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QCD-based description of one-particle inclusive B decays

Xavier Calmet, Thomas Mannel, and Ingo Schwarze
Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D–76128 Karlsruhe, Germany

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Abstract

We discuss one-particle inclusive B decays in the limit of heavy b and c quarks. Using the large-N_C limit we factorize the nonleptonic matrix elements, and we employ a short distance expansion. Modeling the remaining nonperturbative matrix elements we obtain predictions for various decay channels and compare them with existing data.

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1 Introduction

Over the last ten years, methods have been developed to describe the decays of heavy hadrons within a framework based on the heavy mass expansion of QCD. Using this approach, model dependences have been drastically reduced.

The heavy mass expansion has been formulated for exclusive as well as for inclusive decays. While in the former case the heavy mass expansion is the well-known formalism of heavy quark effective theory (HQET) [1], the latter case additionally requires a short distance expansion (SDE) [2] which is very similar to the operator product expansion (OPE) used for deep inelastic scattering. Both approaches are QCD based methods and have a good theoretical foundation.

However, nothing comparable exists for for a theoretical description of one-particle inclusive decays such as $B \to \overset{(-)}{D} X$ or $B \to \overset{(-)}{K} X$. Unlike the fully inclusive case, the expression for the rate always involves a projection on a specific particle, thereby spoiling a straightforward short distance expansion. The semileptonic case has been discussed in [3], and the present paper is devoted to a study of the nonleptonic decays.

We shall concentrate on decays of the form $B \to \overline{D} X$ and $B \to DX$. These decays have already been considered long ago in the context of models [4]. Here we aim at a QCD description and exploit the heavy mass limit for both the b and the c quark. Furthermore we shall use the large-N_C limit to factorize the hadronic matrix element. In this way we can identify the parts which allow a short distance expansion. The remaining contributions have to be parametrized and we shall discuss simple forms of this parametrization.

We first discuss the right charm contribution arising from quark level $b \to cX$ decays and the wrong charm contribution arising from $b \to \overline{c} X$ separately. After recalling some facts about the phenomenology of $\overset{(-)}{D}$ mesons in Sec. 4, we predict rates and spectra for various decay channels in Sec. 5. Finally we compare the results with existing data.

2 The effective Hamiltonian and the right charm contribution

The relevant effective Hamiltonian for the decays $B \to \overset{(-)}{D} X$ is given by

$$H_{eff} = H_{eff}^{(sl)} + H_{eff}^{(nl)}, \tag{1}$$

where the semileptonic and nonleptonic pieces are

$$H_{eff}^{(sl)} = \frac{G_F}{\sqrt{2}} V_{cb} \, (\bar{b} c)_{V-A} \, (\bar{\ell} \nu)_{V-A} + \text{H.c.}, \tag{2}$$

$$H_{eff}^{(nl)}=\frac{G_F}{\sqrt{2}}\sum_{q=d,s}\left(V_{cb}V_{uq}^*\sum_{k=1}^{2}C_k(\mu)O_k^q+V_{cb}V_{cq}^*\sum_{k=1}^{2}C_k(\mu)P_k^q+V_{tb}V_{tq}^*\sum_{k=3}^{6}C_k(\mu)O_k^q\right)+\text{H.c.} \tag{3}$$

$O_{1,2}$ and $P_{1,2}$ are the current-current operators, and $O_{3\ldots6}$ are the QCD penguin operators. For a full list of operators, see, e.g., the review [5]. We shall only consider the Cabibbo-favored decays and neglect penguin contributions. Hence the operators we need are

$$\begin{aligned} O_1^d = O_1 &= (\overline{b}c)_{V-A}(\overline{u}d)_{V-A} \\ O_2^d = O_2 &= (\overline{b}T^a c)_{V-A}(\overline{u}T^a d)_{V-A} \\ P_1^s = P_1 &= (\overline{b}c)_{V-A}(\overline{c}s)_{V-A} \\ P_2^s = P_2 &= (\overline{b}T^a c)_{V-A}(\overline{c}T^a s)_{V-A}. \end{aligned} \tag{4}$$

C_1 and C_2 are the Wilson coefficients encoding the short distance physics and T^a are the generators of color-SU(3). The operators O_1 and O_2 as well as the semileptonic Hamiltonian $H_{eff}^{(sl)}$ contribute to right charm transitions only, while P_1 and P_2 contribute to both right and wrong charm processes.

It is well known that in the large-N_C limit the matrix elements of the four-fermion operators factorize into products of two current matrix elements [6]. Factorization has also been investigated on a phenomenological basis and found to work well for exclusive nonleptonic decays of B mesons [7]. The contributions of O_2 and P_2 vanish in the factorization limit since the currents are color octets. Furthermore, the coefficient C_1 differs from unity only through radiative corrections which are very small and will be neglected.

The semileptonic case has already been studied in [3], so we focus on the nonleptonic modes. We consider the matrix element

$$G(M^2)=\sum_X\left|\langle\overline{D}^{(*)}(p_{\overline{D}})X|H_{eff}|B(p_B)\rangle\right|^2(2\pi)^4\delta^4(p_B-p_{\overline{D}}-p_X) \tag{5}$$

where the states $|X\rangle$ form a complete set of momentum eigenstates with momentum p_X and H_{eff} is the relevant part of the weak Hamiltonian. The function G depends on the invariant mass

$$M^2=(p_B-p_{\overline{D}})^2 \tag{6}$$

of the state $|X\rangle$. It is related to the decay rate under consideration by

$$d\Gamma(B\to\overline{D}^{(*)}X)=\frac{1}{2m_B}\,d\Phi_{\overline{D}}\,G(M^2), \tag{7}$$

where $d\Phi_{\overline{D}}$ is the phase space element of the final state $\overline{D}^{(*)}$ meson.

Due to the different final states $c\overline{u}d$ and $c\overline{c}s$, there are no interference terms between O_1 and P_1. The contribution of the channel $b \to c\overline{u}d$ is

$$G_1(M^2) = \frac{G_F^2}{2} \, |V_{cb}V_{ud}^*|^2 \, |C_1|^2 \, \sum_X (2\pi)^4 \delta^4(M - p_X) \tag{8}$$
$$\langle B(p_B)|(\overline{c}\gamma_\mu(1-\gamma_5)b)(\overline{d}\gamma^\mu(1-\gamma_5)u)|\overline{D}^{(*)}(p_{\overline{D}})X\rangle$$
$$\langle \overline{D}^{(*)}(p_{\overline{D}})X|(\overline{u}\gamma^\nu(1-\gamma_5)d)(\overline{b}\gamma_\nu(1-\gamma_5)c)|B(p_B)\rangle.$$

Using the factorization of the large-N_C limit we have (see Fig. 1)

$$G_1(M^2) = \frac{G_F^2}{2} \, |V_{cb}V_{ud}^*|^2 \, |C_1|^2 \, \sum_X \sum_{X'} (2\pi)^4 \delta^4(M - p_X - p_{X'}) \tag{9}$$
$$\langle B(p_B)|(\overline{c}\gamma_\mu(1-\gamma_5)b)|\overline{D}^{(*)}(p_{\overline{D}})X\rangle \, \langle 0|(\overline{d}\gamma^\mu(1-\gamma_5)u)|X'\rangle$$
$$\langle X'|(\overline{u}\gamma^\nu(1-\gamma_5)d)|0\rangle \, \langle \overline{D}^{(*)}(p_{\overline{D}})X|(\overline{b}\gamma_\nu(1-\gamma_5)c)|B(p_B)\rangle.$$

