Leading mass correction to Cabibbo suppressed τ decays: the perturbation series structure and extraction of m_s

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Abstract

In this talk the recent results on perturbative QCD corrections to the hadronic decay rate of the τ lepton are presented. We discuss the structure of perturbation theory series of the leading mass correction. On the base of this analysis the numerical value of the strange quark mass is extracted.

The determination of the strong coupling constant α_s is a focal point in recent investigations on extraction of the standard model parameters and high order perturbation theory calculations [1]. The small reduction of the Cabibbo suppressed rate of τ lepton (relative to the massless prediction) has been used recently to extract a value for the strange quark mass [2]. The analysis was based on the total rate within a theoretical calculation of quadratic mass term with the coefficient up to α_s^2 order [3]. Some new development calls, however, for a fresh look at the possibility for an m_s determination from τ decays [4, 5].

We report on the theoretical analysis extended and improved in the several ways [6]. QCD corrections to the mass terms of order α_s^3 are calculated for some of the moments of the spectral functions. Analysis of different spin contributions is done separately [7]. Resummation of effects from the running of both the coupling constant and the strange quark mass along the contour of integration in the complex plane through the renormalization group improvement is used for τ lepton observables [8].

Physical τ lepton observables are related to correlators of vector and axial vector currents of light quarks that are defined as follows

$$\Pi^{V/A}_{\mu\nu,ij}(q) = i\int dx e^{iqx}\langle T j^{V/A}_{\mu,ij}(x)(j^{V/A}_{\nu,ij})^\dagger(0)\rangle = g_{\mu\nu}\Pi^{[1]}_{ij,V/A}(q^2) + q_\mu q_\nu \Pi^{[2]}_{ij,V/A}(q^2) \quad (1)$$

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with $j^{V/A}_{\mu,ij} = \bar{q}_i\gamma_\mu(\gamma_5)q_j$ [9]. Here q_i and q_j are two quarks with masses m_i and m_j.

Another useful representation of the tensor $\Pi^{V/A}_{\mu\nu,ij}(q)$ in terms of scalar functions reads

$$\Pi^{V/A}_{\mu\nu,ij}(q, m_i, m_j, m, \mu, \alpha_s) = (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi^{(1)}_{ij,V/A}(q^2) + q_\mu q_\nu \Pi^{(0)}_{ij,V/A}(q^2) \quad (2)$$

where the correlator is decomposed into the components $\Pi^{(0,1)}_{ij,V/A}(q^2)$ that contain contributions of the states with the angular momentum $J = 0$ and $J = 1$ respectively. A direct comparison of (1) and (2) leads to the relations

$$\Pi^{(1)} = -\Pi^{[1]}/q^2, \quad \Pi^{(0)} = \Pi^{[2]} + \Pi^{[1]}/q^2. \quad (3)$$

In general $\Pi^{[1]}(0)$ may be different from zero which implies a kinematical singularity in both $\Pi^{(1)}$ and $\Pi^{(0)}$.

The hadronic decay rate of the τ lepton is obtained by integrating the absorptive parts of the spectral functions with respect to the invariant hadronic mass. Corresponding to two different tensor decompositions (1) and (2), two different integral representations for the decay rate can be obtained. The first one displays the structure of hadronic contributions classified according to their spin

$$R_\tau = R^{(1)}_\tau + R^{(0)}_\tau = \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) R^{(1)}(s) + R^{(0)}(s)\right] \quad (4)$$

where

$$R^{(J)} = |V_{ud}|^2(R^{(J)}_{ud,V} + R^{(J)}_{ud,A}) + |V_{us}|^2(R^{(J)}_{us,V} + R^{(J)}_{us,A}), \quad J = 0, 1. \quad (5)$$

The second one uses the absorptive parts of the structure functions $\Pi^{[1]}$ and $\Pi^{[2]}$ and is more suitable for analytic continuation into the complex plane

$$R_\tau = \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\frac{2s}{M_\tau^2} R^{(1)}(s) + R^{(2)}(s)\right]. \quad (6)$$

Due to the analyticity of $\Pi^{[1,2]}$ in the cut complex s-plane (the absence of singularities away from the physical cut, even of kinematical singularities at the origin) R_τ can be expressed as the contour integral along a circle C of the radius $|s| = M_\tau^2$

$$R_\tau = 6i\pi \int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\Pi^{[2]}(s) - \frac{2}{M_\tau^2}\Pi^{[1]}(s)\right]. \quad (7)$$

