Short-distance tachyonic gluon mass and $1/Q^2$ corrections

K.G. Chetyrkin[1],

Institut für Theoretische Teilchenphysik, Universität Karlsruhe,
Kaiserstrasse 12, 76128 Karlsruhe, Germany.
E-mail: chet@particle.physik.uni-karlsruhe.de

Stephan Narison

Laboratoire de Physique Mathématique et Théorique
UM2, Place Eugène Bataillon
34095 Montpellier Cedex 05, France
E-mail: narison@lpm.univ-montp2.fr

V.I. Zakharov

Max-Planck Institut für Physik,
Föhringer Ring 6, 80805 München, Germany.
E-mail: xxz@mppmu.mpg.de

Abstract

We consider the assumption that a tachyonic gluon mass imitates short-distance nonperturbative physics of QCD. The phenomenological implications include modifications of the QCD sum rules for correlators of currents with various quantum numbers. The new $1/Q^2$ terms allow to resolve in a natural way old puzzles in the pion and scalar-gluonium channels. They lead to a slight reduction of the values of the running light quark masses from the (pseudo)scalar sum rules and of $\alpha_s(M_\tau)$ from τ decay data. Analogously such terms only affect slightly the determinations of the running strange quark mass from e^+e^- and τ decay data. Further tests can be provided by precision measurements of the correlators on the lattice and by the $e^+e^- \to$ hadrons data.

PACS: 12.38.Aw; 12.39.Mk; 11.55.Hx

Keywords: Quantum chromodynamics, QCD sum rules, Gluon mass; Nonperturbative physics

Corresponding author: K.G.Chetyrkin,

INSTITUT FUR THEORETISCHE TEILCHENPHYSIK
UNIVERSITAT KARLSRUHE
Kaiserstrasse 12, Postfach 6980
D-76128 Karlsruhe Germany
Telephone: (0721) 608 33 76; fax: (0721) 370 726
e-mail: chet@ttpux2.physik.uni-karlsruhe.de

---

[1] On leave from Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, 117312, Russia.

1 Introduction

In this paper we will outline phenomenology of nonperturbative short-distance physics in QCD based on introduction of a tachyonic gluon mass.
To set up a theoretical framework, we will consider physical observables characterized by a large mass scale Q, $Q^2 >> \Lambda^2_{QCD}$ where Λ_{QCD} is the position of the Landau pole in the running coupling:

$$\alpha_s(Q^2) \simeq \frac{1}{b_0 \ln\left(Q^2/\Lambda^2_{QCD}\right)}, \tag{1}$$

where:

$$b_0 \equiv -\frac{\beta_1}{2\pi} = \frac{1}{4\pi}\left(11 - \frac{2}{3}n_f\right) \tag{2}$$

for n_f flavours, is the first coefficient of the β function. Then in the limit of $Q^2 \to \infty$, QCD reduces to the parton model while at finite Q^2 there are corrections of order $(1/\ln Q)^k$ which are nothing else but perturbative expansions and power corrections, $(\Lambda_{QCD}/Q)^n$ which are nonperturbative in nature.
To be more specific, concentrate on the correlators of currents J with various quantum numbers:

$$\Pi_J(Q^2) \ = \ i\int e^{iqx}\langle 0|T\{J(x), J(0)\}|0\rangle dx\,, \tag{3}$$

where $Q^2 \equiv -q^2$ and we suppress for the moment the Lorentz indices. Furthermore, to stick to the standard notations [1, 2] we introduce the Borel/ Laplace transformed $\Pi(M^2)$ where:

$$\Pi_J(M^2) \ \equiv \ \frac{Q^{2n}}{(n-1)!}\left(\frac{-d}{dQ^2}\right)^n \Pi_J(Q^2) \tag{4}$$

in the limit where both Q^2 and n tend to infinity so that their ratio $M^2 \equiv Q^2/n$ remains finite. Then a standard representation for correlators (3) at large Q^2 goes back to the QCD sum rules [1, 2] and somewhat schematically reads as:

$$\Pi_J(M^2) \approx (parton\ model)\cdot\left(1 + \frac{a_J}{\ln\left(M^2/\Lambda^2_{QCD}\right)} + \frac{c_J}{M^4} + \mathcal{O}\left(\ln^{-2}\left(M^2/\Lambda^2_{QCD}\right), M^{-6}\right)\right) \tag{5}$$

where the constants a_J, c_J depend on the channel, i.e. on the quantum numbers of the current $J(x)$, and we assumed that the currents have zero anomalous dimensions. Moreover the terms of order $1/\ln M^2$ and M^{-4} are associated with the first perturbative correction and the gluon condensate, respectively.
One of the central points about Eq. (5) is the absence of M^{-2} corrections. The reason is that Eq. (5) utilises the standard operator product expansion (OPE) and there are no gauge invariant operators of dimension $d = 2$ which could have vacuum-to-vacuum matrix elements.
We will consider a modified phenomenology accounting for a correction due to a (postulated) non-vanishing gluon mass squared, λ^2. In this approach Eqs (5) are replaced by

$$\Pi_j(M^2) \ \approx \ (parton\ model)\cdot\left(1 + \frac{a_J}{\ln\left(M^2/\Lambda^2_{QCD}\right)} + \frac{b_J}{M^2} + \frac{c_J}{M^4} + \mathcal{O}\left(\ln^{-2}\left(M^2/\Lambda^2_{QCD}\right), M^{-6}\right)\right) \tag{6}$$

where b_J are calculable in terms of λ^2 and depend on the channel considered. As we shall explain later, the introduction of the gluon mass is a heuristic way to account for possible existence of small-size strings. Moreover, it is easy to realize that in this case we expect a tachyonic gluon mass, $\lambda^2 < 0$. Also, we have to confine ourselves to the cases when the λ^2 correction is associated with short distances.

2 Power corrections beyond the OPE

The validity of the OPE beyond the perturbation theory cannot be proven without identifying more precisely nonperturbative mechanisms. One may discuss both infrared- and ultraviolet-sensitive non-perturbative contributions. As was emphasised rather recently [3] the general arguments based on dispersion relations alone cannot rule out extra $1/Q^2$ pieces. One can substantiate the point by the simple observation that the removal of the Landau pole from the running coupling:

$$\alpha_s(Q^2) \simeq \frac{1}{b_0 \ln\left(Q^2/\Lambda^2_{QCD}\right)} \;\rightarrow\; \frac{1}{b_0 \ln\left(Q^2/\Lambda^2_{QCD}\right)} - \frac{\Lambda^2_{QCD}}{b_0(Q^2-\Lambda^2_{QCD})} \tag{7}$$

does introduce a Λ^2_{QCD}/Q^2 term at large Q^2. Note that the ad hoc modification of the coupling in the IR region cannot be accommodated by the OPE. This simple example emphasises that the standard OPE is a dynamical framework based on a particular analysis of a generic Feynman graph, with large Q flowing in and out, and allowing some of the lines to become soft [1, 2]. In this paper we accept the standard OPE to treat the infrared-sensitive contributions.

As for the ultraviolet-sensitive contributions, an insight into power corrections is provided by renormalons (for review and further references see [4]). Renormalons are a particular set of perturbative graphs which result in a divergent perturbative series, $\sum_n a_n\alpha_s^n(Q^2)$ with $a_n \sim n!(-b_0)^n$. Starting with $n = N_{cr} \equiv 1/b_0\alpha_s(Q^2)$ the product $|a_n\alpha_s^n|$ grows with n despite of the smallness of $\alpha_s(Q^2)$. The most common assumption is that the rising branch of the perturbative expansion is summed up a la Borel:

$$\sum_{N_{cr}}^{\infty} n!(-b_0)^n\alpha_s^n(Q^2) \rightarrow \int dt\ \frac{(-b_0\alpha_s(Q^2)\cdot t)^{N_{cr}} exp(-t)}{1+b_0\alpha_s(Q^2)\cdot t} \sim \frac{\Lambda^2_{QCD}}{Q^2}. \tag{8}$$

