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Top production above threshold: Electroweak and QCD corrections combined[a]

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Top quark production in electron–positron annihilation is one of the benchmark reactions at a future linear collider. Both electroweak and QCD corrections are large, amounting to 10% or even more in specific kinematic regions. In this note we present a method which allows to combine the dominant terms from both sources, thus improving considerably the result based on a simple addition of both corrections.

1 Introduction

For a precise prediction of the continuum cross-section away from threshold both the one-loop electroweak corrections [1] and the QCD corrections of order α_s^2 [2] are needed. Both contributions are sizeable, amounting to 10% for the electroweak corrections at 2 TeV and more than 100% for the QCD corrections at 400 GeV.

While both corrections are available in the literature, so far no attempt has been made to combine both. Simply adding the electroweak and QCD corrections neglects the mixed $\mathcal{O}(\alpha\alpha_s)$ contributions, and performing the complete $\mathcal{O}(\alpha\alpha_s)$ calculation is of course a formidable task. So the question arises: Can one combine the available pieces such that the dominant mixed $\mathcal{O}(\alpha\alpha_s)$ terms are taken into account?

Unfortunately, not even this is straightforward because the QCD corrections are as yet available only for the vector and axial-vector part of the *correlator*, i.e. they are given as a correction factor multiplying the vector and axial-vector part of the *squared* amplitude. It is not clear how to combine these results with those parts of the electroweak amplitude which cannot simply be decomposed into a vector and an axial-vector part. In this paper we discuss a strategy which allows to combine these two ingredients in an approximate way without evaluating the complete mixed corrections of order $\alpha\alpha_s$.

Purely electromagnetic corrections (apart from the initial-state radiation which can be treated separately) are small and are hence omitted from this preliminary

[a] Talk presented by J. Kühn at the Linear Collider Workshop, Sitges, Spain, April 1999.

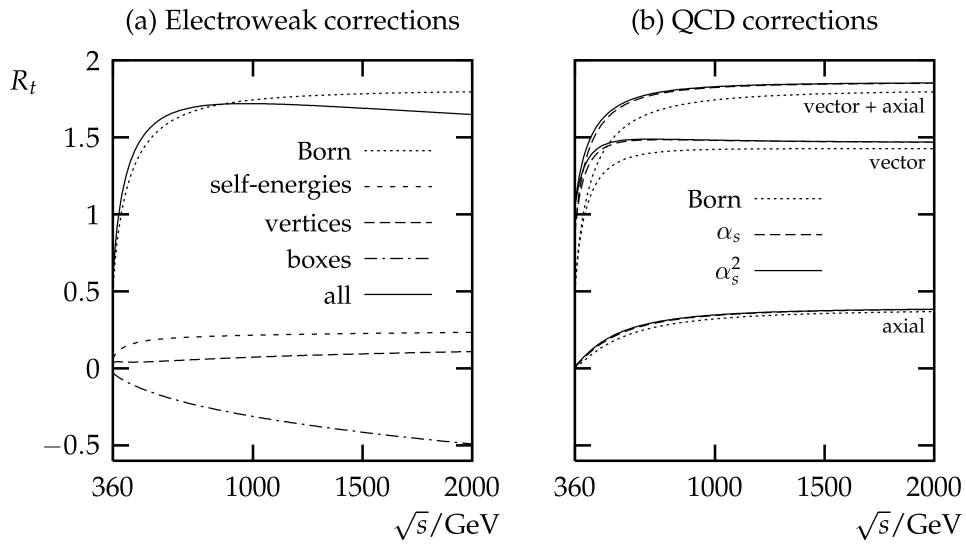

Figure 1: The electroweak (a) and QCD (b) corrections to $e^+e^- \to \bar{t}t$.

analysis. They will be included in a future paper. Very close to threshold the Coulomb enhancement (in other words, bound-state effects) have to be taken into account. To avoid this region we adopt $2m_{\mathrm{t,pole}}+10$ GeV as lower limit. A discussion of the matching between the fixed-order treatment and the one based on Green functions including Coulomb resummation can be found e.g. in [3].

In the following sections we first discuss the electroweak and strong effects separately and then present our recipe for combining both.

2 Electroweak corrections

Figure 1 shows the total $e^+e^- \to \bar{t}t$ cross-section, normalized to $\sigma_{\mathrm{pt}} = 4\pi\alpha^2/3s$, where s is the cms energy squared.

