Description of `Dec1step.m`

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The `Mathematica` file `Dec1step.m` contains decoupling constants for α_s and the light quark masses up to three-loop order for the case of simultaneously decoupling two heavy quarks. They depend on two arbitrary scales μ_c and μ_b according to

$$\begin{aligned} \alpha_s^{(n_l)}(\mu_c) &= \zeta_{\alpha_s}(\mu_c,\mu_b)\,\alpha_s^{(n_l+n_b+n_c)}(\mu_b)\,, \\ m^{(n_l)}(\mu_c) &= \zeta_m(\mu_c,\mu_b)\,m^{(n_l+n_b+n_c)}(\mu_b)\,, \end{aligned} \tag{1}$$

where n_l is the number of massless quarks. There is one quark with mass M_1 and one with mass M_2. Nevertheless we have introduced the labels n_b and n_c, respectively. `Dec1step.m` also contains the inverted relations which are parametrized in terms of $\alpha_s^{(n_l)}$

$$\begin{aligned} \zeta_{\alpha_s}^{-1}(\mu_c,\mu_b)\,\alpha_s^{(n_l)}(\mu_c) &= \alpha_s^{(n_l+n_b+n_c)}(\mu_b)\,, \\ \zeta_m^{-1}(\mu_c,\mu_b)\,m^{(n_l)}(\mu_c) &= m^{(n_l+n_b+n_c)}(\mu_b)\,. \end{aligned} \tag{2}$$

The corresponding symbols are defined in the table:

symbol in `Dec1step.m`	quantity	parametrized by
Zetaa3lmubmuc	ζ_{α_s}	$\alpha_s^{(n_l+n_b+n_c)}$ and $\overline{\rm MS}$ masses
ZetaaInv3lmubmuc	$\zeta_{\alpha_s}^{-1}$	$\alpha_s^{(n_l)}$ and $\overline{\rm MS}$ masses
ZetaaOS3lmubmuc	ζ_{α_s}	$\alpha_s^{(n_l+n_b+n_c)}$ and OS masses
ZetaaInvOS3lmubmuc	$\zeta_{\alpha_s}^{-1}$	$\alpha_s^{(n_l)}$ and OS masses
Zetam3lmubmuc	ζ_m	$\alpha_s^{(n_l+n_b+n_c)}$ and $\overline{\rm MS}$ masses
ZetamInv3lmubmuc	ζ_m^{-1}	$\alpha_s^{(n_l)}$ and $\overline{\rm MS}$ masses
ZetamOS3lmubmuc	ζ_m	$\alpha_s^{(n_l+n_b+n_c)}$ and OS masses
ZetamInvOS3lmubmuc	ζ_m^{-1}	$\alpha_s^{(n_l)}$ and OS masses

In the case of the $\overline{\rm MS}$ scheme the parameters M_1 and M_2 have to be interpreted as the $\overline{\rm MS}$ masses of the $(n_l+n_b+n_c)$-flavour theory.

The meaning of the symbols used in `Dec1step.m` can be found in the following table.

symbol in `Dec1step.m`	function/parameter/constant
Lpx	$L_+(x)$
Lmx	$L_-(x)$
Hp10xm1	$H(\{1,0\},x^{-1})$
Hm10xm1	$H(\{-1,0\},x^{-1})$
Hp10xp1	$H(\{1,0\},x)$
Hm10xp1	$H(\{-1,0\},x)$
l21	$\ln(M_2/M_1)$
lcb	$\ln(\mu_c/\mu_b)$
lb1	$\ln(\mu_b/M_1)$
x	M_2/M_1
api	$\alpha_s^{(n_l+n_b+n_c)}/\pi$
anlpi	$\alpha_s^{(n_l)}/\pi$
M1	M_1
M2	M_2
nb	n_b
nc	n_c
nl	n_l
TF	T_F
CA	C_A
CF	C_F
z2	$\zeta(2)$
z3	$\zeta(3)$
z4	$\zeta(4)$
B4	B_4
log2	$\ln(2)$

$L_+(x)$ and $L_-(x)$ are defined in the paper [1]. $H(\{1,0\},x^{-1})$, $H(\{-1,0\},x^{-1})$, $H(\{1,0\},x)$, $H(\{-1,0\},x)$ denote harmonic polylogarithms, which enter the result via the transformations from $\overline{\text{MS}}$ to OS masses.

`funcLabels` is a list of replacements which expresses the self-defined functions in the decoupling constants by quantities known by `Mathematica`. A second list, `constantLabels`, replaces the symbols (`CA, CF, z2, z3` ...) by their numerical values. Afterwards only the symbols `M1, M2, lcb, lb1` and `nl` are un-specified and have to be provided by the user.

Setting $\mu_c = \mu_b = \mu$ leads to expressions which can be used for the simultaneous decoupling at the scale μ.

The results $\zeta_{\alpha_s}(m_c(\overline{m}_b), \overline{m}_b)$ and $\zeta_m(m_c(\overline{m}_b), \overline{m}_b)$ as given in [1] are obtained with the help of

`Zetaa3lmubmuc/.{lcb->l21,lb1->0}` and
`Zetam3lmubmuc/.{lcb->l21,lb1->0}`, respectively.

If you use any of the contents of this file, please refer to Ref. [1] in the corresponding publication.

References

[1] A.G. Grozin, M. Höschele, J. Hoff, and M. Steinhauser, *Simultaneous decoupling of bottom and charm quarks*, SFB/CPP-11-32, TTP11-07.