Although some further arguments can be given to support this assumption [5], the Borel summation still looks an arbitrary solution and the theory is basically undefined on the level of Λ^2_{QCD}/Q^2 terms. Hence, one may argue that there should be nonperturbative terms of the same order, Λ^2_{QCD}/Q^2 which make the theory uniquely defined. Note that this kind of logic is very common in case of infrared renormalons (see, e.g., [4]). There were a few attempts to develop a phenomenology of unconventional Λ^2_{QCD}/Q^2 corrections [6]. Since there is no means to evaluate the nonperturbative terms in various channels directly the crucial ingredient here is the assumption on the nature of nonperturbative terms. In particular, a chiral-invariant quark mass and dominance of four-fermionic operators a la Nambu-Jona-Lasino have been tried for phenomenology [6].

Here we will explore the phenomenology of the $1/Q^2$ corrections assuming that at short distances they can be effectively described by a tachyonic gluon mass. The assumption is of openly heuristic nature and can be motivated in the following way. Consider the static potential $V(r)$ of heavy quarks and represent it as:

$$V(r) \;\approx\; -\frac{4\alpha_s(r)}{3r} + kr. \tag{9}$$

Such a form of the potential is quite common at large distances r with $k \approx 0.2\ \mathrm{GeV}^2$ representing the string tension. At *short* distances one expects that the linear correction to the Coulomb-like potential is replaced by a r^2 term [7]. The vanishing of the linear correction at short distances is a reflection of the absence of the Q^{-2} correction in the current correlators (see above). And vice versa, if the coupling $\alpha_s(Q^2)$ has an unconventional Q^{-2} piece then the potential $V(r)$ has a linear correction at short distances as well [8].

The power-like behaviour of the gluon propagator was a subject of intense theoretical studies both by means of the lattice simulations (see, e.g., [9] and references therein) and by means of the Schwinger-Dyson equation (see, e.g., [10] and references therein). However, these studies refer to large distances. As for the short distances, the smallest distances where the measurements of the potential on the lattice are available are of order 0.1fm [11]. Remarkably, there is no sign of the vanishing of the kr term so far. Although it is not a proof of the presence of the linear term at $r \to 0$ since the measurements were not specifically dedicated to short distances, such a possibility cannot at all be ruled out and one is free to speculate that the linear term does not vanish at short distances. Thus, let us assume that the form (9) remains indeed true as $r \to 0$. There is still no obvious way to evaluate $1/Q^2$ corrections to other variables in terms of the same k . Note, however, that as far as the kr term is treated as a small correction at short distances, it can be reproduced by a tachyonic gluon mass λ^2 where [12]

$$-\frac{2\alpha_s}{3}\lambda^2 \;\approx\; k. \tag{10}$$

In this framework λ^2 is intended to represent the effect of small-size strings. From a calculational point of view, we simply replace the gluon propagator

$$D^{ab}_{\mu\nu}(k^2) \;=\; \frac{\delta^{ab}\delta_{\mu\nu}}{k^2} \;\rightarrow\; \delta^{ab}\delta_{\mu\nu}\left(\frac{1}{k^2}+\frac{\lambda^2}{k^4}\right) \tag{11}$$

and check that the integral is saturated by large momenta, $k^2 \sim Q^2$. One should notice that, to the approximation we are working, our analysis is gauge invariant. (An attempt to generalise the substitution in Eq. (11) to higher loops can be found in Ref. [13]).
There are obvious limitations to this approach. In particular, we do not include terms of order λ^4 and do not go beyond one loop. Moreover, to use the Eq. (11) consistently at all distances one should have evaluated the anomalous dimension of the string tension k which goes beyond the scope of the present note (see, however, [13]). Also, the effect of strings at large distances cannot be reproduced by any modified propagator [14] and this is one more reason to stick to short distances. Finally, the procedure described above assumes that at short distances the kr piece in the potential is associated with a vector exchange while at large distances it corresponds to a scalar exchange and there is no direct evidence to support such a change [11].
With these reservations in mind, it is clear that we would better concentrate first of all on the qualitative features of the approach with $\lambda^2 \neq 0$. Let us emphasise therefore from the very beginning that there are qualitative effects around. Indeed, the estimate in Eq. (10) implies a large numerical value for λ^2. Applying Eq. (10), for example, at distances corresponding to $\alpha_s = 0.3$ we would find $\lambda^2 \approx -1$ GeV^2. Although this estimate cannot be trusted to the accuracy, say, better than factor of 2 such a big gluon mass might seem to be easily ruled out since it would produce too much distortion on the standard phenomenology. We shall argue that it is actually not the case and $\lambda^2 \approx -0.5$ GeV^2 can well be admittable.