It is convenient to define two tensors

$$K_{\mu\nu}(p_B, M, Q) = \sum_X (2\pi)^4 \delta^4(M - Q - p_X) \tag{10}$$
$$\langle B(p_B)|(\overline{c}\gamma_\mu(1-\gamma_5)b)|\overline{D}^{(*)}(p_{\overline{D}})X\rangle\langle \overline{D}^{(*)}(p_{\overline{D}})X|(\overline{b}\gamma_\nu(1-\gamma_5)c)|B(p_B)\rangle$$

and

$$P_{\mu\nu}(Q) = \sum_{X'} (2\pi)^4 \delta^4(Q - p_{X'}) \langle 0|(\overline{d}\gamma_\mu(1-\gamma_5)u)|X'\rangle\langle X'|(\overline{u}\gamma_\nu(1-\gamma_5)d)|0\rangle \tag{11}$$

in terms of which we obtain for the rate

$$G_1(M^2) = \frac{G_F^2}{2} \, |V_{cb}V_{ud}^*|^2 \, |C_1|^2 \int \frac{d^4Q}{(2\pi)^4} \, K_{\mu\nu}(p_B, M, Q) \, P^{\mu\nu}(Q). \tag{12}$$

The tensor $P_{\mu\nu}$ involves only light quarks and can be rewritten as

$$P_{\mu\nu}(Q) = \int d^4x \, e^{-iQx} \langle 0|(\overline{d}(x)\gamma_\mu(1-\gamma_5)u(x))(\overline{u}(0)\gamma_\nu(1-\gamma_5)d(0))|0\rangle. \tag{13}$$

For sufficiently large Q this quantity has a short distance expansion in inverse powers of Q. However, the momentum Q is not a measurable kinematical quantity. In particular, it is not equal to the recoil M, but in most of the phase space, it should be of the same order as M, namely $\mathcal{O}(m_b - m_c)$. Close to the nonrecoil point, M is large, corresponding to either large Q or a large momentum of the gluon depicted in Fig. 1. While in the former case the OPE treatment is justified, the latter case could be treated perturbatively. In other

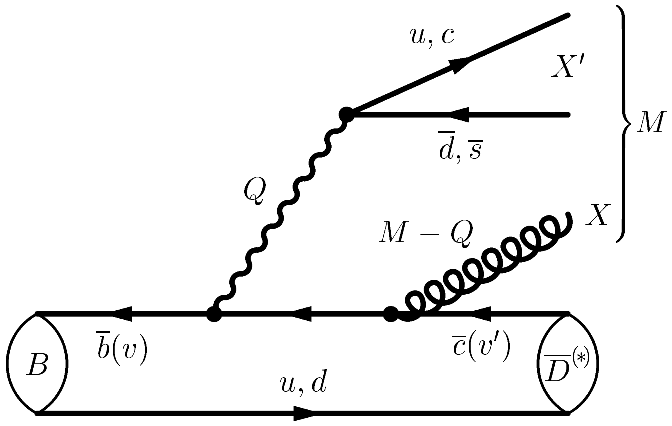

Figure 1: Momenta and velocities used for the description of the right charm contribution.

words, the two expansion parameters for the total rate are $\Lambda_{QCD}/(m_b - m_c)$ and $\alpha_s(m_c)/\pi$. However we also consider the momentum spectra of the $\overset{(-)}{D}$ mesons. In that case the expansion parameter depends on the energy of the $\overset{(-)}{D}$ meson since

$$Q^2 \approx M^2 = (m_b - m_c)^2 - 2m_b(E_c - m_c). \tag{14}$$

The leading term of the short distance expansion yields the partonic result

$$P_{\mu\nu}(Q) = \frac{N_C}{3\pi}\,(Q_\mu Q_\nu - g_{\mu\nu}Q^2)\,\Theta(Q^2) \tag{15}$$

where we have assumed both light quarks to be massless.

For the quantity $K_{\mu\nu}$ it is convenient to use the heavy mass limit for both the bottom and the charm quark. To this end we redefine the phases of the quark fields as

$$b(x) = b_v(x)\,e^{-im_b vx}, \qquad c(x) = c_{v'}(x)\,e^{-im_c v' x}, \tag{16}$$

where the velocities are defined as $p_B = m_B v$ and $p_{\overline{D}} = m_{\overline{D}} v'$. In the following we shall work in the infinite mass limit for the b and the c quark, and we obtain

$$\begin{aligned} K_{\mu\nu}(m_b v, M, Q) = \int d^4 z \sum_X \exp[-i(M-Q)z] \\ \langle B(v)|(\overline{c}_{v'}(z)\gamma_\mu(1-\gamma_5)b_v(z))|\overline{D}^{(*)}(v')X\rangle \\ \langle \overline{D}^{(*)}(v')X|(\overline{b}_v(0)\gamma_\nu(1-\gamma_5)c_{v'}(0))|B(v)\rangle, \end{aligned} \tag{17}$$

where in the heavy mass limit M becomes $m_b v - m_c v'$.

Using the result in Eq. (15) for $P_{\mu\nu}$ we can express the rate in terms of the two quantities

$$K_1(v, v', Q) = \frac{Q_\mu Q_\nu}{Q^2} K^{\mu\nu}(m_b v, M, Q), \tag{18}$$

$$K_2(v, v', Q) = K^\mu{}_\mu(m_b v, M, Q), \tag{19}$$

and obtain

$$\begin{aligned} G_1(M^2) = \frac{G_F^2}{2} \left|V_{cb} V_{ud}^*\right|^2 \left|C_1\right|^2 \frac{N_C}{3\pi} \\ \int \frac{d^4 Q}{(2\pi)^4} Q^2\, \Theta(Q^2) \left[K_1(v, v', Q) - K_2(v, v', Q)\right]. \end{aligned} \tag{20}$$

Not much can be said about the functions K_1 and K_2. They do not have an obvious short distance expansion due to the projection on the final state $\overline{D}$ meson. The only restriction we have is from the spin symmetry of the heavy b and c quarks. To implement these symmetries we use the spin projection matrices for the heavy mesons

$$\begin{aligned} H_B(v) &= \frac{1}{2}\sqrt{m_B}(1 + \not{v})\gamma_5, \\ H_{\overline{D}^{(*)}}(v') &= \frac{1}{2}\sqrt{m_D}(1 + \not{v}') \begin{cases} \gamma_5 & \text{(pseudoscalar meson)}, \\ \not{\epsilon} & \text{(vector meson)}. \end{cases} \end{aligned} \tag{21}$$

In the heavy mass limit, D and D^* become degenerate and constitute the ground state spin symmetry doublet of the D meson system. For phenomenological applications we shall later take the mass splitting between the D and D^* mesons into account, although formally this is a $1/m_c$ effect.

As far as the spinor indices are concerned, the tensor $K_{\mu\nu}$ is given by

$$K_{\mu\nu}(p_B, M, Q) \propto \overline{H}_B(v)\gamma_\mu(1-\gamma_5)H_{\overline{D}^{(*)}}(v') \otimes \overline{H}_{\overline{D}^{(*)}}(v')\gamma_\nu(1-\gamma_5)H_B(v), \tag{22}$$

where the remaining indices are light quark indices which have to be contracted using the most general four-index object.

