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Sudakov logarithms in electroweak processes

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Abstract

The dominant electroweak double logarithmic corrections to the process $e^+e^- \to f\bar{f}$ at high energy are found in all orders of perturbation theory. In contrast to results in simple Yang-Mills theories these corrections do not exponentiate.

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The double logarithmic "Sudakov" corrections in Abelian [1] and non-Abelian [2] gauge theories are essentially determined by the infrared structure of the theory. They dominate in the limit of fixed-angle scattering when all the invariant energy and momentum transfers of the process become much larger than the typical mass scale of the particles running inside the loop.

In electroweak processes at energies far larger than M_Z or M_W the corresponding corrections due to the virtual Z (W) boson exchange scale like a power of $\frac{g^2}{16\pi^2}\log^2\left(\frac{s}{M^2_{Z,W}}\right)$ [3][1]. They grow rapidly with energy and become dominant in the TeV region available at the LHC or the Next Linear Collider.

In this paper we consider the dominant electroweak corrections to the process $e^+e^- \to f\bar{f}$ at high energy. In fact, the results are equally well applicable to any process $f'\bar{f}' \to f\bar{f}$ and we will therefore keep the discussion quite general throughout this paper. The one loop double logarithmic corrections to this process reach $\sim -10\%$ at $s \sim 1$ TeV^2 [4]. Similarly large negative corrections were observed in [5] for quark pair production in hadronic collisions. The two loop terms may well be comparable to the subleading one loop corrections [6] and therefore have to be taken into account to guaranty reasonable accuracy of the theoretical predictions. Though only the two loop corrections are of practical interest we give the general result for the double logarithmic contribution in an arbitrary order of perturbation theory.

In Born approximation the chiral amplitudes of the process $f'\bar{f}' \to f\bar{f}$ can be written in the form

$$M^B_{IJ} = \frac{ig^2}{s}\left(T^3_f T^3_{f'} + t^2_W \frac{Y_f Y_{f'}}{4}\right) \tag{1}$$

where $I,\ J = L,\ R$, $t_W = \tan\theta_W$, T^3_f is the isospin and Y_f is the hypercharge of the fermion.

[1]For the massless photon the corrections are infrared divergent and one has to take into account the real soft photon radiation to obtain an infrared safe cross section. We do not consider this effect and focus on the pure electroweak corrections.

We study the fixed-angle regime and restrict the analysis to the double logarithmic contributions which originate from the interplay between the soft and collinear singularities. All single logarithmic contributions from soft, collinear or ultraviolet divergencies are neglected.

The structure of the electroweak corrections depends crucially on the chirality of the fermions. Let us start with the non-Abelian interaction of left-handed fermions. It has been demonstrated [2, 7] that the leading infrared divergencies exponentiate for Yang-Mills theories and one obtains for each on-shell fermion line the suppression factor

$$\exp\left(-\frac{C_F}{2}L(s)\right) \tag{2}$$

where

$$L(s) = \frac{g^2}{16\pi^2}\log^2\left(\frac{s}{M^2}\right), \tag{3}$$

C_F is the Casimir operator ($C_F = 3/4$ for $SU(2)$) and M is the infrared regulator (the gauge boson mass). The external fermions are assumed to be in the fundamental representation. The fact that the double logarithmic corrections are determined by the external on-shell particles and do not depend on the details of the process has been proven in ref. [7] where the authors demonstrate that, in a physical (Coulomb or axial) gauge, the double logarithms originate from the self energy corrections to the external lines.

However, this result cannot be directly applied to the electroweak case because the Z boson, the mass eigenstate, is a mixture of A^3_μ, the pure $SU(2)_L$, and B_μ, the Abelian hypercharge components. Hence, it is the difference of the coupling of Z and W bosons to fermions, that renders eq. (2) invalid. Nevertheless, it is quite straightforward to obtain the one loop result for the case of interest using this equation. The lowest order double logarithmic contribution in the Coulomb (axial) gauge is determined by the diagram displayed in Fig. 1 [7, 8] and in the absence of mixing ($\theta_W = 0$) the correction reads

