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One-particle inclusive CP asymmetries

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Abstract

One-particle inclusive CP asymmetries in the decays of the type $B \to \overset{(-)}{D}{}^{(*)} X$ are considered in the framework of a QCD based method to calculate the rates for one-particle inclusive decays.

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1 Introduction

One of the main goals in B physics is a detailed study of flavor mixing, which is encoded in the Cabibbo-Kobayashi-Maskawa (CKM) matrix of the standard model. In particular, the violation of the CP symmetry, which the standard model describes by a nontrivial phase in the CKM matrix or equivalently by the angles of the unitarity triangle, will be investigated.

Typically CP asymmetries are expected to be large in some of the exclusive nonleptonic B decays which, however, have only small branching ratios. Examples are the determination of β from $B \to J/\psi\, K_s$ and of α from $B \to \pi\pi$. In addition, in these exclusive nonleptonic decays it is very hard to obtain a good theoretical control over the hadronic uncertainties, in particular due to the presence of strong phases.

On the other hand, inclusive decays have large branching fractions but typically smaller CP asymmetries than exclusive decays [1]. One may use parton hadron duality to obtain a good theoretical description. This has been studied by Beneke, Buchalla and Dunietz who set up a theoretically clean method to calculate the CP asymmetries in inclusive B decays [2]. They still find sizable CP asymmetries, but their measurement would require to identify charmless final states inclusively, which is not an easy task.

One-particle inclusive decays lie somehow between these two cases. This class of decays still has large branching fractions and some of the expected CP asymmetries are sizable. Furthermore, a measurement of these decays is feasible.

For one-particle inclusive decays of the type $B \to \overset{(-)}{D}{}^{(*)} X$, a QCD based description has been developed recently, exploiting factorization and the heavy mass limit for both the b and the c quark [3]. Since the expansion parameters are $\Lambda_{\mathrm{QCD}}/(m_b - m_c)$, $1/N_C$ and $\alpha_s(m_c)$, corrections to the leading term could be fairly large, in the worst case of the order of 30%. Using this method, which unfortunately is not completely model independent, we compute mixing induced time-dependent and time-integrated CP asymmetries in the framework of the standard model.

In view of the considerable uncertainties due to an unknown strong phase, our method cannot yet be used for a competitive determination of the CP violation parameters, in particular compared to a measurement of $\sin(2\beta)$ in the "gold-plated" channel $B \to J/\psi\, K_s$. However, it can be used as an estimate of the one-particle inclusive CP asymmetries, for which we shall use present central values of the CP angles β and γ [4]. Compared to fully inclusive methods, the advantage is that we can predict asymmetries for the various spins and charges of the ground-state charmed mesons separately. This is certainly a worthwhile task, in particular since we are not aware of any previous prediction for these asymmetries, not even in the context of quark models.

After introducing our notations for B mixing in Sec. 2, we calculate the relevant matrix elements in Sec. 3 and model the form factors in Sec. 4. The numerical results are given in Sec. 5.

2 CP asymmetries in $B \to \overset{(-)}{D}{}^{(*)} X$

In Wigner Weisskopf approximation the time evolution of an initially pure B^0 or $\overline{B}^0$,

$$\begin{aligned} \left|B^0_{\rm phys}(t)\right\rangle &= g_+(t)\left|B^0\right\rangle - \frac{q}{p} g_-(t)\left|\overline{B}^0\right\rangle , \\ \left|\overline{B}^0_{\rm phys}(t)\right\rangle &= g_+(t)\left|\overline{B}^0\right\rangle - \frac{p}{q} g_-(t)\left|B^0\right\rangle , \end{aligned} \tag{1}$$

is determined by the time-dependent functions

$$\begin{aligned} g_+(t) &= e^{-iMt-\frac{1}{2}\Gamma t}\left[\cosh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\sinh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right] \\ g_-(t) &= e^{-iMt-\frac{1}{2}\Gamma t}\left[\sinh\frac{\Delta\Gamma t}{4}\cos\frac{\Delta M t}{2} + i\cosh\frac{\Delta\Gamma t}{4}\sin\frac{\Delta M t}{2}\right] , \end{aligned} \tag{2}$$

where $\Delta M = M_H - M_L > 0$ and $\Delta\Gamma = \Gamma_H - \Gamma_L < 0$ are the mass and width differences between the mass eigenstates $|B_H{>} = p|B^0{>} + q|\overline{B}^0{>}$ and $|B_L{>} = p|B^0{>} - q|\overline{B}^0{>}$.

