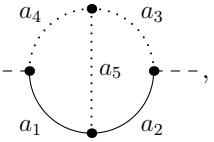
$$[\mathrm{d}l] = \frac{\mathrm{d}^d l}{i\pi^{\frac{d}{2}}} \frac{1}{\Gamma[1+\epsilon]} \,. \tag{2}$$

We define the seven topologies for the non-diagonal correlators up to three loops as follows. Dotted lines denote massless propagators, solid lines massive ones.

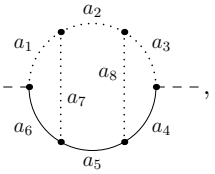
PND1L(a_1, a_2) =



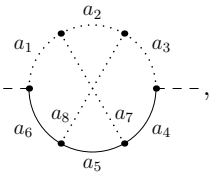
PND2L$(a_1, \ldots, a_5)$ =



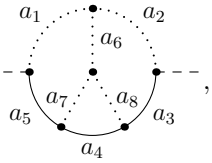
PND3L1$(a_1, \ldots, a_8)$ =



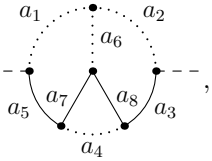
PND3L2$(a_1, \ldots, a_8)$ =



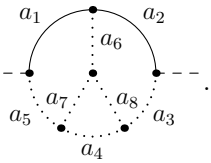
$$\mathrm{PND3L3a}(a_1,\ldots,a_8) = \quad ,$$



$$\mathrm{PND3L3b}(a_1,\ldots,a_8) = \quad ,$$

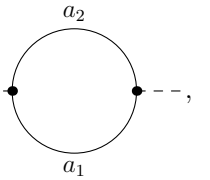


$$\mathrm{PND3L3c}(a_1,\ldots,a_8) = \quad . \tag{3}$$

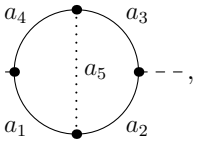


The topologies for the diagonal correlators are

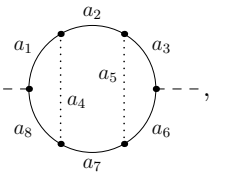
$$\mathrm{P1L}(a_1,a_2) = \quad ,$$



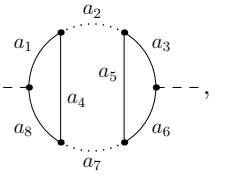
$$\mathrm{P2L}(a_1,\ldots,a_5) = \quad ,$$

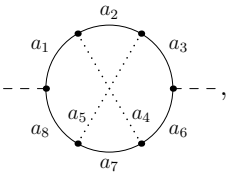


$$\mathrm{P3L1a}(a_1,\ldots,a_8) = \quad ,$$

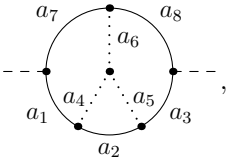


$$\mathrm{P3L1b}(a_1,\ldots,a_8) = \quad ,$$

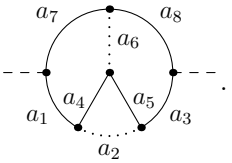


P3L2($a_1,\ldots,a_8$) = 

,

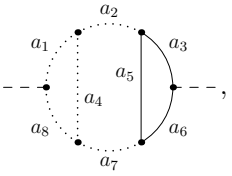
P3L3a($a_1,\ldots,a_8$) = 

,

P3L3b($a_1,\ldots,a_8$) = 

. (4)

In the calculation of the diagonal axial-vector correlator also a light isospin partner for the heavy quark has been taken into account. This leads to the appearance of an additional topology:

P3L1l($a_1,\ldots,a_8$) = 

, (5)

For each topology there are two files containing all master integrals according to the conventions defined above expanded in the low-energy and the high-energy region, respectively. Since some master integrals appear in more than one topology there are double entries. In order to faciliate the identification of master integrals there is an additional pdf file in which all master integrals in a given topology are depicted.

We use the following notation:

`z`	`ep`	`eq`	`z2,...,z8`	`a4, a5`
q^2/m^2	$\epsilon = (4-d)/2$	$\left(-q^2\right)^{-\epsilon}$	$\zeta(2),\ldots,\zeta(8)$	$\mathrm{Li}_4(1/2), \mathrm{Li}_5(1/2)$

`c4`	`log2,log3`	`sqrt3`	`pi`
$24\,\mathrm{Li}_4(1/2) + \log^4(2) - 6\zeta(2)\log^2(2)$	$\log 2, \log 3$	$\sqrt{3}$	π

`ls2,...,ls5`	`ls4p, ls5p`	`LS4, LS5`
$\mathrm{Ls}_2\left(\frac{2\pi}{3}\right),\ldots,\mathrm{Ls}_5\left(\frac{2\pi}{3}\right)$	$\mathrm{Ls}'_4(\frac{2\pi}{3}), \mathrm{Ls}'_5(\frac{2\pi}{3})$	$\mathrm{Ls}_4\left(\frac{\pi}{3}\right), \mathrm{Ls}_5(\frac{\pi}{3})$

with the (generalised) log-sine functions

$$\mathrm{Ls}_n(\theta) = -\int_0^\theta \mathrm{d}\tau\, \log^{n-1}|2\sin\frac{\tau}{2}|\,,$$

$$\mathrm{Ls}_n'(\theta) = -\int_0^\theta \mathrm{d}\tau\, \tau^{n-3} \log^2 |2 \sin \frac{\tau}{2}| \,. \tag{6}$$