General notations.

$$\texttt{ep} = \varepsilon, \quad \texttt{zi} = \zeta_i, \quad \texttt{z62} = \zeta_{62} \ \ \texttt{Ceuler} = \gamma_E.$$

`File pmasters4l.res` contains results (in `Mathematica` form) for all 2,3,and 4-loop masters. Notations correspond to those in the article (with all letters in low case). For example,

$$\texttt{t1} \equiv T_1, \ \ \texttt{n0} \equiv N_0 \ \text{ and } \ \texttt{m61} \equiv M_{61}.$$

For the reader convenience the file `pmasters4l.res` also contains the same integrals in the $\overline{\text{MS}}$-scheme, in this case we add `b` in front:

$$\texttt{bt1} \equiv \overline{T}_1, \ \ \texttt{bn0} \equiv \overline{N}_0 \ \text{ and } \ \texttt{bm61} \equiv \overline{M}_{61}.$$

Note that by $\overline{\text{MS}}$-scheme we understand the choice of the normalizing factor $n(\varepsilon)$ (see the 11th footnote in section 3) as follows:

$$n_{\overline{\text{MS}}} = e^{\gamma_E \varepsilon}.$$

The corresponding factor for the G-scheme is:

$$n_G = 1/(\varepsilon\, G(1,1)) \equiv \Gamma(2-2\varepsilon)/(\Gamma(1+\varepsilon)\Gamma(1-\varepsilon)^2).$$

The conversion factor between the L-loop Feynman integrals defined in the G- and $\overline{\text{MS}}$-schemes is

$$\left(n_{\overline{\text{MS}}}/n_G\right)^L = \left(e^{\gamma_E\varepsilon}\Gamma(1+\varepsilon)\Gamma(1-\varepsilon)^2)/\Gamma(2-2\varepsilon)\right)^L.$$

For L=2,3 and 4 they are available in `pmasters4l.res` as

`G2MSbar2l, G2MSbar3l and G2MSbar4l`

respectively (with ε-accuracy up to and including terms of order ε^7).

File `GG`.

Here the G-functions are defined as follows:

$$\texttt{GG[a\_Integer, a1\_Integer, b\_Integer, b1\_Integer]} \equiv \texttt{GG[a + ep * a1, b + ep * b1]}$$

produces $G(a+\varepsilon\, a1, b+\varepsilon\, b1)$ with the ε-accuracy not less than the value of a parameter, `accu`, which is by default set to 7.

In addition, in the same file a function

$$\texttt{F0[a1\_, a2\_, a3\_, a4\_, a5\_]} \equiv F(1+\varepsilon\, a1, 1+\varepsilon\, a2, 1+\varepsilon\, a3, 1+\varepsilon\, a4, 1+\varepsilon\, a5) + \mathcal{O}(\varepsilon^6)$$

is defined (according to the work D. J. Broadhurst, *Exploiting the 1440 Fold Symmetry of The Master two Loop Diagram*, *Z. Phys.* **C32** (1986) 249–253).

A few useful shortcuts are defined as follows.

```
Unprotect[Ex,Coll,Coef,Exn,Zrule];
Ex[x_]   := Expand[x];
Exn[x_] := Expand[Normal[x]];
Coef[x__] := Coefficient[x];
Coll[x_,y_] := Collect[Ex[x],y];
Zrule = {
Ceuler -> EulerGamma,
z2->Zeta[2],
z3->Zeta[3],
z5->Zeta[5],
z4->Zeta[4],
z6->Zeta[6],
z7->Zeta[7],
z8->Zeta[8],
z9->Zeta[9]  ,
B4->16*PolyLog[4,1/2]+2/3*Log[2]^4-2/3*Pi^2*Log[2]^2-13/180*Pi^4
}       ;
Protect[Ex,Coll,Coef,Exn,Zrule];
```

Examples of use

Below we put input commands to display $G(1+\varepsilon,1)$, the master integral M_{51} and the generalized two-loop FI $F(1,1,1,1,\varepsilon)$ are displayed first analytically and then numerically.

```
<<GG.m;
<<pmasters.res;

Print[GG[1+ep,1]];
Print[GG[1+ep,1]/.Zrule//N];

Print[m51];
Print[m51/.Zrule//N];

Print[F0[0,0,0,0,1]];
Print[F0[0,0,0,0,1]/.Zrule//N];
```