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Measuring $\sigma(e^+e^- \to \mathrm{hadrons})$ using tagged photon

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Abstract

We propose to use events with radiated photons in e^+e^- collisions to measure the total cross section of $e^+e^- \to$ hadrons as a function of the center of mass energy. The Monte Carlo simulation for the collider DAPHNE shows that a competitive accuracy can be achieved with this method.

I. INTRODUCTION

During the past years electroweak precision measurements have become one of the central issues in particle physics. The indirect determination of the mass of the top quark through its impact on quantum corrections prior to its observation in proton-antiproton collisions at the TEVATRON can be considered as one of the triumphs of the present theoretical framework. Similarly the indirect determination of the mass of the W boson is in perfect agreement with the present measurements at LEP and the TEVATRON, which motivates a combined fit to the parameters of the Standard Model.

Recently the measurements have become sufficiently precise to even give first indications for a relatively small mass of the Higgs boson with an upper limit of around 200 GeV. Significantly improved measurements, in particularly of M_W and the left-right asymmetry with polarized beams at the SLC are expected to come in the next couple of years which might lead to a fairly precise indirect determination of the Higgs mass M_H, thus repeating the success of the top quark mass determination.

An important ingredient in all these fits is the fine structure constant at the mass of the Z boson. Its running from the Thompson limit to the high scale M_Z is largely determined by the hadronic contribution to the vacuum polarization which in turn can be expressed via dispersion relation by $\sigma_{\rm had}$, the cross section for $e^+e^- \to$ hadrons . At present the dominant uncertainty in the SM fits is due to the fairly large experimental error in $\sigma_{\rm had}$ if the analysis is based on data only. Also the theoretical interpretation of future, improved determinations of the muon anomalous magnetic moment is affected by the limited knowledge of the hadronic contributions. Significant progress can only be achieved through an improved determination

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of the cross section, either through substitution of inadequate data by theoretical predictions based on perturbative QCD, wherever applicable, or through improved measurements of $\sigma_{\rm had}$ over a wide energy range, or by a combination of both.

All present, data based evaluations make use of a large variety of experiments at different accelerators, a consequence of the large energy range to be spanned by the dispersion integral. Some of the data points, e.g. around the ρ resonance, are now extremely accurate, others, e.g. in the region between 2 and 3 GeV have large errors and are only marginally consistent with the predictions based on pQCD. While a highly precise scan through the whole region would certainly be desirable, one may, as a viable alternative, exploit the high luminosity of oncoming e^+e^- machines which operate at fixed energy and use the radiative return to lower energies. This would lead to an improved determination of $\sigma_{\rm had}$ over a large energy range. The feasibility of this approach is the subject of this work.

Similar considerations can be found in [1–4]. However, in most of these papers only initial state radiation is considered which is a priori legitimate for very small angles. In fact, Ref. [3] uses explicitly the small angle approximation and includes higher order corrections within the same approximation. This is an attractive option *if* photon detection at very small angles is experimentally feasible. Unfortunately, this seems to be not the case for the larger part of the oncoming high luminosity e^+e^- machines. In the present paper we consider photons at somewhat larger angles, $\theta_\gamma > 7^\circ$ and include initial and final state radiation and incorporate in addition collinear initial state radiation through the structure function technique.

For the present purpose it is important to separate contributions from initial and final state radiation. Three techniques can be envisaged:

i) Initial state radiation strongly dominates in events with photons emitted at small angles relative to the beam axis. Imposing at the same time an angular separation between photon and pions decreases the contribution of the final state radiation even further;

ii) the forward-backward asymmetry of the pions which originates from the interference between initial and final state radiation can be measured and used to test the underlying model for final state radiation;

iii) the angular distribution of photons from final state radiation involves terms proportional to $\sin^2\theta_\gamma$ and $\cos^2\theta_\gamma$ only and is distinctly different from the one of initial state radiation with its strong peaking at small angles. By defining suitable projectors it may be possible to distinguish the two components.

As a particular example we consider here the case of collider DAPHNE at Frascati which would operate on ϕ resonance; the total energy of the collision therefore being equal $\sqrt{s} = 1.02$ GeV. The physical program of KLOE collaboration is focused on the precision study of the CP violation in kaon system. However, the events with radiative return to lower energies will be in quantity there. These events can be either considered as a background to an interesting physics or utilized to study the cross section $e^+e^- \to$ hadrons at lower energies. In what follows we show that the second option appears to be rather realistic. The studies presented here are based on the Monte Carlo event generator [5] that has been written for this purpose and which, in addition, should be useful for the analysis of the data.