The electroweak corrections are plotted in Fig. 1(a), where the dotted line is the Born approximation and the solid line is the full $\mathcal{O}(\alpha)$ result. The one-loop diagrams can be separated into self-energy insertions on the γ- and Z-propagator, corrections of the electron and top vertex, and box contributions (see Fig. 1(a)),

$$\mathcal{M}_{\text{1-loop}} = \mathcal{M}_{\mathrm{self}} + \mathcal{M}_{\mathrm{vert},e} + \mathcal{M}_{\mathrm{vert},t} + \mathcal{M}_{\mathrm{box}}\,. \tag{1}$$

The box contributions are sizeable and largely compensate the self-energy and vertex contributions.

These results were obtained using *FeynArts* and *FormCalc* [4]. They agree with the ones in Ref. [1].

3 QCD corrections

The QCD corrections are shown in Figure 1(b). The dotted, dashed, and solid lines denote the Born, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s^2)$ approximation, respectively. Large corrections are observed close to threshold, a remnant of the $1/\beta$ singularity of the QCD corrections. These are particularly important for the vector current and lead to a non-vanishing cross-section for small β.

Technically, the corrections are given by two factors R_V and R_A that multiply the vector and axial-vector part of the lowest-order cross-section, respectively.

4 Combined electroweak and QCD corrections

We now derive a strategy for the proper treatment of the terms of order $\alpha(4\pi\alpha_s/\beta)$ which dominate in the region of small β, still without resummation of these terms, thus requiring formally $4\pi\alpha_s/\beta \ll 1$. We start by decomposing the electroweak one-loop amplitude into vector and axial-vector parts, for which we know how to compute the QCD corrections, and the rest:

$$\begin{aligned} \mathcal{M}_{\text{1-loop}} &= \mathcal{M}^V_{\text{1-loop}} + \mathcal{M}^A_{\text{1-loop}} + \mathcal{M}^{\text{rest}}_{\text{1-loop}} \\ \text{where}\quad \mathcal{M}^V_{\text{1-loop}} &= \mathcal{M}^V_{\text{self}} + \mathcal{M}^V_{\text{vert},e} + \mathcal{M}^V_{\text{vert},t}\,, \\ \mathcal{M}^A_{\text{1-loop}} &= \mathcal{M}^A_{\text{self}} + \mathcal{M}^A_{\text{vert},e} + \mathcal{M}^A_{\text{vert},t}\,, \\ \mathcal{M}^{\text{rest}}_{\text{1-loop}} &= \mathcal{M}^{\text{rest}}_{\text{vert},t} + \mathcal{M}_{\text{box}}\,. \end{aligned} \tag{2}$$

All terms with a V superscript are proportional to $\bar{u}_t\gamma_\mu v_t$ and all terms with an A superscript are proportional to $\bar{u}_t\gamma_5\gamma_\mu v_t$. The only corrections which have other Dirac structures are the top vertex and the boxes. The QCD corrections to $\mathcal{M}^V_{\text{1-loop}}$ and $\mathcal{M}^A_{\text{1-loop}}$ are identical to those to $\mathcal{M}^V_{\text{Born}}$ and $\mathcal{M}^A_{\text{Born}}$, respectively, because they both have the same Dirac structure.

Next, observe that terms of the form $\mathcal{M}^V\mathcal{M}^{A*}$ or $\mathcal{M}^A\mathcal{M}^{V*}$ drop out of the squared amplitude after spin summation and angular integration. This means that the cross-section can be written as

$$\begin{aligned} \sigma \propto \sum_{\text{spins}} \int d\Omega \Big[&\left|\mathcal{M}^V_{\text{Born}}\right|^2 + 2\,\text{Re}\left(\mathcal{M}^V_{\text{1-loop}} + \mathcal{M}^{\text{rest}}_{\text{1-loop}}\right)\mathcal{M}^{V*}_{\text{Born}} + \\ &\left|\mathcal{M}^A_{\text{Born}}\right|^2 + 2\,\text{Re}\left(\mathcal{M}^A_{\text{1-loop}} + \mathcal{M}^{\text{rest}}_{\text{1-loop}}\right)\mathcal{M}^{A*}_{\text{Born}}\Big]. \end{aligned} \tag{3}$$

