Virtual annihilation contribution to orthopositronium decay rate

Gregory Adkins*
Franklin and Marshall College, Lancaster, PA 17604

Kirill Melnikov†
Institut für Theoretische Teilchenphysik,
Universität Karlsruhe, D–76128 Karlsruhe, Germany

Alexander Yelkhovsky‡
Budker Institute for Nuclear Physics,
Novosibirsk, 630090, Russia

(May 1999)

Order α^2 contribution to the orthopositronium decay rate due to one-photon virtual annihilation is found to be $\delta_{\rm ann}\Gamma^{(2)} = \left(\frac{\alpha}{\pi}\right)^2 \left(\pi^2 \ln\alpha - 0.8622(9)\right)\Gamma_{\rm LO}$.

PACS numbers: 36.10.Dr, 06.20.Jr, 12.20.Ds, 31.30.Jv

Positronium, the bound state of an electron and positron, is an excellent laboratory to test our understanding of Quantum Electrodynamics of bound states. Although in the majority of cases the agreement between theory and experiment is very good, the case of orthopositronium (o-Ps) decay into three photons is outstanding, since the theoretical predictions differ by about 6 standard deviations from the most accurate experimental result [1] (see, however, an alternative result in [2]). Provided that the experiment [1] is correct, the theory can only be rescued if the second order correction to the o-Ps lifetime turns out to be $\sim 250(\alpha/\pi)^2\Gamma_{\rm LO}$.

It is however difficult to imagine how such a large number could appear in the perturbative calculations, even if the bound state is involved. This point of view is supported by a recent complete calculation of the $\mathcal{O}(\alpha^2)$ correction to the parapositronium (p-Ps) decay rate into two photons [3]. It has shown that the "natural scale" of the gauge-invariant contributions is [several units]$\times(\alpha/\pi)^2\Gamma_{\rm LO}$. For this reason, it was conjectured in [3] that the $\mathcal{O}(\alpha^2)$ correction to the orthopositronium decay rate o-Ps $\to 3\gamma$ most likely is of the same order of magnitude.

At first sight, the result of Ref. [4],

$$\delta_{\rm ann}\Gamma^{(2)} = \left(\frac{\alpha}{\pi}\right)^2 \left(\pi^2 \ln\alpha + 9.0074(9)\right)\Gamma_{\rm LO}, \qquad (1)$$

for the gauge-invariant contribution to the o-Ps$\to 3\gamma$ decay rate induced by the single-photon virtual annihilation, does not provide much support for this conjecture. In fact, the value of the non-logarithmic constant in Eq.(1) is larger by approximately one order of magnitude than the values of coefficients in gauge-invariant contributions to p-Ps$\to 2\gamma$ decay rate.

In this note we would like to point out that the result for the second order correction to virtual annihilation contribution given in Eq.(1) is incomplete, in that closely related contributions should be included as well. It turns out that if the missing pieces are added to Eq.(1), then the complete result for $\delta_{\rm ann}\Gamma^{(2)}$ decreases and is in accord with the expectations advocated in Ref. [3].

We recall, that in bound state calculations there are two different types of contributions. The hard corrections arise as contributions of virtual photons with momenta $k \sim m$. These contributions renormalize local operators in the non-relativistic Hamiltonian. For this reason they can be computed without any reference to the bound state.

On the contrary, the soft scale contributions come from a typical momenta scale $k \sim m\alpha$ in virtual loops, and for this reason are sensitive to the bound state dynamics. For $\delta_{\rm ann}\Gamma^{(2)}$, it is easy to see that the soft scale contribution reads:

$$\delta_{\rm ann}^{(\rm soft)}\Gamma^{(2)} = \frac{4\pi\alpha}{m^2}G(0,0), \qquad (2)$$

where

$$G(r,r') = {\sum_n}' \frac{|n(r)\rangle\langle n(r')|}{E-E_n}$$

is the reduced Green function of the Schrödinger equation in the Coulomb field.