			Event rates		
Collider	$\sqrt{s}$	Annual luminosity, fb^{-1}	$\theta_{\min} = 5^\circ$	$\theta_{\min} = 7^\circ$	$\theta_{\min} = 10^\circ$
DAPHNE	1.02	1.35	$18 \cdot 10^6$	$16 \cdot 10^6$	$14 \cdot 10^6$
$B-$ factory	10.6	100	$4 \cdot 10^6$	$3.5 \cdot 10^6$	$3 \cdot 10^6$
LEP1	92	0.24	125	109	93
LEP2	183	0.2	27	24	20

TABLE I. Estimated number of radiative events $e^+e^- \to \pi^+\pi^-\gamma$ for different center of mass energies from Eq.(1). The cut on the photon energy is 0.1 GeV. To calculate the annual luminosity we used 1 year = 10^7 sec. For LEP2 we used accumulated luminosity for the 1997 run with at $\sqrt{s} = 183$ GeV.

II. BASICS OF THEORETICAL CONSIDERATION

Before we start with a more detailed discussion of the two pion case at DAPHNE, let us present the order of magnitude of the event rates for a few characteristic configurations. The differential cross section for radiative events with $\theta_{\min} < \theta < 180^\circ - \theta_{\min}$ is given by[1]:

$$Q^2 \frac{d\sigma}{dQ^2} = \frac{4\alpha^3}{3s} R(Q^2) \left\{ \frac{(s^2 + Q^4)}{s(s - Q^2)} \log \frac{1 + \cos\theta_{\min}}{1 - \cos\theta_{\min}} - \frac{(s - Q^2)}{s} \cos\theta_{\min} \right\}. \quad (1)$$

Here Q^2 is the invariant mass of the hadronic system and $R(Q^2)$ is $\sigma(e^+e^- \to \text{hadrons})/\sigma_{\text{point}}$. The value of $Q^2 = s - 2\sqrt{s}\omega_\gamma$ is fixed by ω_γ, the energy of the photon. It is important to note that the function in the curly brackets in Eq.(1) is flat for $Q^2 \ll s$. This is similar to the weight function in the dispersive integral for $\alpha(M_Z)$ and should facilitate the experimental determination of this important quantity. We will return to this issue below.

As an illustrative example we consider the $\pi^+\pi^-\gamma$ final state, a photon angular cut of $5^\circ, 7^\circ, 10^\circ$, a realistic model for the pion form factor [7] and the typical parameters of four colliders: DAPHNE, B-factory, LEP and LEP2. The respective beam energies, the luminosities and the event rates as obtained using Eq.(1), are listed in Table 1.

Another interesting class of events are those with the invariant mass of the hadronic system between 1.5 and 3.5 GeV. The R-ratio in this region is very poorly known and even a measurement with an accuracy of 5% would be considered a significant improvement. On one hand, no collider will operate in this region in the near future and, on the other hand, the R ratio could be well determined with this method at a B-factory. In Table 2 we list typical event rates, adopting for simplicity a constant R value of 2.

It is clear from these tables that large event rates are at hand, even for severe cuts, which confine the photon to the region of relatively large angles. The main issues are the control

[1] Only initial state radiation is taken into account.

	$\theta_{\rm min} = 5°$	$\theta_{\rm min} = 7°$	$\theta_{\rm min} = 10°$
$1.5 \le \sqrt{Q^2} \le 2$	$11 \cdot 10^5$	$9.9 \cdot 10^5$	$8.4 \cdot 10^5$
$2 \le \sqrt{Q^2} \le 2.5$	$9 \cdot 10^5$	$7.9 \cdot 10^5$	$6.7 \cdot 10^5$
$2.5 \le \sqrt{Q^2} \le 3$	$7.6 \cdot 10^5$	$6.6 \cdot 10^5$	$5.6 \cdot 10^5$
$3 \le \sqrt{Q^2} \le 3.5$	$6.7 \cdot 10^5$	$5.8 \cdot 10^5$	$5.0 \cdot 10^5$

TABLE II. Estimated number of radiative events $e^+e^- \to \text{hadrons} + \gamma$ at B-factories for one year of running ($\sqrt{s} = 10.6$ GeV, integrated luminosity 100 fb^{-1}). We use $R = 2$ in the interval $1.5 \le \sqrt{Q^2} \le 3.5$.

of final state radiation and radiative corrections. These questions will now be studied in more detail for the $\pi^+\pi^-\gamma$ state and for the KLOE experiment.