Along "the large circle" of radius $|s| = M_\tau^2$, the functions $\Pi^{[1,2]}(s)$ are assumed to be reliably evaluated within pQCD. Therefore, representation (7) is a well-defined pQCD prediction for R_τ. This is not true for the spin-separated parts. Indeed, the direct use of (3) leads to

$$\begin{aligned} R^{(1)}_\tau &= 6i\pi \int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{[1]}(0)/s\right], \\ R^{(0)}_\tau &= 6i\pi \int_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\Pi^{(0)}(s) - \Pi^{[1]}(0)/s\right] \end{aligned} \quad (8)$$

where the contribution of the singularity at the origin (proportional to $\Pi^{[1]}(0)$) has to be included. A nonvanishing value of $\Pi^{[1]}(0)$ is certainly a nonperturbative constant. Thus, within pQCD we cannot predict the decay rates $R_\tau^{(1,0)}$ separately. In the massless limit $\Pi^{(0)}=0$ within perturbation theory and $R^{(0)}$ is saturated by $\Pi^{[1]}(0)$ corresponding to the massless pion (kaon) pole for the axial part of the correlator. The unknown constant drops out from moments

$$R_\tau^{(1,0)k,l}(s_0)=\int_0^{s_0}\frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^k\left(\frac{s}{M_\tau^2}\right)^l\frac{dR_\tau^{(1,0)}}{ds}, \tag{9}$$

with $k\geq 0,\ \ l\geq 1$. (Note that the moments introduced in [10] are related to ours as $R_\tau^{kl}=R_\tau^{(1)k,l}+R_\tau^{(0)k,l}$.)

We stress that inclusion of Cabibbo suppressed modes into the analysis gives not only an additional set of experimental data but open conceptually new possibilities because the massive piece can be measured in conjunction with the massless contribution thus providing a strict normalization and reducing the systematic errors of the experimental data.

The quark mass corrections for the vector and axial correlators are identical for the case under consideration with $m_i=m_s\neq 0$ and $m_j=m_u=m_d=0$. The perturbative prediction for the quadratic mass corrections up to order α_s^3 and for arbitrary quark masses has been presented in [11]. The results for the mass correction to both polarization functions read

$$\begin{aligned}\Pi^{[1]}_{V,2} = 2m_s^2\Bigg\{ & l_{\mu Q}+\frac{\alpha_s}{\pi}\left[\frac{25}{4}-4\,\zeta(3)+\frac{5}{3}l_{\mu Q}+l_{\mu Q}^2\right]\\
+ & \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{18841}{432}-\frac{1}{360}\pi^4-\frac{3607}{54}\,\zeta(3)+\frac{1265}{27}\,\zeta(5)\right.\\
&\left.\qquad+\frac{4591}{144}l_{\mu Q}-\frac{35}{2}\,\zeta(3)l_{\mu Q}+\frac{22}{3}l_{\mu Q}^2+\frac{17}{12}l_{\mu Q}^3\right]\\
+ & \left(\frac{\alpha_s}{\pi}\right)^3\left[l_{\mu Q}\left(\frac{1967833}{5184}-\frac{1}{36}\pi^4-\frac{11795}{24}\,\zeta(3)+\frac{33475}{108}\,\zeta(5)\right.\right.\\
&\left.\left.\qquad+\frac{4633}{36}l_{\mu Q}-\frac{475}{8}\,\zeta(3)l_{\mu Q}+\frac{79}{4}l_{\mu Q}^2+\frac{221}{96}l_{\mu Q}^3\right)+k_3^{[1]}\right]\Bigg\},\end{aligned} \tag{10}$$

$$\begin{aligned}\Pi^{[2]}_{V,2} = -4m_s^2\Bigg\{ & 1+\frac{\alpha_s}{\pi}\left[\frac{7}{3}+2l_{\mu Q}\right]\\
+ & \left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{13981}{432}+\frac{323}{54}\,\zeta(3)-\frac{520}{27}\,\zeta(5)+\frac{35}{2}l_{\mu Q}+\frac{17}{4}l_{\mu Q}^2\right]\\
+ & \left(\frac{\alpha_s}{\pi}\right)^3\left[l_{\mu Q}\left(\frac{14485}{54}+\frac{3659}{108}\,\zeta(3)-\frac{3380}{27}\,\zeta(5)+\frac{1643}{24}l_{\mu Q}\right.\right.\\
&\left.\left.\qquad+\frac{221}{24}l_{\mu Q}^2\right)+k_3^{[2]}\right]\Bigg\}.\end{aligned} \tag{11}$$

