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# Second Order QCD Corrections to the Top Decay Rate†

K.G. Chetyrkin[1], R. Harlander[1,2], T. Seidensticker[1], M. Steinhauser[3]

[1]University of Karlsruhe, D-76128 Karlsruhe, Germany
[2]Physics Department, Brookhaven National Laboratory, Upton, NY 11973
[3]University of Hamburg, D-22761 Hamburg, Germany

**Abstract**

We report on a recent calculation of the top decay rate up to $\mathcal{O}(\alpha_s^2)$. It is based on asymptotic expansions of the off-shell top propagator, followed by a Padé approximation in order to reach the physically relevant point $q^2 = M_t^2$.

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**Abstract**

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## 1. Introduction

The top quark is currently the heaviest known elementary particle. Thus top physics is a very promising field with regard to physics beyond the Standard Model (SM). For example, its large Higgs coupling makes one hope to learn something about the Higgs spectrum in particular, or mass generating mechanisms in general. Furthermore, its large mass is an important premise for decays into non-standard particles. Another exceptional property of the top quark is its large decay width ($\Gamma_{\rm Born} \approx 1.56$ GeV) as predicted by the SM. Instead of undergoing the process of hadronization, the top quark is hardly affected by the non-perturbative regime of QCD and decays almost exclusively into a bottom quark and a $W$ boson by weak interaction.

## 2. Known results

The $\mathcal{O}(\alpha_s)$ corrections [1] to the decay rate $\Gamma(t \to bW)$ amount to $-9\%$ of the Born result. This is comparable to the expected experimental accuracy at an NLC which is around 10%. Thus one should make sure that the QCD corrections are reliable, in the sense that the series in $\alpha_s$ converges sufficiently fast. This is the main motivation for investigating the $\alpha_s^2$ corrections to this process. The $-9\%$ from above may be split into a contribution for $M_W = 0$ ($-11\%$) and the effects induced by a non-vanishing $W$ mass ($+2\%$). The electroweak corrections [2] at one-loop level are about 2%.

For the $\mathcal{O}(\alpha_s^2)$ corrections there exists a result for $M_W = 0$ [3]. It was obtained by performing an expansion in the limit

$$(M_t^2 - M_b^2)/M_t^2 \ll 1$$

and taking into account a sufficiently large number of terms in the expansion. The result reads

$$\Gamma_t = \Gamma_{\rm Born}(1 - 0.09 - 0.02)\,, \tag{1}$$

where the three numbers correspond to the Born, $\mathcal{O}(\alpha_s)$, and $\mathcal{O}(\alpha_s^2)$ terms, respectively.

The aim of the calculation of [4] was, on the one hand, to perform an independent check of the $\mathcal{O}(\alpha_s^2)$ terms at $M_W = 0$, and, on the other hand, to take into account effects induced by a non-vanishing $W$ mass at this order.

## 3. Method

While in [3] vertex diagrams for $t \to bW$ were computed, in [4] the top quark decay rate was obtained via the optical theorem which relates it to the imaginary part of the top quark propagator $\Sigma = \not{q}\Sigma_{\rm V} + M_t\Sigma_{\rm S}$:

$$\Gamma_t \propto {\rm Im}(\Sigma_{\rm V} + \Sigma_{\rm S})\big|_{q^2=M_t^2}\,. \tag{2}$$

This means that one should calculate $\Sigma$ at the point $q^2 = M_t^2$ up to $\mathcal{O}(G_{\rm F}\alpha_s^2)$. An example for a diagram that contributes to this order is shown in Fig. 1. The $b$ quark mass can safely be set to zero, and for the moment also $M_W = 0$ will be assumed. The analytic evaluation of diagrams like the one shown in Fig. 1 is currently neither available for general $q^2$, nor for the special case of interest, $q^2 = M_t^2$. The only limiting cases that are accessible are $q = 0$ or $M_t = 0$. However, asymptotic expansions provide an efficient tool to obtain

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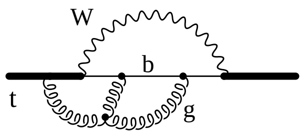