For $\sqrt{s} \le 1$ GeV the dominant hadronic final state produced in e^+e^- annihilation consists of a pair of charged pions. For this reason we limit the discussion to the reaction $e^+e^- \to \pi^+\pi^-\gamma$ and demonstrate below that by measuring the momenta of the final state particles at fixed center of mass energy, one can extract accurate information about the cross section of $e^+e^- \to \pi^+\pi^-$ over a wide range of energy.

In the reaction $e^+e^- \to \pi^+\pi^-\gamma$ the photon can be emitted either by the electron or positron (ISR) or by the pions in the final state (FSR). The corresponding amplitude reads:

$$\mathcal{M} = \mathcal{M}_{\rm ISR} + \mathcal{M}_{\rm FSR}. \tag{2}$$

The calculation of ISR is a straightforward application of the Feynman rules for QED. In contrast, with the pions not being elementary, the calculation of FSR is in general more tricky. In the current version of our Monte Carlo program, we consider pions as point-like particles as far as FSR is concerned and use the standard Feynman rules of scalar QED. Being definitely not rigorous, this approach should provide a reasonable estimate of FSR for photons emitted at small angles relative to pions and an upper limit for photon emission at large angles. Moreover, below we present a useful tool to control the accuracy of this approximation. Final state radiation through the radiative decay $\phi \to \pi^+\pi^-\gamma$ is not included in the analysis (for a recent discussion see [6] and references therein). It can be considered as a special case of FSR and we expect that this effect can also be controlled through its interference with the dominant amplitude.

The interaction of the virtual photon with the $\pi^+\pi^-$ system is described by a pion form factor $F_\pi(Q^2)$, where Q^2 is the virtuality of the photon. A convenient parameterization of this form factor was proposed in Ref. [7]. The pion form factor is described by a sum of the Breit-Wigner resonances with a Q^2 dependent width:

$$F_\pi(Q^2) = \frac{BW_\rho \frac{(1+\alpha BW_\omega)}{1+\alpha} + \beta BW_{\rho'} + \gamma BW_{\rho''}}{1+\beta+\gamma};$$

$$BW_\rho(Q^2) = \frac{m_\rho^2}{m_\rho^2 - Q^2 - i\sqrt{Q^2}\Gamma_\rho(Q^2)}. \quad (3)$$

In the above equation α, β, γ are the parameters of the model (see Ref. [7]). For $Q^2 = 0$ one obtains $F_\pi(0) = 1$; that ensures the proper charge of a pion. In what follows we shall use this parameterization.

In general, by measuring the photon energy ω_γ one determines the invariant mass of the pions. Consider now the square of the ISR amplitude. In this case the differential cross section is evidently proportional to the pion form factor squared at the proper momentum scale:

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}\right)_{\mathrm{ISR}} \sim |F_\pi(Q^2)|^2 \sim \sigma_{e^+e^-\to\pi^+\pi^-}(Q^2). \quad (4)$$

A different situation occurs when the photon is emitted from the final state. In this case the differential cross section is proportional to

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2}\right)_{\mathrm{FSR}} \sim |F_\pi(s)|^2 \neq \sigma_{e^+e^-\to\pi^+\pi^-}(Q^2). \quad (5)$$

Clearly, in this case measuring the energy of the final state photon would not help in determining the proper energy of the collision and therefore the cross section of $e^+e^- \to \pi^+\pi^-$. For this reason FSR and ISR/FSR interference should be regarded as a "background" and the contribution due to ISR as a "signal". It is definitely not possible to "switch off" FSR completely , hence FSR and the interference will lead to a systematic error in the determination of $\sigma(e^+e^- \to \pi^+\pi^-)$ from $\sigma(e^+e^- \to \pi^+\pi^-\gamma)$. The choice of a proper selection criterion is thus the only possibility to suppress the contribution of FSR. In fact, the DAPHNE luminosity is so high, that even after imposing quite severe cuts, the number of radiative events remains huge, rendering the statistical accuracy of the measurement a minor problem. For this reason, it is the level of suppression of FSR and the control of the remainder that determines the accuracy of the measurement. For the $\pi^+\pi^-$ final state, recent measurements in Novosibirsk have pushed the uncertainty in the pion form factor down to one per cent for $\sqrt{s} < 1$ GeV [8]. Hence, for the proposed method to be competitive, the level of the FSR contribution should not exceed this number.

In addition the FSR amplitude can be calibrated experimentally by measuring the ISR-FSR interference which is odd under charge conjugation and thus gives rise to a relatively large forward-backward asymmetry.