This decomposition is exact at $\mathcal{O}(\alpha)$. Now we tack on the QCD corrections as

$$\begin{aligned} \sigma \propto \sum_{\text{spins}} \int d\Omega \Big[&R_V\left(\left|\mathcal{M}^V_{\text{Born}}\right|^2 + 2\,\text{Re}\left(\mathcal{M}^V_{\text{1-loop}} + \mathcal{M}^{\text{rest}}_{\text{1-loop}}\right)\mathcal{M}^{V*}_{\text{Born}}\right) + \\ &R_A\left(\left|\mathcal{M}^A_{\text{Born}}\right|^2 + 2\,\text{Re}\left(\mathcal{M}^A_{\text{1-loop}} + \mathcal{M}^{\text{rest}}_{\text{1-loop}}\right)\mathcal{M}^{A*}_{\text{Born}}\right)\Big]. \end{aligned} \tag{4}$$

Of course, this equation does not really reproduce the contributions of order $\alpha\alpha_s$ and $\alpha\alpha_s^2$. Close to threshold, however, the s-wave contribution is entirely contained in the terms that are proportional to $\mathcal{M}^V_{\text{Born}}$. They dominate over the $\mathcal{M}^A_{\text{Born}}$-terms

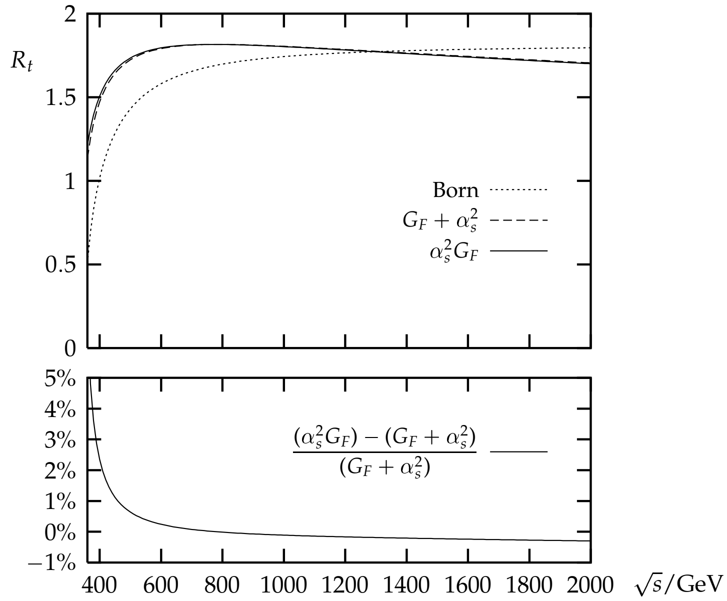

Figure 2: *Upper part:* Comparison of the naive addition of electroweak and QCD corrections (solid line) with the approximate way of including the dominant $\mathcal{O}(\alpha\alpha_s)$ contributions in Eq. (4) (dashed line). *Lower part:* The relative difference of the two methods.

by a factor of $1/\beta^2$. This means that for not-too-large β, the electroweak corrections act at distances that are short compared to the inverse relative t–$\bar{t}$ momentum, and so the QCD corrections, in particular the Coulomb enhancement, can be included as an overall factor multiplying the vector part $\mathcal{M}^{V*}_{\text{Born}}$.

The results of Eq. (4) are plotted in Fig. 2 (solid line). To indicate the significance of this prescription we also give the prediction obtained by simply adding QCD and electroweak corrections (dashed line). As expected, the difference is largest in the low-energy region, amounting to 5% at 360 GeV.

Acknowledgments

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1. W. Beenakker, S.C. van der Marck, and W. Hollik, *Nucl. Phys.* **B365** (1991) 24.
2. K.G. Chetyrkin, J.H. Kühn, and M. Steinhauser, *Nucl. Phys.* **B482** (1996) 213. K.G. Chetyrkin, R. Harlander, J.H. Kühn, and M. Steinhauser, *Nucl. Phys.* **B503** (1997) 339. K.G. Chetyrkin, J.H. Kühn, and M. Steinhauser, *Nucl. Phys.* **B505** (1997) 40. R. Harlander and M. Steinhauser, *Eur. Phys. J.* **C2** (1998) 151.

3. A.H. Hoang and T. Teubner, *Phys. Rev.* **D60** (1999) 114027.
4. J. Küblbeck, M. Böhm, and A. Denner, *Comp. Phys. Commun.* **60** (1990) 165. T. Hahn and M. Pérez-Victoria, *Comp. Phys. Commun.* **118** (1999) 153.