$$-\frac{C_F g^2}{32\pi^2}\log^2\left(\frac{s}{M^2}\right) \tag{4}$$

which is the first non-trivial term in the expansion of eq. (2) in g^2. The neutral Yang-Mills boson A^3_μ gives one third of this term. Due to the trivial structure of the first order correction one obtains the contribution of the Z boson by substituting the A^3_μ coupling gT^3_f by the Z boson coupling $g(T^3_f - s^2_W Q_f)/c_W$ in the expression for the A^3_μ contribution. Here $s_W = \sin\theta_W,\ c_W = \cos\theta_W$ and Q_f is the electric charge of the fermion. The contribution of the W boson amounts to two third of eq. (4). Thus in lowest order the double logarithmic corrections give the factor $1 - F^f_L$ for each pair of the incoming/outgoing left-handed fermion lines where

$$F^f_L = \left(\frac{1}{2} + \frac{1}{4c^2_W} + t^2_W\left(s^2_W Q^2_f - 2T^3_f Q_f\right)\right)L(s). \tag{5}$$

The case of the right-handed fermions is rather simple since their interaction to Z boson is Abelian. Therefore the double logarithms exponentiate to

$$\exp\left(-\frac{t^2_W s^2_W Q^2_f}{2}L(s)\right) \tag{6}$$

for each on-shell fermion leg. In first order this gives the factor $1 - F^f_R$ for each pair of the incoming/outgoing right-handed fermion lines where

$$F^f_R = t^2_W s^2_W Q^2_f L(s). \tag{7}$$

Thus the first order amplitudes in the double logarithm approximation take the form

$$M^{(1)}_{IJ} = M^B_{IJ}\left(1 - F^f_I - F^{f'}_J\right). \tag{8}$$

This result coincides with the more cumbersome expression obtained in ref. [4] by direct evaluating the Feynman diagrams in the eikonal approximation. Note that in [4] the covariant gauge was used for the calculation and the double logarithms originate from both vertex and box diagrams. Anyhow, one loop corrections are known exactly [6] so the first order result in double logarithmic approximation is of no specific interest.

Let us now turn to the two loop analysis. For the right-handed fermions the correction is determined by the second term of the expansion of eq. (6) in g^2. For the left-handed fermions the situation is less trivial. In this order the contribution from the Z boson cannot be obtained by changing the coupling in the contribution from the A^3_μ boson to eq. (2) since exponentiation breaks down explicitly. To see this let us consider the two "rainbow" diagrams in Fig. 2 which give part of the second order double logarithmic contribution [9]. The rest is determined by similar diagrams with two Z or two W bosons in the loops. In the absence of mixing diagrams (a) and (b) give the same result. For non-zero θ_W this is not true due to the presence of the electric charge in the Z boson coupling to fermions. Moreover, the result for the diagram (a) can be obtained from the corresponding $\theta_W = 0$ diagram by the same change of the Z boson coupling $gT^3_f \to g(T^3_f - s^2_W Q_f)/c_W$ as used in the analysis of the first order corrections while the diagram (b) cannot and therefore is non-exponential. In fact, diagram (b) is the only source of the non-exponential corrections in the second order. Writing the sum $(a)+(b)$ as $2\cdot(a)+((b)-(a))$ one finds that $2\cdot(a)$ contribution corresponds to a part of the second term in the expansion of the exponent

$$1 - F^f_L/2 + \frac{\left(F^f_L/2\right)^2}{2!} + \ldots .$$

In this way we find that for each pair of the incoming/outgoing left-handed fermion lines the leading two loop correction reads:

$$\frac{1}{2!} {F^f_L}^2 + \Delta_f \tag{9}$$

where the first term arises from the expansion of the exponent and

$$\Delta_f = \frac{t^2_W}{8} \left(\left(s^2_W Q^2_{\tilde{f}} - 2T^3_{\tilde{f}} Q_{\tilde{f}} \right) - \left(s^2_W Q^2_f - 2T^3_f Q_f \right) \right) \tag{10}$$

is twice the difference $(b) - (a)$. Thus, in two loop approximation the leading logarithmic approximations of the chiral amplitudes read as follows:

$$M_{IJ}^{(2)} = M_{IJ}^{B} \left(1 - \left(F_I^f + F_J^{f'}\right) + \frac{1}{2!}\left(F_I^f + F_J^{f'}\right)^2 + \delta_{IL}\Delta_f + \delta_{JL}\Delta_{f'}\right) \qquad (11)$$

where

$$\delta_{JL} = \begin{cases} 0, & J = R, \\ 1, & J = L \end{cases}$$

Now it is straitforward to obtain the general double logarithmic contribution from the corresponding "rainbow" diagrams. In nth order instead of the term