In practice, the situation is further complicated by additional collinear initial state radiation which changes the total energy of the collision and can not be detected. This effect must be taken into account in any realistic Monte Carlo event generator. As usual, the collinear initial state radiation is described by the structure functions which provide a resummation of the logarithmic corrections $\mathcal{O}(\alpha \log^2(s/m_e^2))$ to all orders in the coupling constant. We also assume that the invariant mass of the $\pi^+\pi^-\gamma$ system can be measured sufficiently accurate. Events with the invariant mass of the final state much smaller than the total energy of the collision squared can thus be rejected without loosing too much luminosity. This allows to reduce significantly the kinematic distortion of the events due to collinear initial state radiation. In the Monte Carlo program this is realized by requiring a minimal invariant mass of

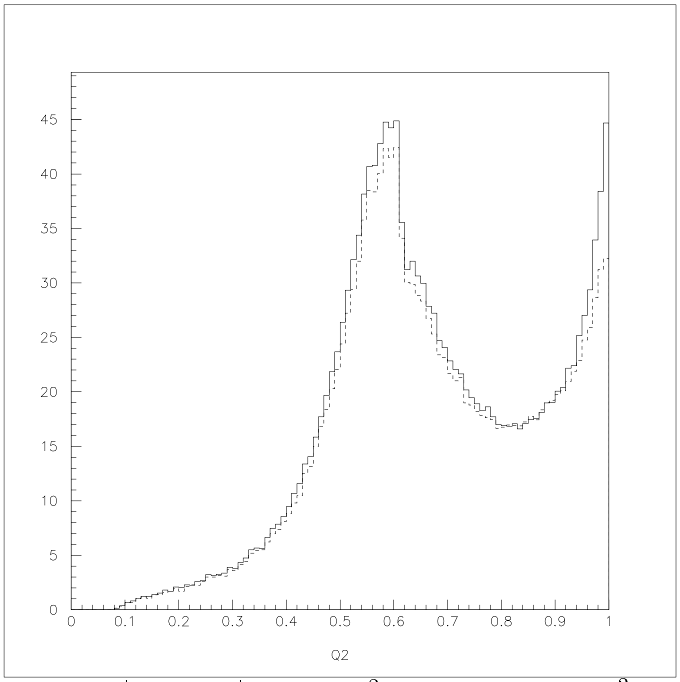

FIG. 1. The cross section $\mathrm{d}\sigma(e^+e^- \to \pi^+\pi^-\gamma)/\mathrm{d}Q^2_{\pi^+\pi^-}$ in nbarn/GeV^2 as a function of $Q^2_{\pi^+\pi^-}$ in GeV^2 for $\sqrt{s} = 1.02$ GeV. The solid line is the distribution without collinear ISR, the dashed line – with the collinear ISR. The cuts are $7^\circ < \theta_\gamma < 173^\circ$, $\omega_\gamma > 0.02$ GeV and the invariant mass of the detected particles in the final state $Q^2_{\pi^+\pi^-\gamma} > 0.9$ GeV^2.

the final state $\pi^+\pi^-\gamma$. For the structure functions the formulas of Ref. [9] are used. For the purpose of illustration, we present the results for the pion invariant mass distribution with and without collinear initial state radiation in Fig.1.

In what follows we discuss two issues essential for a successful measurement. First, we show that by choosing a proper cuts one can significantly suppress the contribution due to FSR. Second, we demonstrate that by measuring the forward-backward asymmetry of the produced pions one can check how realistic the final state radiation amplitude is described. Confronting our predictions for the forward-backward asymmetry with the measurements, one can gain confidence in the predictions based on our Monte Carlo event generator. If not stated otherwise, the additional collinear radiation is incorporated with the structure function technique.

III. ISR DOMINANCE

The spectrum of photons emitted by an ultrarelativistic particle is described by the well-known formula:

$$\mathrm{d}\sigma \propto \mathrm{d}\sigma_0 \frac{\mathrm{d}\omega_\gamma}{\omega_\gamma} \frac{\mathrm{d}\theta^2_\gamma}{\theta^2_\gamma}.$$

Hence the relativistic particle strongly radiates in a small angular cone along its direction of motion. For this reason, a suppression of the FSR is achieved by selecting events where the pions are geometrically separated from the hard photon. Simultaneously, an enhancement of the ISR is achieved if one selects the events where the photon is emitted at small angles relative to the collision axis. In reality, the KLOE detector seems to be able to measure photons down to $\theta_\gamma \sim 7^\circ$. Therefore, an appropriate configuration is, for example, obtained by requiring $30^\circ < \theta_\pi < 150^\circ$ for pions and $7^\circ < \theta_\gamma < 20^\circ$ or $160^\circ < \theta_\gamma < 173^\circ$ for photons. The prediction for the $Q^2_{\pi^+\pi^-}$ invariant mass distribution is shown in Fig.2. Note that we have included events with the photons emitted both in the forward and the backward direction. Nevertheless, a forward backward asymmetry of the π^+ (or, with opposite sign, of the π^-) will be observed, which should not be confused with the trivial kinematic asymmetry if we would consider photons with $7^\circ < \theta_\gamma < 20^\circ$ only.