Here $l_{\mu Q} = \ln(\mu^2/Q^2)$, the mass m_s and QCD coupling constant α_s are taken at a t' Hooft mass μ. All correlators are renormalized within $\overline{\rm MS}$ scheme. Note that terms of order α_s^3 are known only with "logarithmic" accuracy, that is the constant parts $k_3^{[1]}$ and $k_3^{[2]}$ in the large Q behavior of the corresponding correlators are not available[2]. Thus we do not know the $O(m_s^2\alpha_s^3)$ contributions to R_τ; however these constants (as well as the also unknown "low-energy" constant $\Pi^{[1]}(0)$) do not appear in the moments $R_\tau^{(1,0)k,l}$ with $k \geq 0, \quad l \geq 1$.

We present some concrete results. The perturbative result for the m^2 terms does not differentiate between vector and axial vector channels and we consider their sum. The mass corrections to the moments of spin 0 and spin 1 final state distributions and with or without resummation of effects from running are now cast into a general form ($m_s = m_s(M_\tau)$, $\alpha_s = \alpha_s(M_\tau)$)

$$\delta_{us} = b_0 \frac{m_s^2}{M_\tau^2}\{1 + b_1 \frac{\alpha_s}{\pi} + b_2 \left(\frac{\alpha_s}{\pi}\right)^2 + b_3 \left(\frac{\alpha_s}{\pi}\right)^3\} \tag{12}$$

Coefficients b_0, b_1, b_2, b_3 take some numerical values depending on the parity and spin of the channel as well as the method (with or without resummation). In view of the additional nonperturbative contribution $\sim \Pi^{[1]}(0)$ which appears in the lowest order moment ($l = 0$) due to the spin separation only the results for $l \geq 1$ can be obtained.

The rapid change of coefficients of perturbation theory expansions for different moments is caused by the running of the coupling constant and the mass along the contour of integration. The resummation of these effects can be performed in all orders of α_s. The technique for the massless case is described in the literature [8, 10, 12] so we concentrate on the massive case.

The appearance of mass introduces another freedom in the choice of the basic quantities that accumulate the perturbative information. As basic quantities we choose $\Pi_{us}^{[1]}(q^2)$ and $\Pi_{us}^{[2]}(q^2)$. The renormalization of the pieces proportional to m_s^2 is different for $\Pi^{[1]}$ and $\Pi^{[2]}$. $\Pi_{us,2}^{[2]}(Q^2)$ is scale-invariant and the renormalization group improvement can be performed directly

$$Q^2\Pi_{us,2}^{[2]}(Q^2) = k_0^{[2]} m_s^2(Q^2)(1 + k_1^{[2]} a(Q^2) + \ldots) \tag{13}$$

with $a = \alpha/\pi$ and $k_i^{[2]}$ being the known coefficients. $\Pi^{[1]}$ is not renormalized multiplicatively. The problem is solved by introducing the corresponding D-function $D^{[1]}(Q^2)$ by one differentiation with respect to Q^2. The result is

$$D_{us,2}^{[1]}(Q^2) = -\frac{Q^2}{2}\frac{d}{dQ^2}\Pi_{us,2}^{[1]}(Q^2) = m_s^2(Q^2)(1 + d_1^{[1]} a(Q^2) + \ldots).$$

The running of the mass as taken into account through the renormalization group equation

$$Q^2 \frac{d}{dQ^2} m_s(Q^2) = \gamma_m(a(Q^2)) m_s(Q^2), \quad \gamma_m(a) = -\gamma_0 a - \gamma_1 a^2 - \ldots \tag{14}$$

[2]In fact the very calculation of the constant parts is well beyond the present calculational techniques.

with the solution

$$m_s(Q^2) = m_s(\mu^2)\exp\int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} \frac{\gamma(x)dx}{\beta(x)}. \tag{15}$$

Subsequently, the integration along the contour can be performed directly.