$$\frac{(-1)^n}{n!}\left(\frac{C_F}{2}L(s)\right)^n \qquad (12)$$

of the expansion of the exponent (2) we get

$$\frac{(-1)^n}{n!}\left(\frac{C_F x_f L(s)}{6}\right)^n \left(1 + \sum_{m=1}^{n}\sum_{k=0}^{n-m} 2^m C_{n-m+l-k}^{l} C_{m-l-1+k}^{k} \frac{x_{\tilde{f}}^k}{x_f^{k+m}}\right) \qquad (13)$$

where C_i^j are binomial coefficients, l is the integer part of $m/2$ and

$$x_f = 4\left(\frac{T_f^3 - s_W^2 Q_f}{c_W}\right)^2.$$

For $\theta_W = 0$ ($x_f = x_{\tilde{f}} = 1/4$) eq. (13) is reduced to eq. (12). Note that the non-exponential contribution is suppressed by the small quantity $s_W^2 \sim 0.23$ which can be considered as an additional expansion parameter. This would essentially simplify the analysis of the subleading logarithms because in the leading order in s_W^2 the results for the simple Yang-Mills theories [10] are applicable. The exponential form of the corrections can be restored by adding a (hypothetical) contribution of the heavy photon of mass M (we leave aside the Higgs mechanism since the problem of mass generation is irrelevant for the analysis of the infrared properties of the theory). In this case one would find

$$\exp\left(-\frac{C_F + t_W^2 Y_f^2/4}{2}L(s)\right) \qquad (14)$$

instead of eq. (2). The exponentiation holds also for the massless photon if considered separately. So the reason of the absence of exponentiation for the electroweak double logarithmic contributions is twofold:

i) The mass eigenstates, namely photon and Z boson, have no definite gauge transformation properties.

ii) The photon is massless and has to be treated separately with real radiation being taken into account, e.g. in a completely inclusive manner.

The chiral amplitudes determine the differential cross section of the process

$$\frac{d\sigma}{d\Omega} = \frac{N_c^{(f)} s}{(16\pi)^2} \left(\left(|M_{LL}|^2 + |M_{RR}|^2 \right) (1+\cos\theta)^2 + \left(|M_{LR}|^2 + |M_{RL}|^2 \right) (1-\cos\theta)^2 \right)$$

where $N_c^{(f)}$ is 3 for quarks and 1 for leptons. With the expression for the differential cross section at hands we can compute the leading logarithmic corrections to the basic observables for $e^+e^- \to f\bar{f}$.

In the fixed-angle regime and in the leading logarithmic approximation, the invariants s and t are not distinguished since the difference is subleading. Therefore we can integrate the differential cross section over all angles to find the leading result for the total cross sections and for asymmetries, though formally the double logarithmic approximation is not valid for the small angles $\theta < M/\sqrt{s}$. These corrections also do not depend on the choice of mass, used as the scale M. Since the difference between M_W and M_Z is subleading, $M = M_W$ is used throughout. In the two loop approximation we find:

$$\begin{aligned} \sigma/\sigma_B(e^+e^- \to Q\bar{Q}) &= 1 - 1.659L(s) + 1.992L^2(s) \\ \sigma/\sigma_B(e^+e^- \to q\bar{q}) &= 1 - 2.173L(s) + 2.826L^2(s) \\ \sigma/\sigma_B(e^+e^- \to \mu^+\mu^-) &= 1 - 1.392L(s) + 1.495L^2(s) \end{aligned} \quad (15)$$

where $Q = u, c, t,\ q = d, s, b$. Numerically $L(s) = 0.07$ and 0.11 for $\sqrt{s} = 1$ TeV and 2 TeV respectively. Here $M = M_W$ has been choosen for the infrared cutoff and

$g^2/16\pi^2 = 2.7 \cdot 10^{-3}$ for the $SU(2)$ coupling evaluated at $\sqrt{s} = 1$ TeV. Clearly, for energies at 1 and 2 TeV the two loop corrections are huge and amount up to 1% and 4% respectively.