The quantity q/p is given in terms of the off-diagonal elements of the Hamiltonian $H = M - i\Gamma/2$ of the neutral B meson system

$$\frac{q}{p} = \frac{\Delta M - \frac{i}{2}\Delta\Gamma}{2\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)} = \frac{M_{12}^*}{|M_{12}|}\left(1 - \frac{1}{2}a + \mathcal{O}(a^2)\right), \qquad a = \mathrm{Im}\left\{\frac{\Gamma_{12}}{M_{12}}\right\}. \tag{3}$$

In fact, $\Gamma_{12}/M_{12} = \mathcal{O}(m_b^2/m_t^2)$ is very small and hence q/p is to a good approximation a phase factor.

The time-dependent rate for the decay of a B meson into a set of final states $|f\rangle = \sum_i |f_i\rangle$ can be written as

$$\begin{aligned} \Gamma[B(t)\to f] &= \frac{1}{2m_B}\sum_i \int d\phi_i\,(2\pi)^4\delta^4(p_B - p_{f_i})\,\langle B(t)\,|H_{\rm eff}|\,f_i\rangle\,\langle f_i\,|H_{\rm eff}|\,B(t)\rangle \\ &= \frac{1}{2m_B}\int d^4x\,\,\langle B(t)\,|H_{\rm eff}(x)\Pi_f H_{\rm eff}(0)|\,B(t)\rangle\,, \end{aligned} \tag{4}$$

where $d\phi_i$ is the phase space element of the state $|f_i\rangle$ and

$$\Pi_f = \sum_i \int d\phi_i\,\,|f_i\rangle\,\langle f_i| \tag{5}$$

is the projector on the set of final states. Note that both an exclusive final state as well as inclusive states can be treated in this way. Even differential distributions can be considered if the phase spaces $d\phi_i$ are not fully integrated.

The CP asymmetries we are going to consider are of the type

$$\mathcal{A}_{CP}(t) = \frac{\Gamma(B^0(t) \to f) - \Gamma(\overline{B}^0(t) \to \overline{f})}{\Gamma(B^0(t) \to f) + \Gamma(\overline{B}^0(t) \to \overline{f})} \tag{6}$$

which involves the CP conjugate set $|\overline{f}>$ of final states.

Up to here the discussion is completely general. In the following we shall use the above formalism to compute the CP asymmetries for one-particle inclusive final states, for which the projector reads

$$\Pi_f = \sum_X |XY\rangle \langle XY| , \tag{7}$$

where Y can be a D or a $\overline{D}$ meson. Since the sum runs over all possible states X, the CP conjugate of the projector is

$$\Pi_{\overline{f}} = \sum_X \left|X\overline{Y}\right\rangle \left\langle X\overline{Y}\right| . \tag{8}$$

Inserting the time-dependent states (1) we obtain

$$\begin{aligned} \Gamma[B(t){\to}YX] &= |g_+(t)|^2\, \Gamma_Y^{BB} + \left|\frac{q}{p} g_-(t)\right|^2 \Gamma_Y^{\overline{B}\,\overline{B}} - 2\mathrm{Re}\left\{\frac{q}{p} g_+^* g_-(t) T_Y^{B\overline{B}}\right\} , \\ \Gamma[\overline{B}(t){\to}\overline{Y}X] &= |g_+(t)|^2\, \Gamma_{\overline{Y}}^{\overline{B}\,\overline{B}} + \left|\frac{p}{q} g_-(t)\right|^2 \Gamma_{\overline{Y}}^{BB} - 2\mathrm{Re}\left\{\frac{p}{q} g_+^* g_-(t) T_{\overline{Y}}^{\overline{B}B}\right\} , \end{aligned} \tag{9}$$

where the matrix elements are defined by

$$\begin{aligned} \Gamma_Y^{BB} &= \frac{1}{2m_B} \int d^4x\, \langle B\, |H_{\mathrm{eff}}(x) \Pi_Y H_{\mathrm{eff}}(0)|\, B\rangle , \\ T_Y^{B\overline{B}} &= \frac{1}{2m_B} \int d^4x\, \left\langle B\, |H_{\mathrm{eff}}(x) \Pi_Y H_{\mathrm{eff}}(0)|\, \overline{B}\right\rangle . \end{aligned} \tag{10}$$

The $\Delta B = 2$ transition matrix elements representing the interference between the mixed and the unmixed amplitudes are related by CPT symmetry, such that

$$T_Y := T_Y^{B\overline{B}} = \left(T_{\overline{Y}}^{\overline{B}B}\right)^* . \tag{11}$$

