Two-loop master-integrals

We define all integrals in the Euclidean space with $D = 4 - 2\epsilon$ dimensions. In each integral we only compute a piece of the imaginary part, which includes only cuts crossing the line with mass x (corresponding to the normalized Higgs boson mass present in the denominator D_1). Master integrals were identified with the help of Laporta algorithm. Integral U6 is a linear combination of U1, U1a, and U8, other integrals are linearly independent.

The forward scattering kinematics is assumed: the external momenta are on-shell, $p_1^2 = p_2^2 = 0$, normalized by $(p_1+p_2)^2 = -1$. Each integral depends on x, $0 < x < 1$. The topologies are defined as follows:

$$\mathrm{TT}\#(a_1, ..., a_7; x) = \frac{1}{\pi \mathcal{F}^2} \Im^* \int \frac{d^D v_1 \ d^D v_2}{(2\pi)^{2D}} \frac{1}{D_1^{a_1} ... D_7^{a_7}}, \ D_1 = v_1^2 + x, \ D_2 = v_2^2, \ D_i = k_i^2, \ k = 3..7$$

with the loop factor $\mathcal{F} = \frac{\Gamma(1+\epsilon)}{(4\pi)^{2-\epsilon}}$, symbol $\Im^*$ denoting the sum over appropriate cuts, and

#	k_3	k_4	k_5	k_6	k_7
A	$v_1 + v_2 - p_1 - p_2$	$v_1 + v_2 - p_2$	$v_2 - p_1$	$v_2 - p_2$	$v_1 + v_2 - p_1$
B	$v_1 + v_2 - p_1 - p_2$	$v_2 - p_1$	$v_1 - p_1$	$v_1 + v_2 - p_1$	$v_2 - p_2$
C	$v_1 + v_2 - p_1 - p_2$	$v_2 - p_1$	$v_1 - p_1$	$v_1 - p_2$	$v_1 + v_2 - p_1$
D	$v_1 + v_2 - p_1 - p_2$	$v_2 - p_1$	$v_1 - p_1$	$v_1 - p_2$	$v_2 - p_2$
E	$v_2 + p_1$	$v_2 - p_2$	$v_1 + v_2 + p_1$	$v_1 + v_2 - p_2$	$v_1 + v_2$
F	$v_2 + p_1$	$v_2 - p_2$	$v_1 + v_2 + p_1$	$v_1 + p_1 + p_2$	$v_1 + p_1$
G	$v_2 + p_1$	$v_2 - p_2$	$v_1 + v_2 + p_1$	$v_1 + p_1 + p_2$	$v_1 + p_2$
H	$v_2 + p_1$	$v_2 - p_2$	$v_1 + v_2 + p_1$	$v_1 + v_2 - p_2$	$v_1 + v_2 + p_1 - p_2$
J	$v_2 + p_1$	$v_1 + v_2 + p_1 + p_2$	$v_1 + v_2 + p_1$	$v_1 + p_1 + p_2$	$v_1 + p_1$
K	$v_2 + p_1$	$v_1 - p_2$	$v_1 + v_2 + p_1$	$v_1 + v_2 - p_2$	$v_1 + v_2$
L	$v_2 + p_1$	$v_2 - p_2$	$v_1 + v_2 + p_1 - p_2$	$v_1 + v_2 + p_1$	$v_1 + p_1$

For example, using also the Cutkoski relation, integral U7a is

$$U7a(x, \epsilon) \quad = \quad \frac{1}{2\pi \mathcal{F}^2} \int \frac{d^D v_1 \ d^D v_2}{(2\pi)^{2D}} \frac{v_2^2 \ (2\pi)^3 \ \delta\left(v_1^2 + x\right) \delta\left((v_2 - p_2)^2\right) \delta\left((v_1 + v_2 + p_1)^2\right)}{(v_2 + p_1)^2 \ (v_1 + p_2)^2}. \quad (1)$$

We find the integrals by solving the system of differential equations $d\,U_i(x, \epsilon)/d\,x = \sum c_j(x, \epsilon) U_j(x, \epsilon)$, in terms of harmonic polylogarithms (HPLs), and fix the integration constants with the expansion of each integral in $(1 - x) \ll 1$ to a few orders: two in case of U1 (U1a) and U7 (U7a), and one for the remaining integrals. For most integrals, we have managed to over-constrain the derivation and produce three or more expansion terms.

The file `masters.m` contains Mathematica expressions according to the list below. For each integral, we present a few orders in the fixed-order ϵ-expansion, e.g., `U1[x,ep]`, and the singular limit regulated by a factor $(1 - x)^{n\epsilon}$ with integer n, e.g. `U1[x -> 1,ep]`.