The explicit formula in the leading order of the β-function for $\Pi^{[2]}(Q^2)$ is easily found. We have

$$Q^2\Pi^{[2]}_{us,2}(Q^2) = k_0^{[2]} m_s^2(\mu^2)\left(\frac{a(Q^2)}{a(\mu^2)}\right)^{\frac{2\gamma_0}{\beta_0}}(1 + k_1^{[2]}a(Q^2) + \ldots) \tag{16}$$

where $a(Q^2)$ is the running coupling. For another amplitude the result is

$$\Pi^{[1]}_{us,2}(Q^2) = -\frac{m_s^2(\mu^2)}{\beta_0}\left(\frac{a(Q^2)}{a(\mu^2)}\right)^{\frac{2\gamma_0}{\beta_0}}\left(\frac{1}{\left(\frac{2\gamma_0}{\beta_0}-1\right)}\frac{1}{a(Q^2)} + \frac{d_1^{[1]}}{\frac{2\gamma_0}{\beta_0}} + \frac{d_2^{[1]}}{\frac{2\gamma_0}{\beta_0}+1}a(Q^2) + \ldots\right).$$

These formulae should be substituted in (7),(8) and integrated along the contour. The generalization to higher orders of the β- and γ-functions is straightforward. The coefficients $d_i^{[1]}$ and $k_i^{[2]}$ can be inferred from the explicit expression for the polarization function (10). They accumulate the whole perturbative information because the β-function and the quark anomalous dimension necessary for the restoration of the full expressions (10) and (11) are known.

We perform the analysis along these lines for the same moments as in the finite order of perturbation theory with the available coefficients for $\Pi^{[2]}$ and for the D-function in the case of $\Pi^{[1]}$ and with β- and γ-functions from first to fourth order [13, 14, 15].

The main problem of the strange quark determination and even the total perturbative analysis of the Cabibbo suppressed τ lepton decays is the interpretation of the perturbation theory series for m_s^2 corrections. In passing we note that also perturbative corrections to the power suppressed terms proportional to m_s^4 and m_s^6 might provide us with another example of bad behaviour of higher order terms. However, since the $\mathcal{O}(\alpha_s^2)$ corrections to the spin one are available we discuss the m_s^2 terms only.

The problem is fairly obvious from a consideration of the two lowest order moments. We use $\alpha_s(M_\tau) = 0.334$ [16] in computing the perturbative contributions. The order of magnitude of the unknown coefficient $k_3^{[2]}$ in Eq. (11) is estimated on the basis of a geometric series to amount to $k_3^{[2]} = 0 \pm (19.6)^2/2.33 = 0 \pm (k_2^{[2]})^2/k_1^{[2]} = 0 \pm 164.4$. For fixed order we thus find series

$$\begin{aligned}
\delta^{00}_{us,2} &= -8\frac{m_s^2}{M_\tau^2}(1. + 5.33a + 46.0a^2 + 284a^3 + 0.75a^3k_3^{[2]}) \\
&= -8\frac{m_s^2}{M_\tau^2}(1. + 0.567 + 0.520 + 0.341 \pm 0.148) \\
&= -8\frac{m_s^2}{M_\tau^2}(2.4 \pm 0.5),
\end{aligned} \tag{17}$$

where we have assumed the (maximal!) value of the $\mathcal{O}(\alpha_s^3)$ term as an estimate of the theoretical uncertainty. For the "contour improved" series (which is marked with a tilde in our notation) one obtains

$$\begin{aligned}
\tilde{\delta}^{00}_{us,2} &= -8\frac{m_s^2}{M_\tau^2}(1.44 + 3.65a + 30.9a^2 + 72.2a^3 + 1.18a^3k_3^{[2]}) \\
&= -8\frac{m_s^2}{M_\tau^2}(1.44 + 0.389 + 0.349 + 0.0867 \pm 0.234) \\
&= -8\frac{m_s^2}{M_\tau^2}(2.26 \pm 0.32). \qquad (18)
\end{aligned}$$

Comparing the "improved" with the finite order analysis one observes that higher orders give numerically smaller contributions although the apparent convergence is still rather marginal. The ratio between two correction terms $\tilde{\delta}^{00}_{us,2}/\delta^{00}_{us,2} \approx 0.95$ shows a relatively stable behaviour. Eq. (18), with the uncertainty increased by perhaps a factor 2 could be considered as a reasonable estimate of the strange mass corrections.