The direct CP asymmetries in these processes are expected to be tiny. In fact, using the method described in Ref. [3], they turn out to be of higher order in the $1/m$ expansion. Hence we have

$$\Gamma_Y := \Gamma_Y^{BB} = \Gamma_{\overline{Y}}^{\overline{B}\,\overline{B}} = \Gamma(B \to YX), \qquad \Gamma_{\overline{Y}} := \Gamma_{\overline{Y}}^{BB}, \tag{12}$$

$$T_Y = T_{\overline{Y}}. \tag{13}$$

Inserting the time-dependent decay rates in Eq. (9) and neglecting both the width difference and a, such that q/p becomes a phase factor, we obtain for the time-dependent CP asymmetries

$$\mathcal{A}_{CP}(t) = \frac{\sin\left(\Delta M t\right) \mathrm{Im}\left\{\frac{q}{p} T_Y\right\}}{\cos^2\left(\frac{\Delta M t}{2}\right)\Gamma_Y + \sin^2\left(\frac{\Delta M t}{2}\right)\Gamma_{\overline{Y}}}, \tag{14}$$

from which we get the time-integrated asymmetry

$$\mathcal{A}_{CP} = \frac{2\,x\,\mathrm{Im}\left\{\frac{q}{p} T_Y\right\}}{(2+x^2)\,\Gamma_Y + x^2\,\Gamma_{\overline{Y}}}, \tag{15}$$

where $x = \Delta M/\Gamma$ is measured to be $x = 0.73$ [5].

3 Transition matrix elements

In order to compute the CP asymmetries, one has to evaluate the matrix elements in Eq. (10). The total rates Γ_Y have already been discussed in Ref. [3], so we only need to calculate the interference term T_Y.

The relevant pieces of the effective Hamiltonian contributing to this interference are $(\overline{u}b)_{V-A}(\overline{d}c)_{V-A}$ and $(\overline{c}b)_{V-A}(\overline{d}u)_{V-A}$ interfering with each other and $(\overline{c}b)_{V-A}(\overline{d}c)_{V-A}$ interfering with itself, so T_Y is a sum of the two contributions

$$T_Y = T_c + T_u, \tag{16}$$

$$\begin{aligned} T_q = {} & \frac{1}{2m_B}\frac{G_F^2}{2} V_{cb}V_{qd}^* V_{qb} V_{cd}^* \left|C_1\right|^2 \sum_X (2\pi)^4 \delta^4(p_B - p_D - p_X) \\ & \left\langle B^0 \left|(\overline{q}b)_{V-A}(\overline{d}c)_{V-A}\right| DX\right\rangle \left\langle DX \left|(\overline{d}q)_{V-A}(\overline{c}b)_{V-A}\right| \overline{B}^0\right\rangle . \end{aligned} \tag{17}$$

Fierzing the operators into the form $(\overline{d}b)_{V-A}(\overline{u}c)_{V-A}$, $(\overline{d}b)_{V-A}(\overline{c}u)_{V-A}$ and $(\overline{d}b)_{V-A}(\overline{c}c)_{V-A}$ one can reproduce the inclusive results of Ref. [2]. In order to evaluate the interference term for the one-particle inclusive case, we use the method developed in Ref. [3]. It is based on factorization, which holds to leading order in the $1/N_C$ expansion, where N_C is the number of QCD colors. Thus we can write the interference terms as products of two tensors

$$T_q = \frac{1}{2m_B}\frac{G_F^2}{2}V_{cb}V_{qd}^*V_{qb}V_{cd}^*\,|C_1|^2 \int \frac{d^4Q}{(2\pi)^4}\, K_{\mu\nu}(p_B,Q) \int d\phi_D\, P_q^{\mu\nu}(p_D,Q) \quad (18)$$

with

$$\begin{aligned} K_{\mu\nu}(p_B,Q) &= \sum_X (2\pi)^4\delta^4(p_B-p_X-Q) && (19)\\ &\quad \left\langle B^0(p_B)\left|(\overline{d}\gamma_\mu(1-\gamma_5)b)\right| X\right\rangle \left\langle X\left|(\overline{d}\gamma_\nu(1-\gamma_5)b)\right| \overline{B}^0(p_B)\right\rangle, \\ P_q^{\mu\nu}(p_D,Q) &= \sum_{X'}(2\pi)^4\delta^4(Q-p_D-p_{X'}) && (20)\\ &\quad \left\langle 0\left|(\overline{q}\gamma^\mu(1-\gamma_5)c)\right| D^{(*)}(p_D)X'\right\rangle \left\langle D^{(*)}(p_D)X'\left|(\overline{c}\gamma^\nu(1-\gamma_5)q)\right| 0\right\rangle . \end{aligned}$$

The tensor $K_{\mu\nu}(p_B,Q)$ is fully inclusive and one can perform a standard short distance expansion. The resulting $\Delta B = 2$ matrix element can be parameterized by the decay constant f_B of the B meson and the bag factors B and B_s for the axial vector and the scalar current, respectively.