**Figure 1.** Sample diagram contributing to the top quark propagator at $\mathcal{O}(G_{\rm F}\alpha_s^2)$.

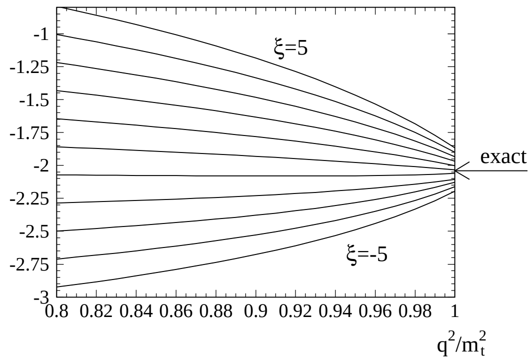


**Figure 2.** $q^2$ dependence of the [4/4] Padé result at $\mathcal{O}(G_{\rm F}\alpha_s)$ for different values of $\xi$.

approximate results also away from these extreme choices. Their application yields series in $q^2/M_t^2$ (or $M_t^2/q^2$), with the individual coefficients containing the non-analytic structures in terms of logarithms of $\mu^2/q^2$ and $\mu^2/M_t^2$ ($\mu$ is the renormalization scale). Employing the analyticity properties of the approximated function, one obtains the region of convergence for the corresponding series. Within this region, the full result can be approximated with arbitrary accuracy by including sufficiently many terms in the expansion. Examples demonstrating the quality of such approximations can be found in [5, 6].

Applying this strategy to top quark decay, in a first step one should compute as many terms as possible in the expansion around $q^2/M_t^2$. Note that one cannot approach the point $q^2 = M_t^2$ from the opposite side (i.e. $q^2/M_t^2 > 1$), because this implies top quarks in the final state which is kinematically forbidden. The second step is to extrapolate the result from the small-$q^2$ region to the point $q^2 = M_t^2$. At $\mathcal{O}(\alpha_s)$ this could be done by explicitly resumming the full series in $q^2/M_t^2$ [7]. At $\mathcal{O}(\alpha_s^2)$, however, a different strategy was pursued in [4] by performing a Padé approximation in the variables $z = q^2/M_t^2$ and $\omega = (1-\sqrt{1-z})/(1+\sqrt{1-z})$.

Effects of a non-vanishing $W$ mass can be taken into account by applying asymptotic expansions w.r.t. the relation $M_t^2 \gg q^2 \gg M_W^2$. In this way one obtains a nested series in $M_W^2/M_t^2$ and $q^2/M_t^2$. The above procedure can then be applied to each coefficient of $M_W^2/M_t^2$ separately.

One of the questions one is faced with when following this approach is gauge (in)dependence. The off-shell fermion propagator is not a gauge invariant quantity, and only in the limit $q^2 = M_t^2$ the QCD gauge parameter $\xi$ formally drops out. Due to the fact that one works with a limited number of terms here, gauge parameter dependence does not vanish exactly but is only expected to decrease gradually as soon as a sufficiently large number of expansion terms is included. Nevertheless, the claim is that by a reasonable choice of the gauge parameter the predictive power of the result is preserved. At $\mathcal{O}(\alpha_s^2)$ the calculation cannot be performed for general $\xi$ due to the enormous increase in required computer resources. Thus one cannot decide upon the choice of $\xi$ *a posteriori*, but has to set it to some definite value from the very beginning. On the other hand, it is expected that the behavior of the $\alpha_s^2$ terms w.r.t. the choice of $\xi$ is similar to the $\mathcal{O}(\alpha_s)$ result. Therefore, in [4] the gauge parameter dependence was studied in some detail at $\mathcal{O}(\alpha_s)$. This study was mainly based on the stability of the Padé results $[m/n]$ upon variation of $m$ and $n$ for different values of $\xi$. Another way to find a "reasonable" choice for $\xi$ is to study the $q^2$-dependence of the Padé results close to the physically relevant point $q^2 = M_t^2$. The extrapolation to $q^2 = M_t^2$ is expected to work best if the variation of the approximating function close to this point is smoothest. For some values of $\xi$, the $q^2$ dependence of the one-loop result near $q^2/M_t^2 = 1$ is shown in Fig. 2. The results of [4] were obtained by setting $\xi = 0$. The validity of this choice is further justified by the perfect agreement of the approximation to the exact result at $\mathcal{O}(\alpha_s)$ (see Fig. 2 and the results in the following section).

Concerning the technical realization of the calculation it heavily relies on automatic Feynman diagram evaluation with the help of algebraic programs [8]. For details we refer to [4].