Since there are many possibilities to contract the indices, the discussion of the general case would leave us with a large number of unknown functions, so we have to make a choice. The matrix elements appearing in $K_{\mu\nu}$ are identical to the semileptonic case. Therefore we shall use the same ansatz for the nonleptonic as for the semileptonic case [3] and write

$$\begin{aligned} K_{\mu\nu}(p_B, M, Q) = (2\pi)^4 \delta^4(M - Q)\, \eta(vv') \\ \mathrm{Tr}[\overline{H}_B(v)\gamma_\mu(1-\gamma_5)H_{\overline{D}^{(*)}}(v')]\ \mathrm{Tr}[\overline{H}_{\overline{D}^{(*)}}(v')\gamma_\nu(1-\gamma_5)H_B(v)]. \end{aligned} \tag{23}$$

In this way we can model the two functions K_1 and K_2 in terms of the single nonperturbative function $\eta(vv')$, the advantage being that spin symmetry relates the rates of $B \to \overline{D}X$ and $B \to \overline{D}^* X$ to each other.

Finally we also have to consider the right charm contribution of the operator P_1, which is

$$
\begin{aligned}
G_2(M^2) = \frac{G_F^2}{2} |V_{cb}V_{cs}^*|^2 \, |C_1|^2 \, \sum_X (2\pi)^4 \delta^4(M - p_X) \\
\langle B(p_B)|(\overline{c}\gamma_\mu(1-\gamma_5)b)(\overline{s}\gamma^\mu(1-\gamma_5)c)|\overline{D}^{(*)}(p_{\overline{D}})X\rangle \\
\langle \overline{D}^{(*)}(p_{\overline{D}})X|(\overline{c}\gamma^\nu(1-\gamma_5)s)(\overline{b}\gamma_\nu(1-\gamma_5)c)|B(p_B)\rangle \, .
\end{aligned} \tag{24}
$$

The calculation is exactly the same as before, but the short distance expansion of $P_{\mu\nu}$ yields a different result since now a massive charm quark is involved. We have

$$
P'_{\mu\nu}(Q) = \sum_{X'} (2\pi)^4 \delta^4(Q - p_{X'}) \langle 0|(\overline{s}\gamma_\mu(1-\gamma_5)c)|X'\rangle\langle X'|(\overline{c}\gamma_\nu(1-\gamma_5)s)|0\rangle \tag{25}
$$

from which we have the leading order contribution

$$
P'_{\mu\nu}(Q) = \Big(A(Q^2)\, Q_\mu Q_\nu - B(Q^2)\, Q^2 g_{\mu\nu} \Big)\, \Theta(Q^2 - m_c^2), \tag{26}
$$

with

$$
\begin{aligned}
A(Q^2) &= \frac{N_C}{3\pi} \left(1 - \frac{m_c^2}{Q^2}\right)^2 \left(1 + 2\frac{m_c^2}{Q^2}\right), \\
B(Q^2) &= \frac{N_C}{3\pi} \left(1 - \frac{m_c^2}{Q^2}\right)^2 \left(1 + \frac{1}{2}\frac{m_c^2}{Q^2}\right),
\end{aligned} \tag{27}
$$

and we get

$$
\begin{aligned}
G_2(M^2) = \frac{G_F^2}{2} |V_{cb}V_{cs}^*|^2 \, |C_1|^2 \int \frac{d^4Q}{(2\pi)^4} \, \Theta(Q^2 - m_c^2)\, Q^2 \\
[A(Q^2)K_1(v, v', Q) - B(Q^2)K_2(v, v', Q)].
\end{aligned} \tag{28}
$$

Thus the same two nonperturbative functions appear in G_2.

3 The wrong charm contribution

Wrong charm decays can only be mediated by the operators P_1 and P_2. In the large-N_C limit, the contribution of P_2 can be neglected and we have

$$
\begin{aligned}
G_3(p_B, p_D) = \frac{G_F^2}{2} |V_{cb}V_{cs}^*|^2 \, |C_1|^2 \, \sum_X (2\pi)^4 \delta^4(p_B - p_D - p_X) \\
\langle B(p_B)|(\overline{c}\gamma_\mu(1-\gamma_5)b)(\overline{s}\gamma^\mu(1-\gamma_5)c)|D^{(*)}(p_D)X\rangle \\
\langle D^{(*)}(p_D)X|(\overline{c}\gamma^\nu(1-\gamma_5)s)(\overline{b}\gamma_\nu(1-\gamma_5)c)|B(p_B)\rangle.
\end{aligned} \tag{29}
$$

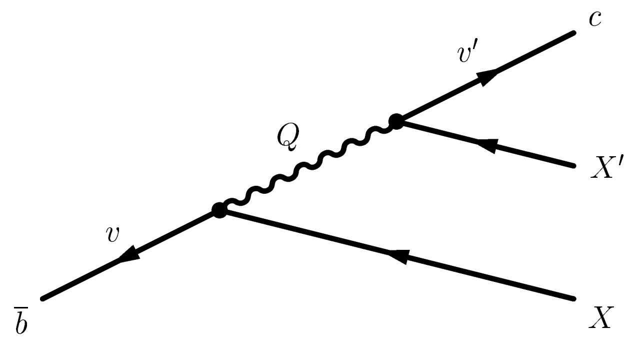

Figure 2: Momenta and velocities used for the description of the wrong charm contribution.

Again using the factorization of the large-N_C limit we get (see Fig. 2)

$$G_3(p_B, p_D) = \frac{G_F^2}{2} |V_{cb}V_{cs}^*|^2 \, |C_1|^2 \sum_X \sum_{X'} (2\pi)^4 \delta^4(p_B - p_D - p_X - p_{X'}) \tag{30}$$
$$\langle B(p_B)|(\bar{c}\gamma_\mu(1-\gamma_5)b)|X\rangle \, \langle X|(\bar{b}\gamma_\nu(1-\gamma_5)c)|B(p_B)\rangle$$
$$\langle 0|(\bar{s}\gamma^\mu(1-\gamma_5)c)|D^{(*)}(p_D)X'\rangle \, \langle D^{(*)}(p_D)X'|(\bar{c}\gamma^\nu(1-\gamma_5)s)|0\rangle.$$

In a similar way as before it is convenient to define two tensors

$$K'_{\mu\nu}(p_B, Q) = \sum_X (2\pi)^4 \delta^4(p_B - p_X - Q) \tag{31}$$
$$\langle B(p_B)|(\bar{c}\gamma_\mu(1-\gamma_5)b)|X\rangle\langle X|(\bar{b}\gamma_\nu(1-\gamma_5)c)|B(p_B)\rangle$$

and

$$R_{\mu\nu}(p_D, Q) = \sum_{X'} (2\pi)^4 \delta^4(Q - p_D - p_{X'}) \tag{32}$$
$$\langle 0|(\bar{s}\gamma_\mu(1-\gamma_5)c)|D^{(*)}(p_D)X'\rangle\langle D^{(*)}(p_D)X'|(\bar{c}\gamma_\nu(1-\gamma_5)s)|0\rangle,$$

in terms of which the rate becomes

$$G_3(p_B, p_D) = \frac{G_F^2}{2} |V_{cb}V_{cs}^*|^2 \, |C_1|^2 \int \frac{d^4Q}{(2\pi)^4} \, K'_{\mu\nu}(p_B, Q) \, R^{\mu\nu}(p_D, Q). \tag{33}$$

The quantity $K'_{\mu\nu}$ is fully inclusive and one may perform a short distance expansion. In the heavy mass limit for the b quark it is convenient to rescale the b quark field as in Eq. (16), and we obtain

$$K'_{\mu\nu}(p_B, Q) = \int d^4y \, \exp[-i(m_b v - Q)y] \tag{34}$$
$$\langle B(v)|(\bar{c}(y)\gamma_\mu(1-\gamma_5)b_v(y))(\bar{b}_v(0)\gamma_\nu(1-\gamma_5)c(0))|B(v)\rangle.$$