The other tensor $P_q^{\mu\nu}(p_D,Q)$ involves a projection on a one-particle inclusive charmed meson state and hence we cannot perform a short distance expansion. We proceed along the same lines as in Ref. [3], where the rates for wrong charm decays have been modeled. Heavy quark symmetry yields the Dirac matrix structure

$$P_q^{\mu\nu}(p_D,Q) \propto \overline{H}_{D^{(*)}}(p_D)\gamma^\mu(1-\gamma_5)\otimes\gamma^\nu(1-\gamma_5)H_{D^{(*)}}(p_D), \quad (21)$$

where the representation matrices for the charmed mesons are

$$H_D = \sqrt{m_D}\,\frac{1+\not{v}_D}{2}\,\gamma_5, \qquad H_{D^*} = \sqrt{m_{D^*}}\,\frac{1+\not{v}_{D^*}}{2}\,\not{\epsilon}. \quad (22)$$

In principle, all possible contractions of the light quark indices may contribute, giving rise to several form factors. For a first estimate, it is sufficient to use only the simplest one of these contractions,

$$P_q^{\mu\nu}(p_D,Q) = 2\pi\,\delta\Big((Q-p_D)^2-m_q^2\Big)\,\mathrm{Tr}\Big\{\not{p}_D\,\gamma^\mu(1-\gamma_5)\,(\not{Q}-\not{p}_D)\,\gamma^\nu(1-\gamma_5)\Big\}\,\tilde{f}_{qY}, \quad (23)$$

corresponding to a replacement of the $D^{(*)}X$ final state by a pair of free quarks, rescaled by an operator- and decay-channel-specific form factor $\tilde{f}_{qY}$, where Y is one of the ground state D mesons. In the following, we call this contraction "partonic."

Using this ansatz and the heavy mass limit, the transition matrix elements read

$$T_c = -\frac{G_F^2 m_B^3 f_B^2}{24\pi}(V_{cb}V_{cd}^*)^2 \left|C_1\right|^2 \sqrt{1-4z}[(1-4z)B+2(1+2z)B_S]\tilde{f}_{cY}, \quad (24)$$

$$T_u = -\frac{G_F^2 m_B^3 f_B^2}{24\pi}V_{cb}V_{ud}^*V_{ub}V_{cd}^* \left|C_1\right|^2 (1-z)^2[(1-z)B+2(1+2z)B_S]\tilde{f}_{uY}, \quad (25)$$

where $z = (m_c/m_b)^2$ and C_1 is the Wilson coefficient of the effective Hamiltonian in the notation of Ref. [3]. Equations (24) and (25) correspond to the expression for the width difference of neutral heavy meson systems [6].

In the standard CKM parametrization, the phases of the transition matrix elements are

$$\arg(T_c) = 0, \quad (26)$$

$$\arg(T_u) = \arg(-V_{ub}) = -\gamma, \quad (27)$$

$$\arg(q/p) = \arg(-V_{td}^2) = -2\beta, \quad (28)$$

such that

$$\mathrm{Im}\left\{\frac{q}{p}T_Y\right\} = \sin(2\beta)\left|T_c\right| + \sin(2\beta+\gamma)\left|T_u\right|. \quad (29)$$

4 Modeling the form factors

We assume that the form factors $\tilde{f}_{qY}$ do not vary strongly over the accessible phase space and hence we approximate them by constants. For the case $q = c$, these constants have been fitted to the wrong charm yield in B decays [3]. Operators analogous to the case $q = u$ are Cabibbo suppressed when calculating wrong charm rates, so they did not appear in Ref. [3]. Assuming that all charm quarks eventually hadronize to D mesons, we use

$$\tilde{f}_{uD^0} + \tilde{f}_{uD^+} = 1. \quad (30)$$

To resolve the spin and charge counting, we first discuss the heavy mass limit where the pseudoscalar and vector charmed mesons form a degenerate ground state doublet. The decay of vector to pseudoscalar mesons will be discussed below. In the following, D_{dir} refers to those D mesons that do not result from D^* decays, and $D^{(*)}$ can be either D_{dir} or D^*.