In fixed order approximation the moments with $l \geq 1$ are independent of the constant $k_3^{[2]}$; a residual dependence remains, however, in the "contour improved" treatment. For the fixed order we find

$$\begin{aligned}
\delta^{01}_{us,2} &= -\frac{5}{3}\frac{m_s^2}{M_\tau^2}(1 - 4.17a - 113.a^2 - 1820.a^3) \\
&= -\frac{5}{3}\frac{m_s^2}{M_\tau^2}(1 - 0.443 - 1.27 - 2.19) \\
&= -\frac{5}{3}\frac{m_s^2}{M_\tau^2}(-2.9 \pm 2.2), \qquad (19)
\end{aligned}$$

while the "contour improved" method gives

$$\begin{aligned}
\tilde{\delta}^{01}_{us,2} &= -\frac{5}{3}\frac{m_s^2}{M_\tau^2}(-2.26 - 14.7a - 204.a^2 - 171.a^3 - 13.3a^3k_3^{[2]}) \\
&= -\frac{5}{3}\frac{m_s^2}{M_\tau^2}(-2.26 - 1.56 - 2.3 - 0.206 \mp 2.62) \\
&= -\frac{5}{3}\frac{m_s^2}{M_\tau^2}(-6.3 \pm 2.8). \qquad (20)
\end{aligned}$$

The $(0,1)$ moments thus exhibit a rapid growth of the coefficients and, at the same time, with $\tilde{\delta}^{01}_{us,2}/\delta^{01}_{us,2} = 2.2$ a strong dependence on the improvement procedure. This comparison shows that there is no consistent prediction in $\overline{\mathrm{MS}}$ scheme for this observable – the first moment of the differential rate.

Now we turn to the contributions of spin one and spin zero separately. As stated before, the lowest moments ($l = 0$) of the spin-dependent functions depend on the nonperturbative quantity $\Pi^{[1]}(0)$. For the spin one part and for $(k, l) =$

$(0,1)$ we find within finite order perturbation theory

$$\begin{aligned}
\delta^{(1)01}_{us,2} &= -5\frac{m_s^2}{M_\tau^2}(1.+4.83a+35.7a^2+276.a^3) \\
&= -5\frac{m_s^2}{M_\tau^2}(1.+0.514+0.404+0.331) \\
&= -5\frac{m_s^2}{M_\tau^2}(2.25\pm 0.33)
\end{aligned} \tag{21}$$

and with resummation

$$\begin{aligned}
\tilde{\delta}^{(1)01}_{us,2} &= -5\frac{m_s^2}{M_\tau^2}(1.37+2.55a+16.1a^2+135a^3) \\
&= -5\frac{m_s^2}{M_\tau^2}(1.37+0.271+0.182+0.163) \\
&= -5\frac{m_s^2}{M_\tau^2}(2.0\pm 0.2).
\end{aligned} \tag{22}$$

Note that the spin 1 contribution is determined by the component $\Pi^{[1]}$ alone and is known up to third order. Clearly, this series is decreasing in a reasonable way (comparable to the behaviour of $\tilde{\delta}^{00}_{us,2}$) and, at the same time, is only moderately dependent on the improvement prescription with $\tilde{\delta}^{(1)01}_{us,2}/\delta^{(1)01}_{us,2} = 0.89$. On the basis of Eq. (22) this moment might well serve for a reliable m_s determination, with a sufficiently careful interpretation of the theoretical uncertainty.

The corresponding spin 0 part is proportional to m_s^2 and is ideal for a measurement of m_s. However, the behaviour of the perturbative series

$$\begin{aligned}
\tilde{\delta}^{(0)01}_{us,2} &= \frac{3}{2}\frac{m_s^2}{M_\tau^2}(3.19+11.2a+126.a^2+289.a^3+6.63a^3k_3^{[2]}) \\
&= \frac{3}{2}\frac{m_s^2}{M_\tau^2}(3.19+1.19+1.42+0.347\pm 1.31) \\
&= \frac{3}{2}\frac{m_s^2}{M_\tau^2}(6.14\pm 1.6)
\end{aligned} \tag{23}$$

shows a rapid growth of the coefficients. The series is not expected to provide an accurate prediction for the mass effects.

Besides the series for observables themselves one has also to look at convergence of the β- and γ-functions that determine the running along the contour. In the present case

$$\beta(a) = -2.25a^2(1+1.78a+4.47a^2+21.0a^3)$$

or at $a = 0.1$

$$\beta(0.1) = -0.0225(1+0.18+0.045+0.021)$$

which is quite good. For the γ-function, however,

$$\gamma(a) = -a(1 + 3.79a + 12.4a^2 + 44.3a^3)$$

and the convergence is marginally acceptable.