In the region where the momentum $m_b v - Q$ is large, we can perform a short distance expansion. As before, $m_b v - Q$ is not an observable, but it is of the order of $m_b - m_c$ in most of the phase space. The leading term is the dimension three operator $\overline{b}_v b_v$, the matrix elements of which are normalized due to heavy quark symmetry. Thus we obtain

$$\begin{aligned} K'_{\mu\nu}(m_b v, Q) &= 2\pi\delta\left((m_b v - Q)^2 - m_c^2\right) \\ &\qquad \mathrm{Tr}[\overline{H}_B(v)\gamma_\mu(1-\gamma_5)(m_b \not{v} - \not{Q})\gamma_\nu(1-\gamma_5)H_B(v)]. \end{aligned} \tag{35}$$

The other factor $R_{\mu\nu}$ involves a projection on a $D^{(*)}$ meson in the intermediate state and has to be parametrized. Heavy quark spin symmetry for the c quark implies that $R_{\mu\nu}$ is of the form

$$R_{\mu\nu}(m_c v', Q) \propto \overline{H}_{D^{(*)}}(v')\,\gamma_\mu(1-\gamma_5) \otimes \gamma_\nu(1-\gamma_5)\,H_{D^{(*)}}(v'), \tag{36}$$

where the remaining light quark indices have to be contracted using the most general four-index object.

Again there is quite a large number of possibilities to contract the indices, making it useless to discuss the general case. Hence we shall only give two physically motivated ways for modeling this quantity.

The first model ansatz corresponds to factorization:

$$\begin{aligned} R_{\mu\nu}(m_c v', Q) &= 2\,F(v'Q, Q^2) \\ &\qquad \mathrm{Tr}[\overline{H}_{D^{(*)}}(v')\,\gamma_\mu(1-\gamma_5)]\;\mathrm{Tr}[\gamma_\nu(1-\gamma_5)\,H_{D^{(*)}}(v')], \end{aligned} \tag{37}$$

the second one is inspired by the parton model and is defined through

$$\begin{aligned} R_{\mu\nu}(m_c v', Q) &= 2\,\tilde{F}(v'Q, Q^2) \\ &\qquad \mathrm{Tr}[\overline{H}_{D^{(*)}}(v')\gamma_\mu(1-\gamma_5)(\not{Q} - m_c \not{v}')\gamma_\nu(1-\gamma_5)H_{D^{(*)}}(v')]. \end{aligned} \tag{38}$$

We shall discuss the functions F and $\tilde{F}$ using data.

4 Effects of $D^* \to D$ decays

The right charm decays of $b = +1$ mesons are the ones into $\overline{D}^0$ or D^-

$$\begin{array}{ll} B^+ \to \overline{D}^{(*)0}\ X, & B^0 \to \overline{D}^{(*)0}\ X, \\ B^+ \to D^{(*)-}\ X, & B^0 \to D^{(*)-}\ X, \end{array} \tag{39}$$

while the wrong charm decays are

$$\begin{array}{ll} B^+ \to D^{(*)0}\ X, & B^0 \to D^{(*)0}\ X, \\ B^+ \to D^{(*)+}\ X, & B^0 \to D^{(*)+}\ X. \end{array} \tag{40}$$

For the semileptonic case the charge of the the lepton tags the b flavor of the decaying B meson. Wrong charm semileptonic decays are suppressed by the large charm mass and will be ignored in our discussion.

Off the heavy mass limit the degeneracy between $\overset{(-)}{D}$ and $\overset{(-)}{D}{}^*$ mesons is removed. In fact the mass difference is large enough to allow strong decays into pions. Thus the rate for decays into $\overset{(-)}{D}$ mesons is the sum of a direct contribution and the contribution arising from the decay chain $B \to \overset{(-)}{D}{}^* X' \to \overset{(-)}{D} X$. In the narrow width approximation, the latter is obtained by weighting the rate for $B \to \overset{(-)}{D}{}^* X$ by the branching ratios $D^* \to DY$ where Y is either a pion or a photon.

While $D^{*\pm}$ decays are governed by the isospin Clebsch-Gordan coefficients receiving only tiny corrections from phase space effects and from the radiative process,

$$\mathrm{Br}(D^{*+} \to D^+ Y) = 0.32, \qquad \mathrm{Br}(D^{*+} \to D^0 Y) = 0.68 \tag{41}$$

in D^{*0} decays isospin invariance is maximally broken by phase space effects [8] such that

$$\mathrm{Br}(D^{*0} \to D^0 Y) = 1. \tag{42}$$

Consequently the total branching ratios to pseudoscalar $\overline{D}$ mesons are

$$\begin{aligned} \mathrm{Br}(B \to D^- X) &= \mathrm{Br}(B \to D^-_{\mathrm{dir}} X) + 0.32\,\mathrm{Br}(B \to D^{*-} X) \\ \mathrm{Br}(B \to \overline{D}^0 X) &= \mathrm{Br}(B \to \overline{D}^0_{\mathrm{dir}} X) + \mathrm{Br}(B \to \overline{D}^{*0} X) + 0.68\,\mathrm{Br}(B \to D^{*-} X) \end{aligned} \tag{43}$$

where $\overline{D}_{\mathrm{dir}}$ refers to the direct contribution, i.e., the contribution where no $\overline{D}^*$ meson appears in an intermediate state.

We shall later consider spin and charge counting in one-particle inclusive B decays and define the following ratios:

$$r_S = \frac{\mathrm{Br}(B \to \overset{(-)}{D}{}^* X)}{\mathrm{Br}(B \to \overset{(-)}{D}_{\mathrm{dir}} X)}, \tag{44}$$

$$r_Q = \frac{\mathrm{Br}(B \to \overset{(-)}{D}{}^0 X)}{\mathrm{Br}(B \to D^{\pm} X)}. \tag{45}$$

We assume that $\overset{(-)}{D}{}^*$ decay is the only relevant isospin violating effect. Under this assumption the charge counting ratio is governed by the spin counting ratio:

$$r_Q = \frac{1 + 1.68 r_S}{1 + 0.32 r_S}. \tag{46}$$

Assuming equal rates for the pseudoscalar mesons and for each polarization state of the vector mesons, the naive expectations for these two ratios are $r_S = 3$ and $r_Q = 3$.

Apart from the ground states $\overset{(-)}{D}$ and $\overset{(-)}{D}{}^*$, exited $\overset{(-)}{D}$ mesons may be produced in B meson decays. Even in the heavy mass limit a finite splitting to the ground state mesons remains, which means that these states will decay into $\overset{(-)}{D}$ or $\overset{(-)}{D}{}^*$. In order to analyze decays into higher resonances, one would need to study the state $|X\rangle$, which we take fully inclusive. Thus we may rely on parton hadron duality to obtain a correct description of the $\overset{(-)}{D}$ and $\overset{(-)}{D}{}^*$ production up to leading order in the heavy mass expansion.

Charmed mesons can also be produced via charmonium intermediate states. These states show up in the invariant mass distribution $d\Gamma/dm_{D\overline{D}}$ as more or less pronounced resonances. The distribution $d\Gamma/dm_{c\bar{c}}$ is certainly different, but by global parton hadron duality and the assumption that all charm quarks hadronize into $\overset{(-)}{D}$ mesons, their integrals are equal, so the charmonium contributions are included in our calculation.

5 Results

Even after restricting the number of possible form factors by the ansätze in Eqs. (23), (37) and (38), we are still left with unknown nonperturbative functions. As far as the right charm contributions are concerned we shall first consider the decays into $\overline{D}^*$ mesons. According to Eq. (23), these are described by a single function $\eta(vv')$. Spin symmetry relates these decays to the ones into $\overline{D}$ mesons, however, this is only true in the heavy quark limit, where again $\eta(vv')$ is the nonperturbative input. In reality one has to take into account that $\overline{D}^*$ mesons decay into $\overline{D}$'s, and thus one has to add this contribution using Eqs. (43).