As long as the light quark spin indices of the $D^{(*)}$ meson representation matrices are contracted with each other, Eq. (21) reproduces the naive spin counting

$$\tilde{f}_{qD^{*0}} = 3\tilde{f}_{qD^0_{\mathrm{dir}}}, \qquad \tilde{f}_{qD^{*+}} = 3\tilde{f}_{qD^+_{\mathrm{dir}}}. \quad (31)$$

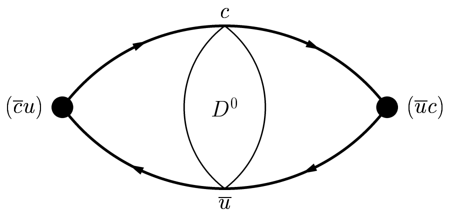

Figure 1: Topology yielding $|a_1|^2$ in Eq. (33).

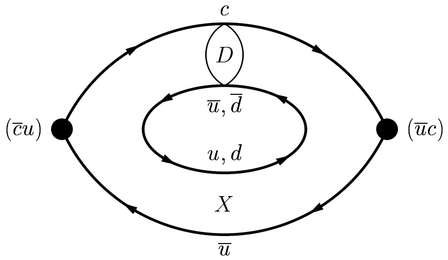

Figure 2: Topology yielding $|a_2|^2$ in Eq. (33).

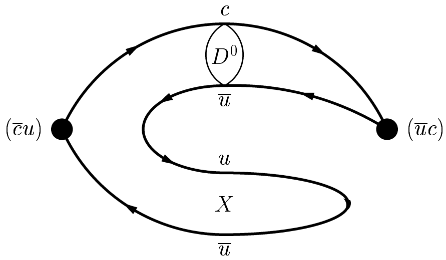

Figure 3: Interference topology for Eq. (33).

Different contractions yield results of comparable size. The experimental spin counting factor appears to be smaller by roughly a factor of two [3]. Since this effect is not yet understood, we treat it as an uncertainty.

Concerning charge counting, we argued by isospin symmetry [3] that in the case $q = c$ we have

$$\tilde{f}_{cD^{(*)0}} = \tilde{f}_{cD^{(*)+}}. \tag{32}$$

In the case $q = u$, two topologies can contribute to the decay amplitude: the charm quark can either hadronize with the u quark from the weak effective current, in which case the isospin of the state $|X\rangle$ is $I_X = 0$, or with a u or d quark from vacuum, which contains both $I_X = 0$ and $I_X = 1$ contributions. In the case $I_X = 0$, both amplitudes can interfere, so there are three contributions to the decay rate

$$\begin{aligned} \tilde{f}_{uD^{(*)0}} &= |a_1 + a_2|^2 = |a_1|^2 + |a_2|^2 + 2\mathrm{Re}\,\{a_1^* a_2\} \\ \tilde{f}_{uD^{(*)+}} &= |a_2|^2, \end{aligned} \tag{33}$$

see Figs. 1–3.

One might doubt whether using the partonic contraction given in Eq. (23) is justified for all the topologies, as it appears to correspond to the topology in Fig. 2, while the topology in Fig. 1 should rather be described by the contraction

$$P_q^{\mu\nu}(p_D, Q) \propto \mathrm{Tr}\Big\{\overline{H}_{D^{(*)}}(p_D)\gamma^\mu(1-\gamma_5)\Big\}\mathrm{Tr}\Big\{\gamma^\nu(1-\gamma_5)H_{D^{(*)}}(p_D)\Big\}. \tag{34}$$

This is not a problem for three reasons. First, we do not claim to be able to accurately model the matrix element, but we only give the simplest possible ansatz by rescaling the partonic result. In particular, it is clearly not yet feasible to model particular contributions individually. We only use the three topologies to estimate the integrated relative magnitudes of the two main contributions and to bound the magnitude of their interference term. Secondly, neither the time-dependent nor the time-integrated asymmetries depend on the choice of the contraction unless studied differentially in the momentum of the charmed meson, which so far we do not attempt to do. Finally, as noted in Ref. [3], the choice of the wrong charm contraction appeared to have little influence even on differential observables.