It follows from Fig.2 that for the chosen cuts the contribution of the FSR radiation to the total cross section is of the order of 1 per cent only. Also, one does not loose too much in the event rate since the pion angular distribution is relatively uniform and the photon angular distribution is strongly peaked at small angles.

Note that the contribution of the FSR to such a configuration comes only from the emission of photons by pions at *large* angles. In this case the pions in the intermediate state are far off-shell. For this reason, one expects that the point-like interaction of pions to photons is not likely to be an adequate description. However, we believe, that in this case the point-like interaction *overestimates* the strength of the interaction and therefore gives a larger contribution due to the FSR. Still, it is desirable to check how reasonable the interaction of photons to pions is described by our ansatz. A possible check is considered in the next Section.

The distribution shown in Fig.2 may be compared with the distribution when no geometric separation is imposed on photons and pions Fig.3. In this case, one finds a significant FSR contribution. An interesting fact to note, is that the FSR contribution decreases for smaller $Q^2_{\pi^+\pi^-}$. This implies that possibly better results can be achieved if a combined $Q^2_{\pi^+\pi^-}$ - $\cos\theta_\gamma$ cut is applied. Large Q^2, on the other hand, correspond to soft photons where our description of FSR is expected to work reasonably well.

Let us imagine that the application of the proper selection criteria resulted in a suppression of the final state radiation. In this case, even measuring the total cross section integrated over the invariant masses of pions could provide useful information for the determination of the fine structure constant.

To illustrate this point, we consider the expression for the cross section due to ISR:

$$\frac{\mathrm{d}\sigma_{ee\to\pi\pi\gamma}}{\mathrm{d}Q^2\mathrm{d}\cos\theta_\gamma} = \frac{\alpha^3}{3s^2}|F_\pi(Q)|^2\left(1-\frac{4m_\pi^2}{Q^2}\right)^{3/2}\left\{\frac{(s^2+Q^4)}{Q^2(s-Q^2)}\frac{1}{1-\cos^2\theta_\gamma}-\frac{(s-Q^2)}{2Q^2}\right\}. \tag{6}$$

Consider now the case $s \gg Q^2$, i.e. a creation of the pion pair with the invariant mass much smaller than the energy of the collision. The above equation is then simplified:

$$\frac{\mathrm{d}\sigma_{ee\to\pi\pi\gamma}}{\mathrm{d}Q^2\mathrm{d}\cos\theta_\gamma} \to \frac{\alpha}{2\pi s}\sigma_{ee\to\pi\pi}(Q^2)\frac{(1+\cos\theta_\gamma^2)}{(1-\cos\theta_\gamma^2)}. \tag{7}$$

The total cross section for $e^+e^- \to \pi^+\pi^-\gamma$ is obtained if one integrates over the invariant mass of pions and the photon production angle:

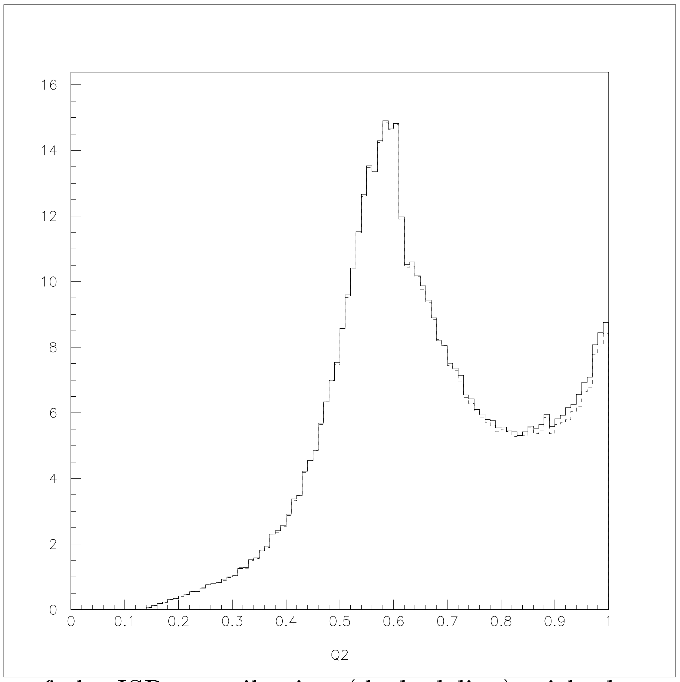

FIG. 2. A comparison of the ISR contribution (dashed line) with the complete result (solid line). Plotted is the cross section $\mathrm{d}\sigma(e^+e^- \to \pi^+\pi^-\gamma)/\mathrm{d}Q^2_{\pi^+\pi^-}$ in nbarn/GeV^2 as a function of $Q^2_{\pi^+\pi^-}$ in GeV^2 for $\sqrt{s} = 1.02$ GeV. The cuts are $7^\circ < \theta_\gamma < 20^\circ$ or $160^\circ < \theta_\gamma < 183^\circ$, $30^\circ < \theta_\pi < 150^\circ$, $\omega_\gamma > 0.02$ GeV and the invariant mass of the detected particles in the final state $Q^2_{\pi^+\pi^-\gamma} > 0.9$ GeV^2.

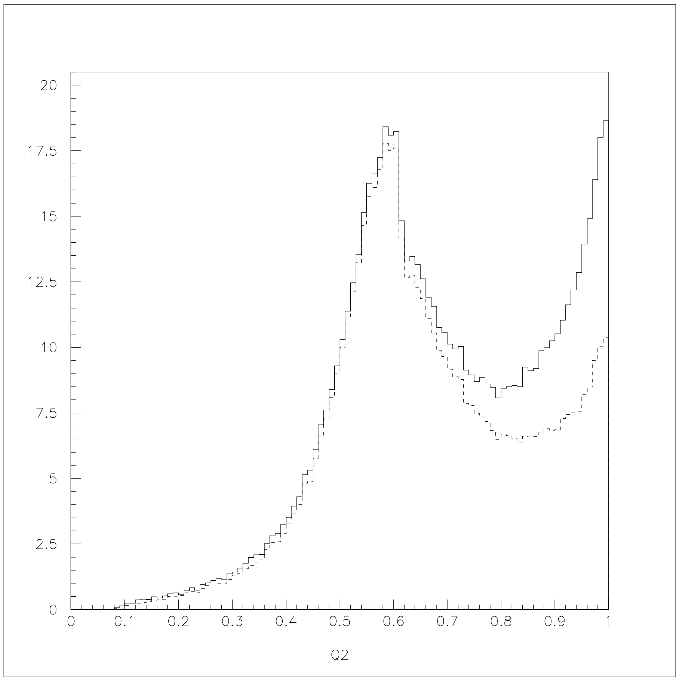

FIG. 3. The same as in Fig.2, but for angular cuts $30^\circ < \theta_\gamma, \theta_\pi < 150^\circ$.

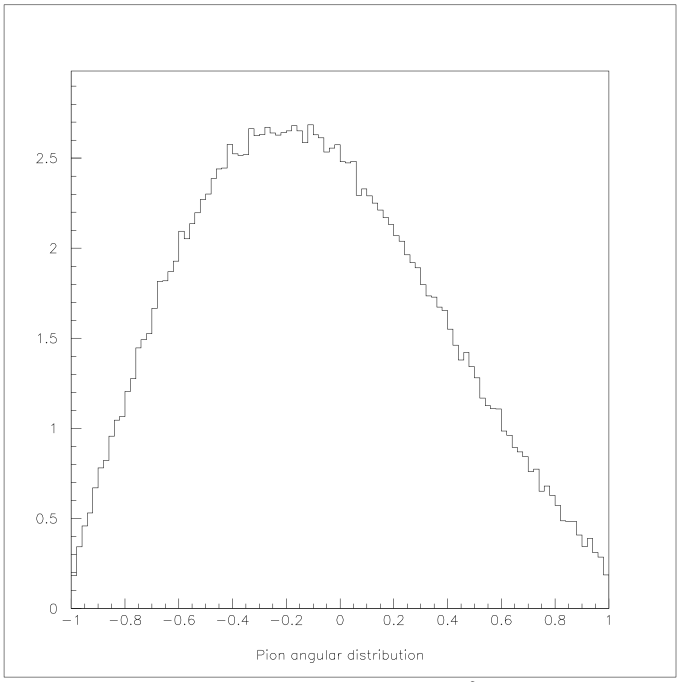

FIG. 4. Pion (π^+) angular distribution in the laboratory frame. The cut on the photon angle is $60^\circ < \theta_\gamma < 120^\circ$.