Still all right charm decays are given in terms of the single function $\eta(vv')$. Comparing the present case to the semileptonic one, we shall use the same saturation assumption as in [3] and write

$$\eta(vv') = |\xi(vv')|^2. \tag{47}$$

For numerical calculations we use the measurement [9] of the Isgur-Wise function

$$\xi(vv') = 1 - a(vv' - 1), \qquad a = 0.84. \tag{48}$$

Since the exclusive semileptonic decays are spectatorlike and since in the limit of factorization, the matrix elements of the heavy quark current are the same in the semi- and nonleptonic cases, dominance of spectatorlike decays is a natural assumption for the nonleptonic right charm case as well. Therefore we use the results of Sec. 2 for the channels

$$\begin{array}{ll} B^+ \to \overline{D}^0_{\rm dir}\, X, & B^0 \to D^-_{\rm dir}\, X, \\ B^+ \to \overline{D}^{*0}\, X, & B^0 \to D^{*-}\, X, \end{array} \tag{49}$$

neglecting possible contributions from nonspectator channels such as $B^+ \to D^- X$. Another class of decays allow to obtain a factorizable expression for $H_{\rm eff}$ after a Fierz transformation. However, the color indices need to be rearranged as well, yielding a suppression by one power of $1/N_C$ in the amplitude. Since we are working to leading order in $1/N_C$, we find a vanishing rate for $B^0 \to \overline{D}^{*0} X$, whereas the channel $B^0 \to \overline{D}^0 X$ is fed by the decay chain via the D^{*-} meson.

For the contribution of the quark level decay $b \to c\bar{c}s$, we assume the same number of right charm quarks to hadronize as $\overline{D}^{(*)}$ as wrong charm quarks hadronize as either $D^{(*)}$ or $D_s^{(*)}$. Using the CLEO wrong charm measurement $\mathrm{Br}(B \to DX) = (7.9 \pm 2.2)\%$ [10] and the Particle Data Group average $\mathrm{Br}(B \to D_s^{\pm} X) = (10.0 \pm 2.5)\%$ [11] and neglecting a possible right charm $B \to D_s^-$ contribution, we obtain a total right charm contribution from $b \to c\bar{c}s$ of about 18%.

It is known that the channel $b \to c\bar{c}s$ receives large radiative corrections computed in [12]. We shall not include these corrections in their detailed form, rather we shall take into account their bulk effect by adjusting the charm quark mass in Eqs. (26)–(28). Inserting an "effective" mass $m_c^{\rm eff} = 1.0\,\mathrm{GeV}$ into the tree level relation, the measured rate is reproduced.

Data are sparse for the wrong charm part, therefore we replace the unknown functions in Eqs. (37) and (38) by

$$\begin{aligned} F(v'Q, Q^2) &= f\ 2\pi\delta\left((Q - m_c v')^2\right), \\ \tilde{F}(v'Q, Q^2) &= \tilde{f}\ 2\pi\delta\left((Q - m_c v')^2\right), \end{aligned} \tag{50}$$

where f and $\tilde{f}$ are constants. This parametrization using a delta function is motivated by the negligible mass of the strange quark against which the two charmed quarks recoil. It can actually be checked by measuring the invariant mass $m_{X'}$ of the additional decay products in $B \to D\overline{D}X'$ which should turn out to be small. Setting $\tilde{f}$ to one, Eq. (38) reproduces the well-known parton model result. The fits yield

$$f = 0.147\,\mathrm{GeV} \qquad \text{and} \qquad \tilde{f} = 0.121. \tag{51}$$

Note that one naively expects $\tilde{f} = 1/8$ when assuming isospin invariance in $B \to D^* X$ and $B \to D_{\rm dir} X$. Under this assumption, we have two charge states and four spin states, hence in total eight states contributing.

For the semileptonic contributions, we use the results of [3],[1] but apply the same approximations as for the nonleptonic channels. We neglect the

[1] Note the sign error afflicting one term in [3], Eq. (43), which should read $E_V^{B^0\overline{D}^0}(y) = -\frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} \mathrm{Br}(D^{*-} \to \overline{D}^0 X) \frac{1}{2}(y^2-1)|X(y)|^2 + C_{18}(y,\Lambda)\frac{1}{N_C}$.

renormalization group improvement and find

$$G_{sl}(M^2) = \frac{G_F^2}{2}|V_{cb}|^2 \frac{1}{3\pi}(M^2 g_{\mu\nu} - M_\mu M_\nu)\Theta(M^2)\,\eta(vv') \qquad (52)$$
$$\mathrm{Tr}[\overline{H}_B(v)\gamma^\mu(1-\gamma_5)H_{\overline{D}^{(*)}}(v')]\ \mathrm{Tr}[\overline{H}_{\overline{D}^{(*)}}(v')\gamma^\nu(1-\gamma_5)H_B(v)].$$

The τ contribution follows from Eqs. (26)–(28) replacing V_{cs}, N_C and C_1 by one and the charm quark mass by the τ mass.

Table 1 shows the predictions for the total rates of these two ansätze for the wrong charm piece. For the numerical calculation, we used $|V_{cb}| = 0.04$.

6 Comparison with Data

Data on one-particle inclusive B decays are available from $\Upsilon(4S)$ machines and also from the CERN electron positron collider LEP. At the $\Upsilon(4S)$, the total production rates of $D^{(*)+}$ and $D^{(*)-}$ or $\overline{D}^{(*)0}$ and $D^{(*)0}$ are measured on the resonance, from which one can only deduce the rates for B admixture

$$\begin{aligned}\Gamma(B \to D^{(*)\pm}X) &= \frac{1}{2}\left[\Gamma(B^+ \to D^{(*)\pm}X) + \Gamma(B^0 \to D^{(*)\pm}X)\right],\\ \Gamma(B \to \overset{(-)}{D}{}^{(*)0}X) &= \frac{1}{2}\left[\Gamma(B^+ \to \overset{(-)}{D}{}^{(*)0}X) + \Gamma(B^0 \to \overset{(-)}{D}{}^{(*)0}X)\right].\end{aligned} \qquad (53)$$

Table 1 lists the available data. In the fourth row of the table we list the CLEO measurement of wrong charm decays $B \to DX$ which is again the average over B^+ and B^0.

Although we expect our method to work best near the nonrecoil point $vv' = 1$, we integrate the spectra in order to obtain total rates, assuming we can still get some insight into the bulk features. Since we calculate the right and wrong charm contributions separately, we have to add them in order to obtain the number of $\overset{(-)}{D}$ mesons produced in B meson decays. This procedure corresponds to the usual multiplicity definition of the branching ratio [11, p. 570].