The topologies in Figs. 1 and 2 also occur in wrong charm production in B decays. Figure 1 corresponds to the process $B \to D_s^{(*)+}X$, Fig. 2 to the process $B \to D^{(*)}X$, where $D^{(*)}$ can be either $D^{(*)0}$ or $D^{(*)+}$. Both contributions are experimentally known to be of similar size, i.e., $(10 \pm 2.5)\%$ [5] and $(7.9 \pm 2.2)\%$ [7], respectively, such that

$$|a_1|^2 = 2\,|a_2|^2\,. \tag{35}$$

The relative phase of the two contributions is unknown. Therefore, although it may be large, we have to treat the interference part as a theoretical uncertainty. This is acceptable since the $q = u$ contribution is smaller than the $q = c$ contribution according to

$$\left|\frac{T_u}{T_c}\right| = \left|\frac{V_{ub}}{V_{cb}}\frac{V_{ud}}{V_{cd}}\right| \frac{(1-z)^2(1+z)}{\sqrt{1-4z}}\frac{\tilde{f}_u}{\tilde{f}_c} \propto \left|\frac{V_{ub}}{V_{cb}}\right| \frac{(1+z)}{|V_{cd}|} \approx 0.4. \tag{36}$$

Off the heavy mass limit, $D^* \to D$ decay has to be taken into account. In the same way as in Ref. [3], we get

$$\begin{aligned} \tilde{f}_{qD^+} &= \tilde{f}_{qD^+_{\rm dir}} + \mathrm{Br}\left(D^{*+} \to D^+ X\right) \tilde{f}_{qD^{*+}} \\ \tilde{f}_{qD^0} &= \tilde{f}_{qD^0_{\rm dir}} + \tilde{f}_{qD^{*0}} + \mathrm{Br}\left(D^{*+} \to D^0 X\right) \tilde{f}_{qD^{*+}}. \end{aligned} \tag{37}$$

The coefficients obtained from Eqs. (30)–(37) and Ref. [3] are summarized in Table 1. The ranges given result from varying the spin counting factor in Eq. (31) from 3 down to 3/2 and the interference in Eq. (33) from the central value of vanishing interference to full constructive and destructive interference.

5 Results

We have computed the parameters for the time-dependent CP asymmetries as well as the time-integrated asymmetries. We have inserted recent val-

operator / channel	$q=c$ central	$q=c$ r. to	$q=u$ central	$q=u$ range
$\tilde{f}_{qD^+_{\rm dir}}$	2/16	0.2	1/16	0.04–0.34
$\tilde{f}_{qD^0_{\rm dir}}$	2/16	0.2	3/16	0.34–0.04
$\tilde{f}_{qD^{*+}}$	6/16	0.3	3/16	0.09–0.64
$\tilde{f}_{qD^{*0}}$	6/16	0.3	9/16	0.64–0.09
$\tilde{f}_{qD^+}$	4/16	0.3	2/16	0.07–0.51
$\tilde{f}_{qD^0}$	12/16	0.7	14/16	0.93–0.49

Table 1: Operator- and channel-specific form factors.

ues for $\sin 2\beta = 0.75$ and $\gamma = 68^\circ$ [4]. In addition, we use $V_{cb} = 0.04$, $V_{ub} = 0.08\,V_{cb}$, $z = 0.09$, $x = 0.73$, $f_B = 180$ MeV, $\mathrm{Br}\,(D^{*+} \to D^0 Y) = 1 - \mathrm{Br}\,(D^{*+} \to D^+ Y) = 0.683$ and $C_1 = B = B_S = 1$. The results of the calculations can be found in Fig. 4 and Table 2.

To assess the uncertainties involved in Fig. 4, note that according to Eq. (14) the shapes of the time-dependent asymmetries are determined by the ratios of the wrong to right charm rates $\Gamma_{\overline{Y}}/\Gamma_Y$. We checked numerically that the shapes would hardly change even if these ratios were off by 30%. The dominant contribution to the uncertainty of the amplitudes arises from the transition matrix elements T_Y and is directly proportional to the uncertainties of the time-integrated asymmetries given in Table 2.

Suppose N perfectly tagged B^0 decays are recorded in an experiment. In order to establish the asymmetry in a channel with a branching ratio b on the 3σ level,

$$\frac{\mathcal{A}}{3} \geq \Delta\mathcal{A} = \frac{1}{\sqrt{2bN}} \tag{38}$$

has to be satisfied. The necessary numbers of tagged B^0 decays are given in the last column of Table 2. Since the asymmetry tends to be roughly inversely proportional to the branching ratio by Eq. (15), we obtain from Eq. (38)

$$N \propto \frac{1}{\mathcal{A}^2 b} \propto b, \tag{39}$$

such that rare channels are advantageous for observing one-particle inclusive asymmetries.