$$\sigma_{ee\to\pi\pi\gamma} = \int\limits_{Q^2_{\rm min}}^{Q^2_{\rm max}} {\rm d}Q^2 \int\limits_{\cos(\theta)_{\rm min}}^{\cos(\theta)_{\rm max}} {\rm d}\cos\theta_\gamma \frac{{\rm d}\sigma_{ee\to\pi\pi\gamma}}{{\rm d}Q^2{\rm d}\cos\theta_\gamma} \Rightarrow \frac{\alpha}{4\pi s}\mathcal{F}(\cos(\theta)_{\rm max,min}) \int\limits_{Q^2_{\rm min}}^{Q^2_{\rm max}} {\rm d}Q^2 \sigma_{ee\to\pi\pi}(Q^2). \tag{8}$$

This expression should be compared with the small s contribution to $\alpha_{\rm had}$. Considering the contribution of the low-energy region, $s \ll M_Z^2$, one obtains:

$$\delta\alpha_{\rm had}(M_z) \approx \frac{\alpha(0)}{3\pi} \int\limits_{4m_\pi^2}^{s_{\rm max}} {\rm d}s\ \sigma_{ee\to{\rm hadrons}}(s) \propto \sigma_{e^+e^-\to\pi^+\pi^-\gamma}.$$

We therefore conclude, that even the integrated cross section for $e^+e^- \to$ hadrons $+\,\gamma$ can be effectively used to obtain the contribution of the low energy region to the value of the fine structure constant at the Z resonance, provided that the suppression of the final state radiation is achieved using appropriate selection criteria.

In a realistic situation, the accurate determination of $\sigma(e^+e^- \to \pi^+\pi^-)$ from $\sigma(e^+e^- \to \pi^+\pi^-\gamma)$ requires the knowledge of the contribution due to FSR and the effect of the collinear ISR. As we have explained above, FSR can be significantly suppressed using appropriate cuts on the photon and pion emission angles. After this is achieved, an appropriate strategy which can be used to extract the pion form factor would be to fit the parameters of the model for the pion form factor described in [7] to an observed $Q^2_{\pi^+\pi^-}$-distribution using our Monte Carlo event generator which incorporates all the effects related to the collinear initial state radiation.

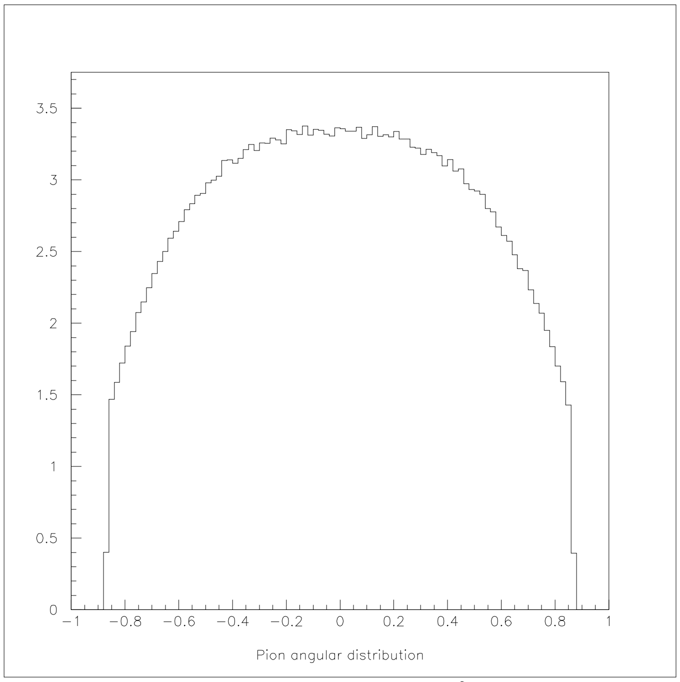

FIG. 5. Pion (π^+) angular distribution in the laboratory frame. The photon angle is restricted to $7° < \theta_\gamma < 20°$ or $160° < \theta_\gamma < 183°$. The cut on the pion angle is $30° < \theta_{\pi^+,\pi^-} < 150°$.

IV. TESTING THE MODEL FOR THE FSR

The cross section for $e^+e^- \to \pi^+\pi^-\gamma$ can be written as:

$$\mathrm{d}\sigma \propto |\mathcal{M}|^2 = |\mathcal{M}_{\rm ISR}|^2 + |\mathcal{M}_{\rm FSR}|^2 + 2\mathrm{Re}\left[\mathcal{M}_{\rm ISR}\mathcal{M}^*_{\rm FSR}\right]. \tag{9}$$

We have explicitly separated the contribution due to ISR, FSR and the interference of the two amplitudes.