The first observable one can study is the total number of $\overset{(-)}{D}$ mesons in B decays. This quantity is considerably smaller than the usually performed charm counting since it does not include the production of D_s mesons, charmed baryons and charmonium, the sum of which amounts to about 20% of the total charm production in B decays. Using the saturation assumption in Eq. (47), our model yields a total $\overset{(-)}{D}$ rate about 20% below the experimental result. However, Eq. (47) was taken from the analysis of the semileptonic

Mode	Model 1	Model 2	Experiment	
$B \to \overset{(-)}{D} X$	68.8%	68.8%	$(87.2 \pm 3.5)\%$	$\overset{(-)}{D}{}^0 + D^\pm$
$B \to \overset{(-)}{D}{}^* X$	51.8%	52.7%	$(48.7 \pm 3.1)\%$	$\overset{(-)}{D}{}^{*0} + D^{*\pm}$
$B \to \overset{(-)}{D}_{\rm dir} X$	17.1%	16.1%	$(38.5 \pm 4.7)\%$	$\overset{(-)}{D} - \overset{(-)}{D}{}^*$
$B \to D X$	7.9%	7.9%	$(7.9 \pm 2.2)\%$	[10] (input)
$B \to D^* X$	5.0%	5.9%		
$B \to \overset{(-)}{D}{}^0 X$	52.0%	52.3%	$(63.1 \pm 2.9)\%$	[11, 14]
$B \to D^\pm X$	16.8%	16.5%	$(24.1 \pm 1.9)\%$	[11, 14]
$B \to \overset{(-)}{D}{}^{*0} X$	25.9%	26.3%	$(26.0 \pm 2.7)\%$	[11, 14]
$B \to D^{*\pm} X$	25.9%	26.3%	$(22.7 \pm 1.6)\%$	[11, 14, 15]
$B \to \overset{(-)}{D}{}^0_{\rm dir} X$	8.5%	8.1%	$(21.7 \pm 4.1)\%$	$\overset{(-)}{D}{}^0 – \overset{(-)}{D}{}^{*0} – 0.68 D^{*\pm}$
$B \to D^\pm_{\rm dir} X$	8.5%	8.1%	$(16.8 \pm 2.9)\%$	$D^\pm - 0.32 D^{*\pm}$
$B \to D^- \ell^+ \nu_\ell X$	2.0%	2.0%	$(2.7 \pm 0.8)\%$	[11, 16]
$B \to \overline{D}{}^0 \ell^+ \nu_\ell X$	6.5%	6.5%	$(7.0 \pm 1.4)\%$	[11, 16]
$B \to D^{*-} \ell^+ \nu_\ell X$	3.3%	3.3%	$(2.8 \pm 0.4)\%$	[11, 9]
$B \to \overline{D}{}^{*0} \ell^+ \nu_\ell X$	3.3%	3.3%	$(3.2 \pm 0.7)\%$	[11, 9]
$B \to D^- \tau^+ \nu_\tau X$	0.6%	0.6%		
$B \to \overline{D}{}^0 \tau^+ \nu_\tau X$	2.0%	2.0%		
$B \to D^{*-} \tau^+ \nu_\tau X$	1.0%	1.0%		
$B \to \overline{D}{}^{*0} \tau^+ \nu_\tau X$	1.0%	1.0%		
$B^0 \to \overline{D}{}^0 X$	31.8%	31.8%		
$B^0 \to D^- X$	29.1%	29.1%		
$B^+ \to \overline{D}{}^0 X$	60.9%	60.9%		
$B^+ \to D^- X$	0%	0%		
$B^0 \to D^0 X$	5.7%	6.0%		
$B^0 \to D^+ X$	2.2%	1.9%		
$B^+ \to D^0 X$	5.7%	6.0%		
$B^+ \to D^+ X$	2.2%	1.9%		
$B^0 \to \overline{D}{}^{*0} X$	0%	0%		
$B^0 \to D^{*-} X$	46.8%	46.8%		
$B^+ \to \overline{D}{}^{*0} X$	46.8%	46.8%		
$B^+ \to D^{*-} X$	0%	0%		
$B^0 \to D^{*0} X$	2.5%	3.0%		
$B^0 \to D^{*+} X$	2.5%	3.0%		
$B^+ \to D^{*0} X$	2.5%	3.0%		
$B^+ \to D^{*+} X$	2.5%	3.0%		

Table 1: Comparison of our results with data. Model 1 uses Eq. (37), model 2 uses Eq. (38). Branching ratios are computed using $\tau_{B^+} = \tau_{B^0} = 1.55$ ps.

$r_S = \dfrac{\mathrm{Br}(B \to \overset{(-)}{D}{}^{*} X)}{\mathrm{Br}(B \to \overset{(-)}{D}_{\mathrm{dir}} X)}$	All channels	Semi-leptonic
Naive spin counting	3	3
Model 1 [see Eq. (37)]	3.04	3.36
Model 2 [see Eq. (38)]	3.26	
Fit all data using $\overset{(-)}{D}{}^{(*)0}$, $D^{(*)\pm}$	1.39±0.27	1.58±0.70
Data using $\overset{(-)}{D}{}^{*}$ and $\overset{(-)}{D}_{\mathrm{dir}}$ only	1.26±0.22	1.62±0.91
Data using $\overset{(-)}{D}{}^{0}$ and $D^{\pm}$ only	1.92±0.46	1.87±1.74

Table 2: Predicted and measured spin counting ratio r_S. For the last line and for the fit, the relation (46) between charge and spin counting has been used as a constraint. For the column "all channels," the error of the fit has been scaled by $\sqrt{\chi^2/N_f}$, where $\chi^2 = 4.2$ and $N_f = 2$.

decays where the $\overline{D}$ and $\overline{D}^*$ exclusive final states saturate only $(71 \pm 13)\%$ of the inclusive rate $B \to \overline{D}\ell^+\nu_\ell X$. Thus Eq. (47) should be replaced by

$$\eta(vv') = |\xi(vv')|^2 + \delta\eta(vv'), \tag{54}$$

where $\delta\eta$ accounts for the remaining contributions. Because we use factorization, this would enhance the nonleptonic channels by a similar amount and hence improve the prediction for the total number of $\overline{D}$ mesons.

Keeping this in mind, we still stick to the saturation assumption in Eq. (47) since $\delta\eta$ is unknown and the naive use of Eq. (47) reproduces the data on $B \to \overset{(-)}{D}{}^{*} X$ decays. This could be accidental, as the real problem is spin counting. The ratio of vector to direct pseudoscalar mesons r_S defined in Eq. (44) is expected to be about three, but experiment yields a ratio barely above one (see Table 2). Even in the semileptonic case, the experimental value is slightly lower than expected, although errors are large. Since the rates of pseudoscalar and vector $\overset{(-)}{D}$ mesons are connected by heavy quark spin symmetry, this large discrepancy is difficult to understand.

Although the effect is less pronounced than the spin counting problem, neutral $\overset{(-)}{D}$ mesons tend to occur slightly more often than expected, see Table 3. Assuming the measured value for the spin counting ratio r_S, the measured charge counting ratios r_Q are generally one or two standard deviations above expectations. If confirmed, this kind of effect would point to an additional contribution which could for instance arise from a nonfactorizing topology as depicted in Fig. 3. Nevertheless, current data are still consistent with the relation (46) between charge and spin counting, see the fit in Table 2.

Channel Model	Naive	1	2	Data
$\mathrm{Br}(B \to \overset{(-)}{D}{}^{*0} X)/\mathrm{Br}(B \to D^{*\pm} X)$	1	1	1	1.15±0.14
$\mathrm{Br}(B \to \overset{(-)}{D}{}^{0}_{\mathrm{dir}} X)/\mathrm{Br}(B \to D^{\pm}_{\mathrm{dir}} X)$	1	1	1	1.29±0.17
$\mathrm{Br}(B \to \overset{(-)}{D}{}^{0} X)/\mathrm{Br}(B \to D^{\pm} X)$	3	3.09	3.17	2.62±0.24
$\mathrm{Br}(B \to \overset{(-)}{D}{}^{*0} \ell\nu X)/\mathrm{Br}(B \to D^{*\pm} \ell\nu X)$	1	1		1.14±0.30
$\mathrm{Br}(B \to \overset{(-)}{D}{}^{0}_{\mathrm{dir}} \ell\nu X)/\mathrm{Br}(B \to D^{\pm}_{\mathrm{dir}} \ell\nu X)$	1	1		1.05±0.89
$\mathrm{Br}(B \to \overset{(-)}{D}{}^{0} \ell\nu X)/\mathrm{Br}(B \to D^{\pm} \ell\nu X)$	3	3.20		2.59±0.93

Table 3: Predicted and measured charge counting ratios r_Q. Using the measured value $r_S \approx 3/2$, naive charge counting yields $r_Q = 7/3$ instead of 3.