The channel $B^0 \to \overline{D}^{*0} X$ deserves a further comment. Looking at Fig. 4, there is an obvious problem at small proper decay times. The reason for this problem is that we have discussed all the rates only to leading order in the combined $1/N_C$ and $1/m_Q$ expansions. However, this leading term vanishes

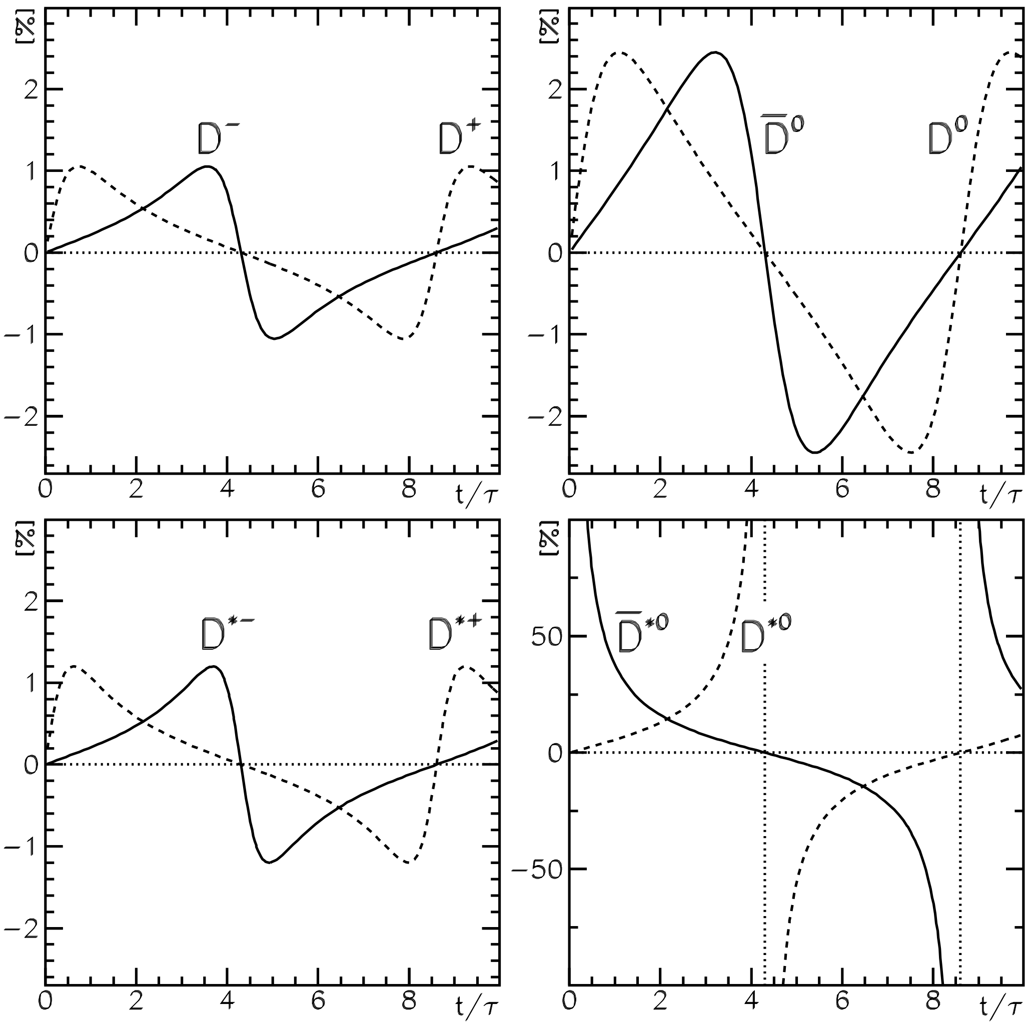

Figure 4: Time-dependent CP asymmetries in $B^0 \to \overset{(-)}{D}X$ for pseudoscalar (above), vector (below), charged (left), neutral (right), right charm (solid), and wrong charm (dashed) $\overset{(-)}{D}$ mesons.

decay channel	Br [3] (%)	$\mathcal{A}$ (%)	$\mathcal{A}$ range (%)	necessary B^0 decays
$B^0 \to D^- X$	29.1	0.16	0.15–0.29	6.000.000
$B^0 \to \overline{D}^0 X$	31.8	0.58	0.59–0.46	400.000
$B^0 \to D^+ X$	2.2	0.58	0.54–1.04	6.000.000
$B^0 \to D^0 X$	5.7	1.53	1.56–1.23	350.000
$B^0 \to D^{*-} X$	46.8	0.16	0.12–0.23	4.000.000
$B^0 \to \overline{D}^{*0} X$	(0)	(20)	(21)–(10)	$>$ 80.000
$B^0 \to D^{*+} X$	2.5	0.61	0.45–0.89	5.000.000
$B^0 \to D^{*0} X$	2.5	4.17	4.40–2.20	100.000

Table 2: Branching ratios, integrated CP asymmetries and numbers of necessary tagged B^0 decays for the one-particle inclusive $B^0 \to \overset{(-)}{D}{}^{(*)} X$ decay channels. Concerning $\overline{D}^{*0}$, see the text.

for the channel $B^0 \to \overline{D}^{*0} X$ and thus subleading terms become relevant. On the other hand, the numerator T_Y of the CP asymmetries is given by a matrix element of a dimension six operator and hence is suppressed compared to the leading terms of most of the rates. In other words, while in most of the rates the asymmetries are of subleading order f_B^2/m_B^2, this is not the case for the channel $B^0 \to \overline{D}^{*0} X$.