If the photon is emitted from the initial(final) state, the pion pair is produced with charge parity $C = -1(+1)$. Therefore, the contribution of the third term in Eq.(9) vanishes if one integrates over the kinematic variables of the pions. However, this term causes a significant charge asymmetry and, correspondently, a forward-backward asymmetry of π^+ and π^-.

As usual, the asymmetry is defined as:

$$A = \frac{\int_{-1}^{0} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_{\pi^+}} \mathrm{d}\cos\theta_{\pi^+} - \int_{0}^{1} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_{\pi^+}} \mathrm{d}\cos\theta_{\pi^+}}{\int_{-1}^{0} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_{\pi^+}} \mathrm{d}\cos\theta_{\pi^+} + \int_{0}^{1} \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_{\pi^+}} \mathrm{d}\cos\theta_{\pi^+}}. \tag{10}$$

The asymmetry is also a function of the cuts on the photon angle and energy which is implicitly assumed in the above equation. This asymmetry should not be confused with the trivial asymmetry from kinematics which arises if one selects photons in one hemisphere only.

For symmetric cuts on the photon angular distribution, the numerator of this expression is non zero only due to the interference term in Eq.(9), the denominator equals to the total cross section. Given the dominance of the initial state radiation in the total cross section, the forward-backward asymmetry is (roughly) proportional to the ratio $\mathcal{M}_{\mathrm{FSR}}/\mathcal{M}_{\mathrm{ISR}}$ and thus is a direct measure of the final state radiation. For this reason, measuring the asymmetry would give an effective control over the accuracy of the ansatz used to describe the interaction of photons to pions.

To demonstrate the magnitude of the effect consider the situation, where the standard cuts on the photon energy and the invariant mass of $\pi^+\pi^-\gamma$ are applied. The photon angular cuts are such that $60^\circ \leq \theta_\gamma \leq 120^\circ$, also $\omega_\gamma > 0.02$ GeV. The angular distribution for positively charged pions is then presented in Fig.4. One sees, that the distribution is not symmetric: positively charged pions are mainly emitted in the backward direction. Using Eq.(10), we then obtain that the asymmetry A equals 20%. The total cross section in this case equals 3.29 nbarn which implies that up to $3.29 \cdot 10^6$ events will be observed in a year of running. Without any doubt, the twenty percent effect will be clearly visible with such a statistic.

Alternatively, we present in Fig.5 the angular distribution of pions for the case where the photon is emitted at small angles, well separated from pions: $7^\circ < \theta_\gamma < 20^\circ$ or $160^\circ < \theta_\gamma < 183^\circ$, $30^\circ < \theta_{\pi^+,\pi^-} < 150^\circ$ and $\omega_\gamma > 0.02$ GeV. In this case the asymmetry is reduced to $A = 1.5\%$, demonstrating again the smallness of final state radiation for this case.

The separation between initial and final state radiation is also possible for the symmetric piece on the basis of the distinctly different angular distributions. Let us for the moment ignore the folding with the radiator functions and consider strictly the reaction $e^+e^- \to \pi^+\pi^-\gamma$. Final state radiation alone leads necessarily to a photon angular distribution of the form $\alpha + \beta\cos^2\theta_\gamma$, initial state radiation on the other hand is described by Eq.(6). Given sufficient statistics, it is clear that a combined fit will allow the separation of the two components.

V. CONCLUSIONS

In this paper we have argued, that one can effectively use the radiative return to low energies in order to measure the cross section of $e^+e^- \to$ hadrons at variable center of mass energy, of course lower than the energy of the colliding beams. We have studied the feasibility of this approach with particular emphasis on the two pion final state – a typical situation for the DAPHNE ring. We have produced a realistic tool for these studies, a Monte Carlo event generator, which provides an option for choosing the cuts and includes both initial, final and collinear initial radiation.

By using special selection criteria and measuring the energies and momenta of the particles in the final state one can suppress the contribution of the final state radiation to the one per cent level. We have also demonstrated that there will be a sizable forward-backward asymmetry, which can be used to check the quality of our description of the final state radiation.

For the forward-backward asymmetry to be significant and to allow for this check, one should keep the photon angle sufficiently large, otherwise the ratio $\mathcal{M}_{\mathrm{ISR}}/\mathcal{M}_{\mathrm{FSR}}$ becomes too small. The opposite requirement must be fulfilled for the accurate extraction of $\sigma(e^+e^- \to$ hadrons), as discussed in Sect.3. For this reason, all events with photons appear to be useful for extracting interesting information.

The proposed technique was mainly discussed in connection with the DAPHNE collider. We would like to stress, however, that the use of the radiation return to lower energies can be used in a more general context and the same technique should, in principle, be applicable also for other "particle factories".

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