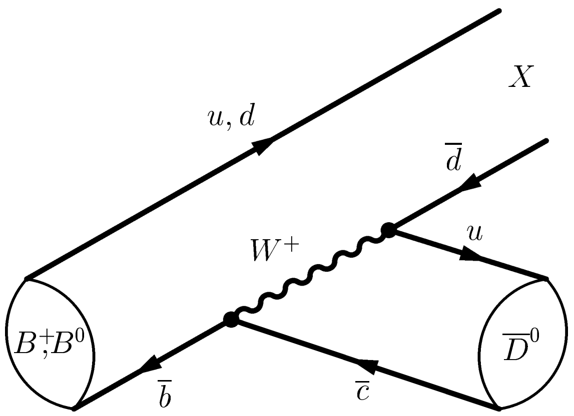

Figure 3: Nonfactorizing isospin-violating topology, supplying a possible explanation for an enhanced $\overline{D}^0$ rate.

Another piece of information for these decays are the $\overset{(-)}{D}$ momentum spectra. These have been measured by ARGUS [13] and CLEO [14], we use the data from CLEO which is more recent and more precise. The spectra are momentum distributions in the rest frame of the $\Upsilon(4S)$. However, the effect of the motion of the B mesons produces only a negligible smearing of these spectra, and we can safely ignore this effect here. In Fig. 4 we compare the data obtained by CLEO [14] with our theoretical prediction.

The spectra show that there is indeed a problem with the ratio of pseudoscalar to vector mesons. Using the saturation assumption in Eq. (47), the spectra of the vector mesons are described within experimental uncertainties and the problem appears with the low momentum region of the decay spectra for the pseudoscalar mesons. Our theoretical ansatz, especially the SDE, should work best in this region of small $\overset{(-)}{D}$ momentum. In addition, the shape of the spectra in this region is mainly determined by phase space, which yields a behavior proportional to p_D^2 for small p_D and constant matrix element. The steep rise of the momentum spectra for the pseudoscalar $\overset{(-)}{D}$ mesons is thus difficult to understand. An investigation of slow direct pseudoscalar $\overset{(-)}{D}$ mesons, i.e., those that do not originate from intermediate $\overset{(-)}{D}{}^*$ vector mesons, would be desirable.

This problem appears to be limited to nonleptonic decays. In Fig. 5 we compare the semileptonic $\overline{D}$ meson momentum spectrum measured by CLEO [10] with the theoretical predictions from [3] as well as with the sum of the first six contributing exclusive channels [17] and find no significant disagreement.

There is also a measurement of the wrong charm D spectrum by CLEO [10]. Unfortunately only four bins could be measured having still substantial uncertainties. In Fig. 6 we see that the parton-model inspired model 2 fits the data better than model 1, although evidence is not yet conclusive due to the quality of the data.

7 Conclusions

In this paper we have developed a QCD based description of one-particle inclusive decays of the type $B \to \overline{D}X$ and $B \to DX$. The method we suggest is based on the large-N_c limit of QCD, allowing us to factorize certain matrix elements. Once factorization has been performed, one can identify pieces in the rates which can be treated by a short distance expansion, assuming the bottom and the charm quarks to be heavy. This yields a series in inverse powers of the parameter $m_b - m_c$. The numerator of the expansion parameter is a typical QCD scale for the light degrees of freedom. Thus the expansion

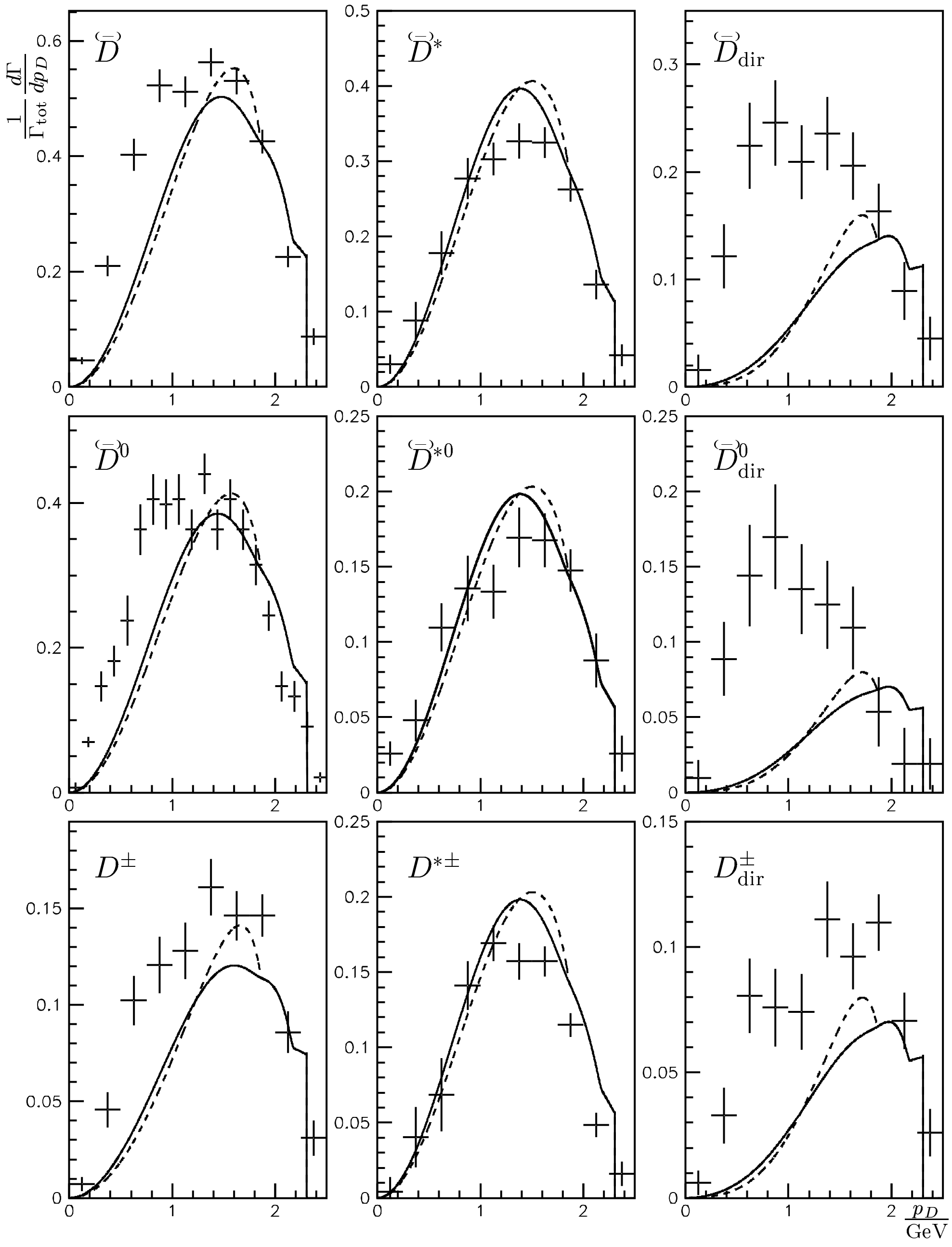

Figure 4: Momentum spectra $1/\Gamma_{\rm tot}\, d\Gamma/dp_D$ of $B \to \overset{(-)}{D}X$ in comparison with theoretical predictions. The solid line is model 2, the dashed one model 1. The columns refer to $\overset{(-)}{D}$, $\overset{(-)}{D}{}^*$ and direct $\overset{(-)}{D}$ mesons, the rows to a charge sum, neutral and charged $\overset{(-)}{D}{}^{(*)}$ mesons.