Let us write the expansion of the Green function in a series over the Coulomb potential:

$$G(0,0) = G_0(0,0) + G_1(0,0) + G_{\rm multi}(0,0). \qquad (3)$$

The first two terms in this expansion are divergent and require regularization. If we use dimensional regularization, then the $G_0(0,0)$ piece delivers a finite contribution, since it has only a power divergence. The second term $G_1(0,0)$ is logarithmically divergent. It can be easily seen

*e-mail: g_adkins@acad.fandm.edu
†e-mail: melnikov@particle.physik.uni-karlsruhe.de
‡e-mail: yelkhovsky@inp.nsk.su

that just this term was accounted for in the calculation of Ref. [4], and it is precisely the term that delivers the $\ln\alpha$ in Eq.(1). However, the contributions of G_0 and $G_{\rm multi}$ were not calculated there.

Both additional terms can be easily extracted from Ref. [5]. We then obtain

$$\delta_0\Gamma = \frac{4\pi\alpha}{m^2} G_0(0,0) = \frac{1}{2}\alpha^2\Gamma_{\rm LO} \qquad (4)$$

and

$$\delta_{\rm multi}\Gamma = \frac{4\pi\alpha}{m^2}\sum_{n=2}^{\infty} G_n(0,0) = -\frac{3}{2}\alpha^2\Gamma_{\rm LO}, \qquad (5)$$

consistent with the results of Ref. [6].

If we now add Eqs.(4), (5) and (1), we obtain the complete $\mathcal{O}(\alpha^2)$ correction to the o-Ps$\to 3\gamma$ decay rate due to single-photon virtual annihilation:

$$\delta_{\rm ann}\Gamma^{(2)} = \left(\frac{\alpha}{\pi}\right)^2 \left\{\pi^2\ln\alpha - 0.8622(9)\right\}\Gamma_{\rm LO}. \qquad (6)$$

One sees that the non-logarithmic contribution is in fact of order one times $(\alpha/\pi)^2\Gamma_{\rm LO}$, in accord with the conjecture in Ref. [3]. Its value is similar in magnitude to the known results for other gauge-invariant $\mathcal{O}(\alpha^2)$ corrections to the decay rate of orthopositronium [6,7].

The only known exception from this "rule" is provided by the square of the $\mathcal{O}(\alpha)$ corrections to the o-Ps decay amplitude. This (gauge-invariant) contribution has an anomalously large coefficient: $28.860(2)(\alpha/\pi)^2\Gamma_{\rm LO}$ [8]. In this respect, we would like to note that there may be an enhancement factor due to a larger (by about a factor of 3) number of diagrams contributing to o-Ps decay as compared to p-Ps decay. This enhancement is seen already in the magnitude of the $\mathcal{O}(\alpha)$ corrections and translates naturally to the large value of the $\mathcal{O}(\alpha^2)$ contribution originating from the square of the $\mathcal{O}(\alpha)$ corrections to the o-Ps decay amplitude. However, unless this enhancement is dramatic, it is hard to believe that this fact alone can explain the discrepancy between theoretical and experimental results on o-Ps decay rate.

This research was supported in part by the National Science Foundation under grant number PHY-9722074, by BMBF under grant number BMBF-057KA92P, by Graduiertenkolleg "Teilchenphysik" at the University of Karlsruhe, by the Russian Foundation for Basic Research under grant number 99-02-17135 and by the Russian Ministry of Higher Education.

[1] J. S. Nico, D. W. Gidley, A. Rich, and P. W. Zitzewitz, Phys. Rev. Lett. **65**, 1344 (1990).

[2] S. Asai, S. Orito, and N. Shinohara, Phys. Lett. **B357**, 475 (1995).

[3] A. Czarnecki, K. Melnikov, and A. Yelkhovsky, hep-ph/9904478.

[4] G.S. Adkins and M. Lymberopoulos, Phys. Rev. **A 51**, R875 (1995); 2908 (1995).

[5] A. Czarnecki, K. Melnikov, and A. Yelkhovsky, hep-ph/9901394, in press in Phys. Rev. **A**.

[6] S. G. Karshenboim, Phys. Atom. Nucl. **56**, 1710 (1993).

[7] A. P. Burichenko and D. Y. Ivanov, Phys. Atom. Nucl. **58**, 832 (1995);
G. S. Adkins and Y. Shiferaw, Phys. Rev. **A52**, 2442 (1995).

[8] A. P. Burichenko, Phys. Atom. Nucl. **56**, 640 (1993);
G. S. Adkins, Phys. Rev. Lett. **76**, 4903 (1996).