## 4. Results

The result for the top decay rate to $\mathcal{O}(\alpha_s^2)$ will be written in the following way ($y \equiv M_W^2/M_t^2$, $y_0 = (80.4/175)^2$):

$$\Gamma_t = \Gamma_0 \left( \delta^{(0)}(y) + \frac{\alpha_s}{\pi}\delta^{(1)}(y) + \left(\frac{\alpha_s}{\pi}\right)^2 \delta^{(2)}(y) \right) , \tag{3}$$

with $\delta^{(0)}(y_0) = 0.885\ldots$. Following the method described in Section 3, one obtains $\delta^{(1)}(y_0) =$

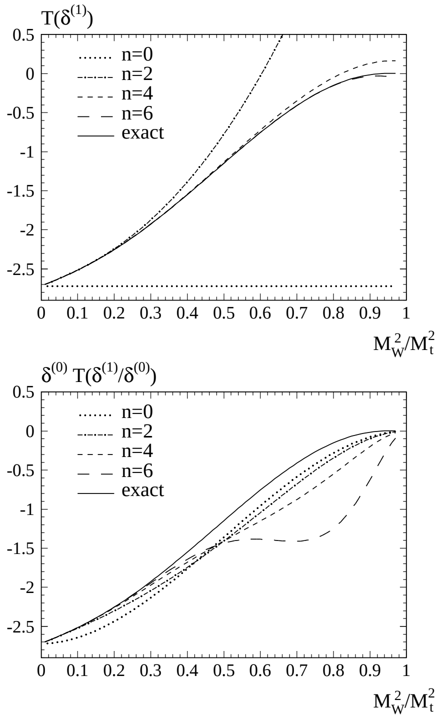


**Figure 3.** T($f$) means Taylor expansion of $f$ around small $y = M_W^2/M_t^2$ up to order $n$ (only even orders are shown). The solid line represents the exact result for $\delta^{(1)}$.

$-2.20(3)$ which is to be compared with the exact number, reading $-2.220\ldots$. This demonstrates the validity and the accuracy of the underlying approach.

At $\mathcal{O}(\alpha_s^2)$, the result of [4] reads

$$\delta^{(2)}(y) = -16.7(8) + 5.4(4)\,y + y^2\,(11.4(5.0) + 7.3(1)\,\ln y) + \mathcal{O}(y^3) \overset{y=y_0}{=} -15.6(1.1). \quad (4)$$

The first number corresponds to the case of vanishing $W$ mass. It agrees perfectly with the result of [3] which is $-16.7(5)$. The uncertainty quoted in [4] is slightly larger than the one of [3] but is still negligible as compared to the expected experimental accuracy at future colliders. The series in $M_W^2/M_t^2$ is converging very quickly, and the total effect of the $M_W$-suppressed terms is small. The errors in (4) are added linearly, yielding a bigger uncertainty for the sum than for the $M_W = 0$ result. Adding the errors quadratically instead, the uncertainty again would be 0.8.

## 5. Estimate for $b \to ul\bar{\nu}$

The integration of Eq. (3) over $y$ from 0 to 1 is directly related to the decay rate for $b \to ul\bar{\nu}$ at $\mathcal{O}(\alpha_s^2)$ (and with obvious modifications to the two-loop QED corrections to $\Gamma(\mu \to e\nu\bar{\nu})$). However, since the $y$ dependence of $\delta^{(2)}$ is only known up to $y^2$, it is more promising to pull out $\delta^{(0)}(y)$ and to consider the Taylor expansion of $\delta^{(i)}/\delta^{(0)}$ which is expected to vary only slightly in the relevant $y$-range [3]. On the other hand, if more terms in $y$ are included, the Taylor series of $\delta^{(i)}/\delta^{(0)}$ becomes ill-behaved, and one should directly expand $\delta^{(i)}$. These observations are illustrated for $i = 1$ in Fig. 3. In [4] both methods were applied, and the results for $\Gamma(b \to ul\bar{\nu})$ and $\Gamma(\mu \to e\nu\bar{\nu})$ agree with the ones of [9] to about 10%. This constitutes a stringent check on both calculations.

Also a direct application of the method described in Section 3 to the (four-loop!) diagrams corresponding to $b \to ul\bar{\nu}$ and $\mu \to e\nu\bar{\nu}$ was performed in [10], and full agreement with [9] was found.

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