The resumed series behave in general better than those of finite order. However, for the mass corrections they still do not satisfy the heuristic criteria of convergence. In practice the resummation maintains the convergence pattern of the corresponding D-function. For the mass corrections the D-functions themselves exhibit rapidly growing coefficients of the perturbative series, whence the resummation does not lead to a significant improvement. Numerically the convergence for D-functions in $\overline{\rm MS}$ scheme is marginal. For the present case the D-function for $\Pi^{[1]}(Q^2)$ is given by

$$\begin{aligned} D^{[1]}(Q^2) &= m_s^2(Q^2)(1 + \frac{5}{3}a + a^2\left(\frac{4591}{144} - \frac{35}{2}\zeta(3)\right) \\ &+a^3\left(\frac{1967833}{5184} - \frac{\pi^4}{36} - \frac{11795}{24}\zeta(3) + \frac{33475}{108}\zeta(5)\right) \\ &= m_s^2(Q^2)(1 + 1.67a + 10.84a^2 + 107.53a^3) \end{aligned}$$

whereas a much better pattern of convergence is observed for massless part

$$\begin{aligned} D_0(Q^2) &= 1 + a + a^2\left(\frac{299}{24} - 9\zeta(3)\right) + a^3\left(\frac{58057}{288} - \frac{779}{4}\zeta(3) + \frac{75}{2}\zeta(5)\right) \qquad (24) \\ &= 1 + a + 1.64a^2 + 6.37a^3\,. \end{aligned}$$

This is the reason why the precise determination of the strong coupling constant from the τ lepton lifetime is possible.

Resummation leads to a modest improvement of the apparent convergence of the series, such that the lowest moments can be predicted with acceptable accuracy. Assuming sufficiently precise data, a combined analysis of the spin zero and one moments might thus lead to a reliable determination of the strange quark mass.

As we mentioned the inclusion of Cabibbo suppressed decay modes into the analysis leads to much stronger effects than only providing an additional set of observables. The whole system of τ lepton physics becomes bound stricter. Experimentally, new data on the massive piece can be obtained in conjunction with the massless one that greatly reduces the systematic errors. Theoretically, pQCD series for several independent observables of the same system become available that allows one to reduce the renormalization ambiguity considerably. Indeed, the prospective observables for experimental study are, for instance, the total rate R_{tot} and the first moments of spin separated rates $R^1_{J=1}$ and $R^1_{J=0}$. They have a representation in terms of theoretical pQCD series

$$R_{tot} = C_0(\alpha_s) - \frac{m_s^2}{M_\tau^2}C_m^{tot}(\alpha_s),$$

$$R^1_{J=1} = C^1_0(\alpha_s) - \frac{m_s^2}{M_\tau^2} C_m^{J=1}(\alpha_s),$$

$$R^1_{J=0} = \frac{m_s^2}{M_\tau^2} C_m^{J=0}(\alpha_s). \quad (25)$$

There are three independent perturbative coefficients from the system of five $C^\#_\#(\alpha_s)$. They are generated by (and reflect the perturbative structure of) the massless part and two pieces of the massive part of the current correlator (1). Explicit expressions are given by Eq. (24) for the massless part and by Eqs. (10) and (11) for the massive contributions. The remarkable features of theoretical analysis of the τ system with mass corrections are now explicit. Because the relative relations between $C^\#_\#(\alpha_s)$ are renormalization scheme independent one cannot make the corresponding perturbative series well convergent (in a heuristic sense) by just playing with redefinition of the coupling constant and the mass parameter m_s. The $\overline{\rm MS}$ scheme definition apparently must not be the optimal (or in any sense preferable) when mass corrections are included. We remind the reader that the $\overline{\rm MS}$ scheme is essensially the definition of the coupling constant for the massless approximation in e^+e^- annihilation.

With all these features in mind, one way to interprete the wild behaviour of perturbation theory series in the system of τ lepton observables would be to consider it as a revelation of the real divergence of PT (not an artefact of unproperly chosen scheme) and an indication to some nonperturbative dynamics. This interpretation, though qualitatively attractive, requires further quantitative study.

To conclude, we note that the large value for the correction factor $\tilde\delta^{00}_{us,2}/(-8)$ leads us to a reduction of the m_s value as determined from the data [2] on the basis of the earlier calculation [3]. The reduction by about 15% leads to $m_s(M_\tau) = (150 \pm 30_{exp} \pm 20_{th})$ MeV. In view of the large corrections the theoretical uncertainity can just be considered as a guess based on Eq. (18) and the subsequent discussion. This corresponds to $m_s(1 \text{ GeV}) = 200 \pm 40_{exp} \pm 30_{th}$ MeV in good agreement with other determinations (see, e.g. [17, 18, 19]).

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