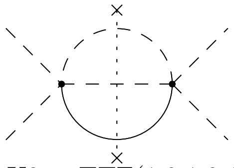
In order to use those expression, one has to add and subtract the singularity. E.g. if the limit is $U(x \to 1, \epsilon) = C(\epsilon)(1 - x)^{k - m\epsilon}$, the regularized solution is

$$U_r(x) = U(x) + C(\epsilon)(1 - x)^{k+1} \left[\left\{ \frac{\delta(1 - x)}{m\epsilon} + \left\lfloor \frac{1}{1 - x} \right\rfloor_+ + ... \right\} - \left\{ \frac{1}{1 - x} - \frac{m \ln 1 - x}{1 - x} + ... \right\} \right]. \quad (2)$$

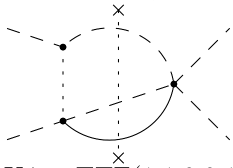
The expression in the square brackets represents the difference $\left[(1 - x)^{-1-m\epsilon} - (1 - x)^{-1-m\epsilon}\right]$, where the first term is expanded in plus-distributions and the second term is expanded "naively", canceling the non-integrable divergences in $U(x)$.

Integrals U17 and U12 have two possible cuts, presented separately as U12d (two-particle) and U12t (three-particle cut), and U17d and U17t.

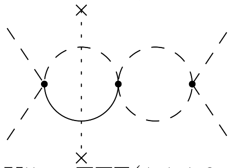
U1 = TTA(1,1,1,0,0,0,0)



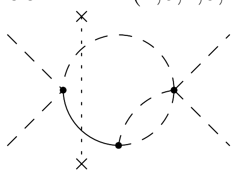
U1a = TTA(1,1,1,-1,0,0,0)



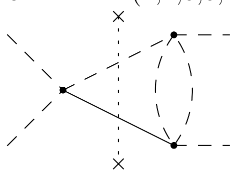
U2 = TTF(1,0,1,1,0,1,0)



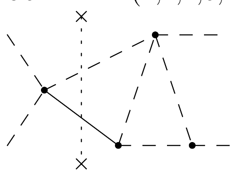
U3 = TTF(1,0,1,0,1,1,0)



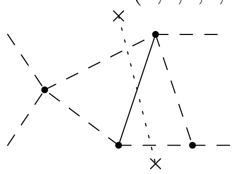
U4 = TTF(1,1,0,0,1,1,0)



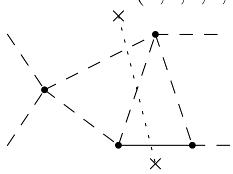
U5 = TTF(1,1,1,0,1,1,0)



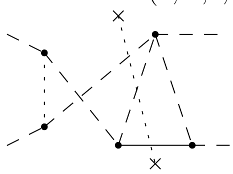
U6 = TTE(1,0,1,1,1,0,1)



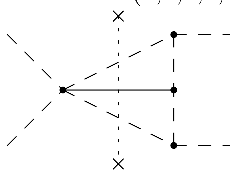
U7 = TTG(1,0,1,1,1,0,1)



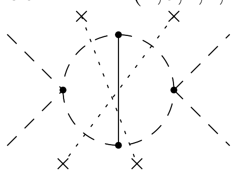
U7a = TTG(1,-1,1,1,1,0,1)



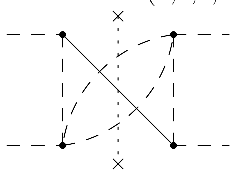
U8 = TTA(1,1,1,1,0,1,0)



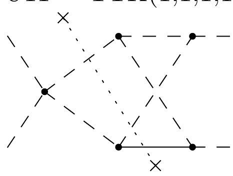
U9 = TTE(1,0,1,1,1,1,0)



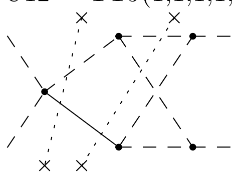
U10 = TTC(1,1,1,0,1,1,0)



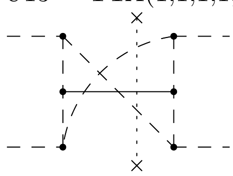
U11 = TTK(1,1,1,1,1,1,0)



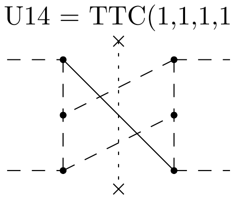
U12 = TTJ(1,1,1,1,1,1,0)



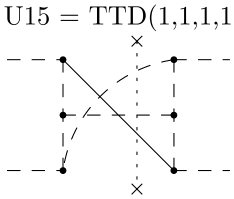
U13 = TTA(1,1,1,1,1,1,1)



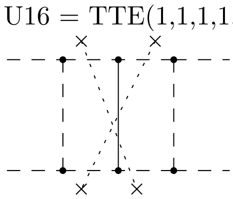
U14 = TTC(1,1,1,1,1,1,1)



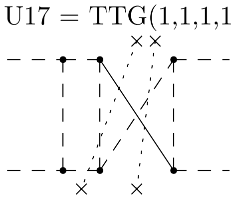
U15 = TTD(1,1,1,1,1,1,1)



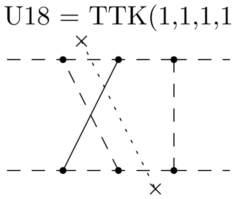
U16 = TTE(1,1,1,1,1,1,1)



U17 = TTG(1,1,1,1,1,1,1)



U18 = TTK(1,1,1,1,1,1,1)



U19 = TTH(1,1,1,1,1,1,1)