Results from the paper

Light quark mass effects in the on-shell renormalization constants

by

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1 Software

The files `zm.m` and `Z2OS.m` contain the results for the relation between the on-shell- and pole quark mass, z_m, and the on-shell wave function renormalization constant Z_2^{OS} in `Mathematica` format. The files have been created with `Mathematica` version 5.2. For the numerical evaluation the package `HPL.m` is needed (cf. hep-ph/0507152) which can be obtained at `http://www-theorie.physik.unizh.ch/~maitreda/HPL/`.

2 Symbols

The above mentioned quantities are described in Sections 3 and 4 of the paper. The results are decomposed into the colour factors and labels for different quark contributions. In the following table the used symbols are explained.

`cf`	$C_F = (N_c^2 - 1)/(2\,N_c)$;eigenvalue of the quadratic casimir operator of the fundamental representation of $SU(N_c)$
`ca`	$C_A = N_c$; the same for the adjoint representation
`tf`	index of the fundamental representation (T_F)
`nh`	label for closed loops of the heavy quark (n_h)
`nl`	label for closed loops of the massless quarks (n_l)
`nm`	label for closed loops of the light quark (n_m)
`x`	ratio of the light and heavy pole masses $x = M_f/M_q$
`api`	α_s/π, strong coupling constant divided by π
`M`	heavy quark mass
`my`	renormalization scale from the running coupling
`e`	dimensional regularization parameter ε

In the case of QCD we have $N_c = 3, C_F = 4/3, C_A = 3, T_F = 1/2$.
α_s is understood to be $\alpha_s^{(n_f)}$, the renormalized coupling with $n_f = n_l + n_m + n_h$ active flavours.

3 Mass renormalization

The file `zm.m` contains the function `zm[x]` which delivers the $\overline{\mathrm{MS}}$-on-shell relation z_m as a function of the mass ratio x, where the renormalization scale μ is assumed to be the heavy quark mass. It is also possible to specify a Mass M and a scale μ using `zm[x,M,my]`. The dependence on M and μ is through terms in $\log(\mu^2/M^2)$ which are suppressed in `zm[x]`.
The data are valid for $0 \leq x \leq 5$. For $x \leq 0.4$ the expansion of z_m in powers of x is used while for $x > 0.4$ the result consists of analytical functions in terms of Harmonic Polylogarithms and interpolation functions based on numerical calculations.
The expansion for small x is also available via the functions `zmxe[x]` and `zmxe[x,M,my]`. The exact results can be obtained from `zman[x]` and `zman[x,M,my]`. The file also contains the functions `zmap[x]` and `zmap[x,M,my]` which implement handy approximation formulae for z_m (cf. Section 3.2 of the paper). They are valid for $0 \leq x \leq 1$ with an accuracy of better than 1 percent.

4 Wave function renormalization

The file `Z2OS.m` contains the functions `Z2OS[x]`, `Z2OS[x,M,my]`, `Z2OSxe[x]`, `Z2OSxe[x,M,my]`, `Z2OSan[x]` and `Z2OSan[x,M,my]`, which can be used analogously to the corresponding functions `zmxx[]`. They allow to calculate the wave function renormalization constant Z_2^{OS} taking into account light quark mass effects.
In the functions `Z2OSap[x]` and `Z2OSap[x,M,my]` we provide approximation formulae which are valid for $0 \leq x \leq 1$ for the QCD case, i.e. the colour factors are chosen as described in Section 2. n_h and n_m are set to 1 while n_l is left as a free parameter.