Unfortunately we cannot compute this possibly large asymmetry, since this would involve to compute subleading terms for the decay rate. Hence we try to estimate the asymmetry by varying $\mathrm{Br}\left(B^0 \to \overline{D}^{*0} X\right)$ in Eq. (15) and show the reaction of the asymmetry in Fig. 5 and of the necessary number of tagged B^0 events in Fig. 6. The wrong charm asymmetry is practically unaffected by $\mathrm{Br}\left(B^0 \to \overline{D}^{*0} X\right)$ since the pole occurs near four average lifetimes where most of the B mesons have already decayed, but the right charm asymmetry turns out to be extremely sensitive. Therefore we cannot predict the latter quantitatively, but it can be as large as several percent, and it will be measurable with a few 100 000 tagged B^0 events.

6 Conclusion

Motivated by the work on fully inclusive CP asymmetries and the question how to measure them, we studied one-particle inclusive CP asymmetries. In the final state only a $\overset{(-)}{D}{}^{(*)}$ meson has to be identified and thus they are experimentally more easily accessible than the fully inclusive CP asymmetries.

We have used a similar method as in in Ref. [3] to calculate the time-dependent and time-integrated CP asymmetries for one-particle inclusive

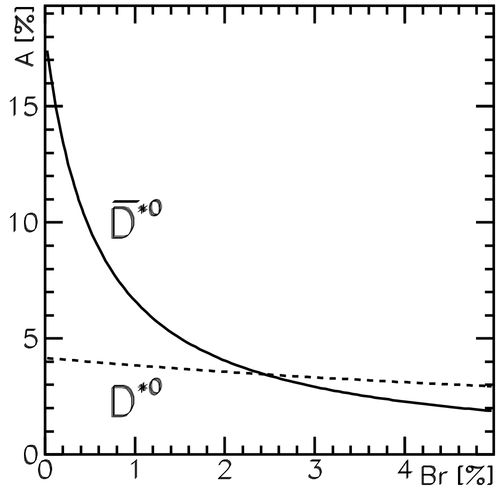

Figure 5: Time-integrated asymmetry in $B^0 \to \overset{(-)}{D}{}^{*0}X$ as a function of $\mathrm{Br}(B^0 \to \overline{D}^{*0}X)$.

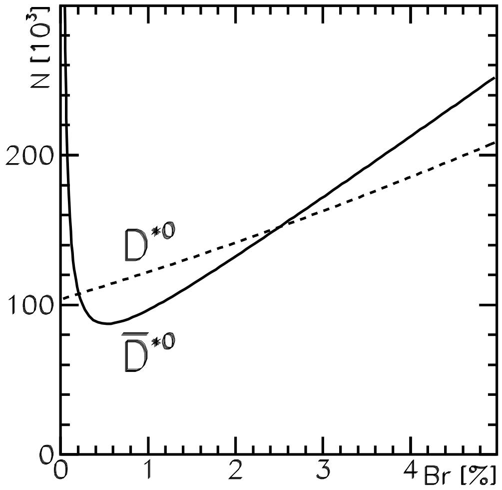

Figure 6: Necessary number of tagged B^0 events in $B^0 \to \overset{(-)}{D}{}^{*0}X$ as a function of $\mathrm{Br}(B^0 \to \overline{D}^{*0}X)$.

$B \to \overset{(-)}{D}{}^{(*)}X$ decays. It turns out that, as in Ref. [3], one cannot avoid to introduce some model dependence. Furthermore, there is also some dependence on an unknown relative phase, which we treat as an uncertainty. Due to these uncertainties we cannot expect our method to compete with proposed methods using "gold-plated" channels for determining CKM parameters, but we can still give estimates for the expected CP asymmetries of the different ground state $\overset{(-)}{D}$ mesons.

For most of the asymmetries we find results of a few 10^{-3}, but some are expected to be as large as several percent. These effects should be observable at the B factories. The channels involving right and wrong charm neutral vector mesons turn out to be most promising: they are expected to have the largest asymmetries, and the theoretical method yields the best results for the production rates and spectra of the vector mesons [3].

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