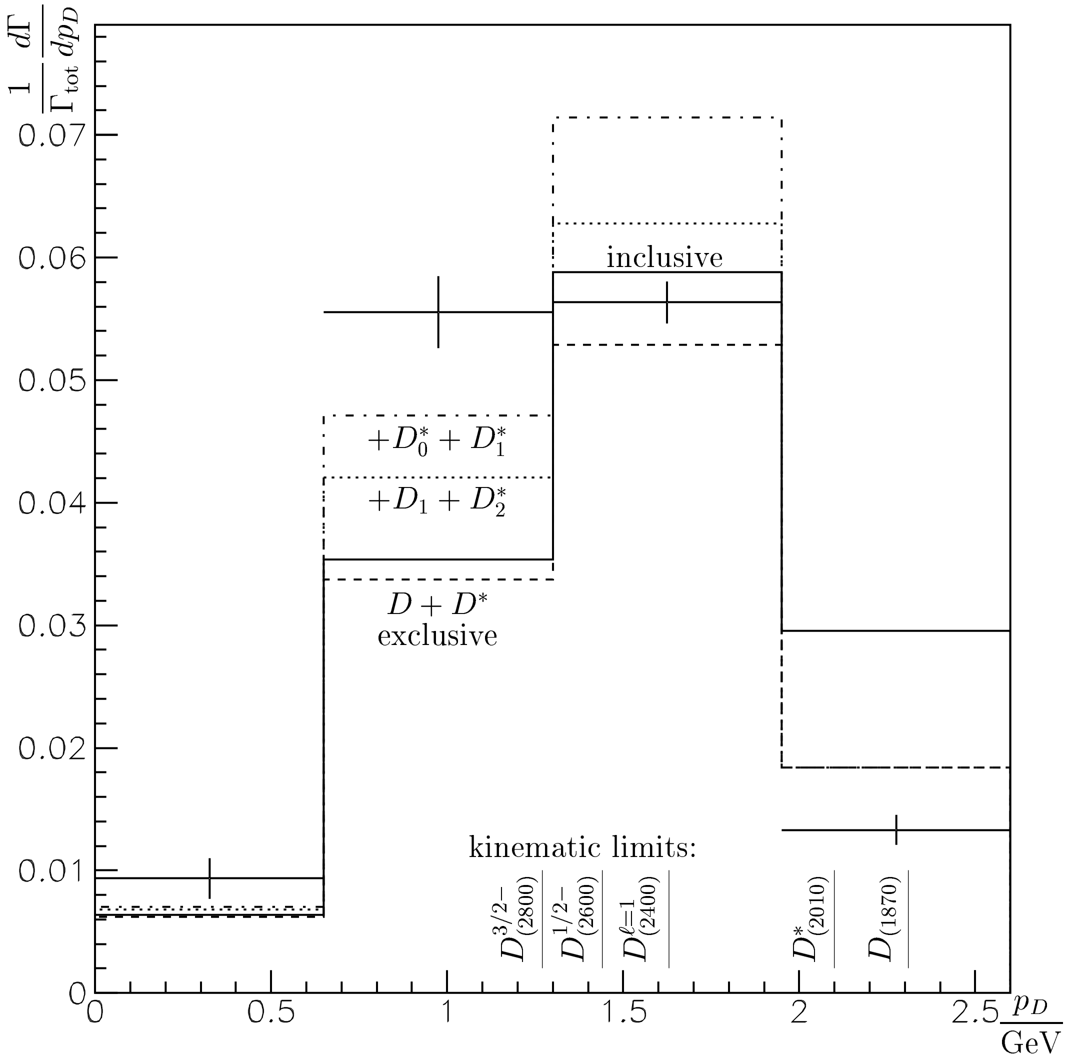

Figure 5: One-particle inclusive semileptonic $\overline{D}$ momentum spectrum in $B \to \overline{D}\ell^+\nu_\ell X$ measured by CLEO [10]. The solid line is the prediction of [3]; the dashed lines are sums of predictions for exclusive channels following [17].

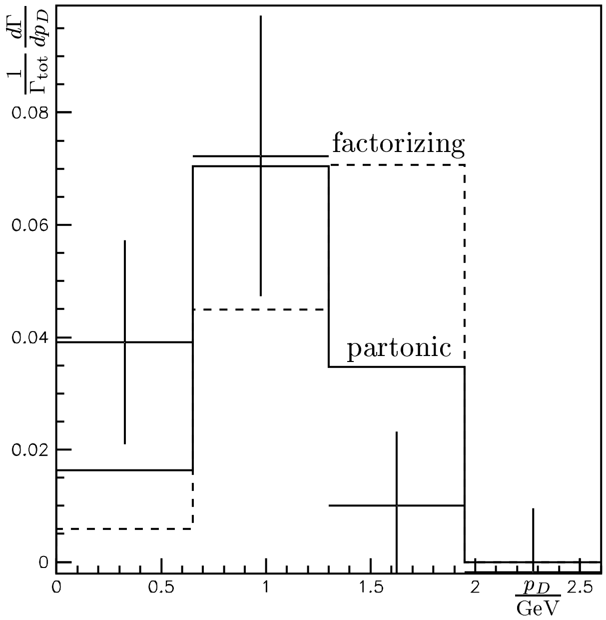

Figure 6: One-particle inclusive wrong charm D momentum spectrum in $B \to DX$ measured by CLEO [10] in comparison with theoretical predictions. The solid line is model 2; the dashed one model 1.

parameters are $\Lambda_{QCD}/(m_b - m_c)$, $1/N_C$ and $\alpha_s(m_c)$ and hence corrections to our calculation could be fairly large, in the worst case of the order of 30%.

We have studied the leading term of this expansion which still contains a number of unknown nonperturbative functions. These functions have to be parametrized. In the same way as in the semileptonic case, we reduce the number of functions appearing in the right charm contributions to a single nonperturbative function which can be related to the Isgur Wise function, once we assume that most of the rate is saturated by the decays into the two ground state mesons $\overset{(-)}{D}$ and $\overset{(-)}{D}{}^*$. For the wrong charm case we suggest two models corresponding to two different ways of contracting the spinor indices. Both models have a single nonperturbative form factor, which we adjust to the experimental wrong charm yield.

Although our method should work best close to the nonrecoil point, we also calculate total rates in order to discuss the total number of $\overset{(-)}{D}$ mesons in B decays and the spin and charge counting. While the well-known problem of spin counting, see, e.g., the review [18], is not solved by our ansatz, charge counting seems to work well, once we properly take the $D^* \to D$ decays into account.

The shapes of the decay spectra into vector mesons are already described quite satisfactorily, in particular in the low momentum region, while the $\overset{(-)}{D}$ meson spectra are off in this region. Furthermore, our model reproduces the normalization of the $\overset{(-)}{D}{}^*$ spectra, such that the spin counting problem manifests itself in a deficit of $\overset{(-)}{D}$ mesons. Since the experimental rates are above the theoretical ones, it would be interesting to investigate which exclusive channels contribute in the small momentum region.

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