For the forward-backward asymmetry (the difference of the cross section averaged over forward and backward semispheres in respect to the electron beam direction divided by the total cross section) we get

$$\begin{aligned} A_{FB}/A^B_{FB}(e^+e^- \to Q\bar{Q}) &= 1 - 0.090L(s) + 0.120L^2(s) \\ A_{FB}/A^B_{FB}(e^+e^- \to q\bar{q}) &= 1 - 0.140L(s) + 0.024L^2(s) \\ A_{FB}/A^B_{FB}(e^+e^- \to \mu^+\mu^-) &= 1 - 0.039L(s) + 0.281L^2(s) \end{aligned} \tag{16}$$

For the left-right asymmetry (the difference of the cross sections of the left and right particles production divided by the total cross section) we obtain

$$\begin{aligned} A_{LR}/A^B_{LR}(e^+e^- \to Q\bar{Q}) &= 1 - 1.129L(s) + 0.821L^2(s) \\ A_{LR}/A^B_{LR}(e^+e^- \to q\bar{q}) &= 1 - 4.551L(s) + 1.123L^2(s) \\ A_{LR}/A^B_{LR}(e^+e^- \to \mu^+\mu^-) &= 1 - 13.744L(s) + 0.399L^2(s) \end{aligned} \tag{17}$$

For the loops with a top quark running inside one may replace the square of logarithm in eq. (3) by

$$\log^2\left(\frac{s}{m_t^2}\right) + 4\log\left(\frac{s}{m_t^2}\right)\log\left(\frac{m_t}{M}\right), \tag{18}$$

a form valid with logarithmic accuracy for $m_t >> M$. Clearly, the difference between logarithm factors in eq. (3) and eq. (18) is also subleading. Numerically various definitions of $L(s)$ differ by $\sim 10\%$ at $s \sim 1$ TeV^2 *i.e.* the uncertainty is of the order of the generic non-enhanced electroweak corrections in each order. For the physical applications this is important in one loop approximation. Fortunately the first order corrections are known exactly beyond the double logarithmic approximation. At the same time this difference is small in the two loop order double logarithmic corrections which are of the main interest because they are supposed to dominate the (still unknown) total two-loop electroweak corrections.

To conclude, we have found the dominant double logarithmic electroweak Sudakov corrections to the process $e^+e^- \to f\bar{f}$ at high energy in all orders of perturbation theory. In contrast to results in simple Yang-Mills theories these corrections do not exponentiate. The explicit expression for the two loop corrections to the total cross sections and asymmetries has been obtained. These corrections reach a few percents size in the TeV region and are crucial for high precision tests of the electroweak model at future colliders.

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Figure captions

Fig. 1. Self-energy correction determining the first order double logarithm contribution in the Coulomb or axial gauge.

Fig. 2. "Rainbow" diagrams giving a part of the second order double logarithm contribution. The bold line corresponds to the fermion isospin partner $\tilde{f}$. The fact that the diagram (b) is not equal to the diagram (a) destroys exponentiation.

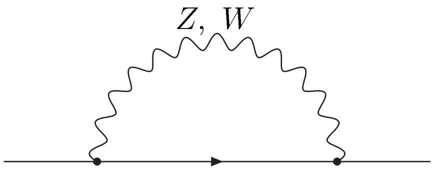

Fig. 1

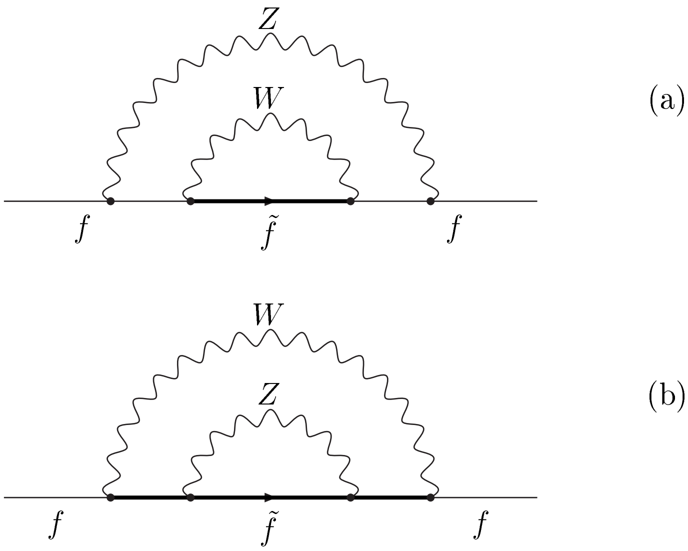

Fig. 2