3 Puzzles to be resolved

Turning to the phenomenology, let us first note that not all the $1/Q^2$ corrections in QCD are associated with short distances. Respectively, not all the terms proportional to λ^2 can be used to mark short-distance contributions. For example, in case of DIS the $1/Q^2$ corrections are coming from the IR region and perfectly consistent with the OPE. Thus, the class of theoretical objects for which an observation of the $1/Q^2$ corrections would signify breaking of the OPE is limited. One example was the potential $V(r)$ discussed above. Other examples are the correlator functions (3) where the power corrections start with Q^{-4} terms. In terms of the expansion in λ^2, we are guaranteed then that terms proportional to λ^2 are associated with short distances. If such terms arose from the infrared region, then the final result would contain nonanalytical in λ^2 terms, $\lambda^2 \ln \lambda^2$ [15] and we shall see that it is not indeed the case.
Even this limited set of variables exhibits actually a remarkable variety of scales which are not explained by the standard OPE [16]. We will briefly review these puzzles since it is an important question, whether a novel phenomenology we are going to explore can resolve the puzzles of the existing phenomenology.
One of the basic quantities to be determined from the theory is at which scale the parton model predictions for the correlators in Eq. (3) get violated considerably via the power corrections. To quantify the scales inherent to various correlators in Eq. (3) one may introduce the notion of M^2_{crit} which is defined as the value of M^2 at which the power corrections become 10 per cent from the unit term [16]. While choosing just 10 per cent is a pure convention, the meaning of M^2_{crit} is that at lower M^2 the power corrections blow up. Moreover, the values of M^2_{crit} can be evaluated using either experimental or theoretical input. In the latter case the calculation is usually under control only as far as the power correction is relatively small and that is why M^2_{crit} is chosen to refer to a 10 per cent, not 100 per cent correction.

Consider first the best studied ρ-channel. On the theoretical side, Π_ρ is represented as:

$$\Pi_\rho(M^2) \;\approx\; (parton\ model)\cdot\left(1+\frac{\alpha_s(M^2)}{\pi}+\frac{\pi^2}{3M^4}\langle\frac{\alpha_s}{\pi}(G^a_{\mu\nu})^2\rangle+...\right) \tag{12}$$

The value of M^2_{crit} is related then to the value of the gluon condensate, $\langle\frac{\alpha_s}{\pi}(G^a_{\mu\nu})^2\rangle$:

$$M^2_{crit}(\rho - channel) \;\approx\; \sqrt{10\frac{\pi}{3}\langle\alpha_s G^2\rangle} \;\approx\; (0.6 \;-\; 0.8)\ \mathrm{GeV}^2 \tag{13}$$

where the factor of 10 in the r.h.s. reflects the 10 per cent convention introduced above. An independent information on M^2_{crit} in this channel can be obtained by using the dispersion relations for $\Pi_\rho(M^2)$ and integrating over the corresponding experimental cross section of e^+e^- annihilation into hadrons, or by optimising the M^2-dependence of the Borel/Laplace sum rules. The value of M^2_{crit} is then most sensitive to parameters of the ρ-meson and comes out about the same 0.6 GeV^2 as indicated by Eq. (13). Since the gluon condensate parametrises contribution of the infrared region one may say that at least in the ρ-channel the breaking of the asymptotic freedom is due to infrared phenomena. Recent detailed experimental studies of the τ-decays did not contradict this picture [17].
If one proceeds to other channels, in particular to the π-channel and to the $0^\pm$-gluonium channels, nothing special happens to M^2_{crit} associated with the infrared-sensitive contribution parametrised by the gluon condensate. However, it was determined from independent arguments [16] that the actual values of M^2_{crit} do vary considerably in these channels:

$$M^2_{crit}(\pi - channel) \;\geq\; 2\ \mathrm{GeV}^2 \tag{14}$$

$$M^2_{crit}(0^\pm - gluonium\ channel) \;\approx 15\ \mathrm{GeV}^2. \tag{15}$$

In the pion channel, the lower bound on M^2_{crit} can be determined by evaluating the pion contribution to the dispersive integral and equating it to the parton-model result. Since all other hadronic states give positive-definite contributions, at lower M^2 the parton model is certainly violated. In more detail, one gets:

$$M^2_{crit}(\pi - channel) \;\geq\; \sqrt{\frac{16\pi^2}{3}\frac{f_\pi^2 m_\pi^4}{(\bar{m}_u + \bar{m}_d)^2}} \approx 1.8\ \mathrm{GeV}^2, \tag{16}$$

where the running light quark masses are evaluated at the scale M^2, and $f_\pi = 93$ MeV. A detailed M^2 optimisation analysis of the pion sum rule gives a value [2]:

$$M^2_{crit}(\pi - channel) \approx (2 \sim 2.5)\ \mathrm{GeV}^2. \tag{17}$$

Although the difference of factor (3-4) in the values of M^2_{crit} in the ρ- and π-channels might seem not so big, it cannot be matched by the standard IR contributions.

Note also that the difference in M^2_{crit} in the ρ- and π-channels was confirmed by the direct measurements of the corresponding correlators on the lattice [18]. These measurements also provide with an independent support to the idea that asymptotic freedom is violated at moderate $M^2 \sim 1\ GeV^2$ sharply, due to power corrections. It is most important in case of channels with large perturbative corrections, the fact which might give rise to doubts in relevance of the notion of M^2_{crit}
In case of the scalar-gluonium channel, the value of M^2_{crit} follows [16] from a low-energy theorem for the correlator associated with the current $\alpha_s(G^2_{\mu\nu})^2$. This low-energy theorem can be translated into correction to the parton model predictions at large M^2:

$$\Pi_G(M^2) \approx (parton\ model) \cdot \left(1 + \left(\frac{4}{b_0}\right)\left(\frac{\pi}{\alpha_s}\right)^2 \frac{\langle\alpha_s G^2\rangle}{M^4} + ...\right), \tag{18}$$

where b_0 is defined in Eq. (2). Thus, the low-energy theorem brings in a large numerical factor $12/(b_0\pi)\left(\pi^2/\alpha_s^2\right) \approx 400$ in front of the M^{-4} correction as compared with the ρ-channel. This changes dramatically the estimate of M^2_{crit} to:

$$M^2_{crit}(0^+ - gluonium) \;\approx\; 20 M^2_{crit}(\rho - channel) \;\approx\; 15\ \mathrm{GeV}^2\ , \tag{19}$$

and of the resonance properties, respectively (see Ref.[20, 19]), in rough agreement with the M^2-stability analysis of the scalar [20, 19] and pseudoscalar [21, 19] gluonia sum rules. Moreover, the huge power correction (18) remains of a uniquely large scale and is not supported by any other large-scale correction which makes it very difficult to interpret the breaking of the asymptotic freedom in terms of resonances [16].

Also, the phenomenology build on the IR power corrections cannot resolve the problem of η'-meson. As is known from general considerations [22] a dynamical resolution of this problem is not possible without accounting for field configurations of nontrivial topology. On the other hand, the standard OPE is based on standard, i.e. perturbative Feynman graphs and cannot account for a nontrivial topology. Hence, one invokes direct instantons when the currents J interact at large Q^2 with small-size instantons [16]. The instanton-induced contributions to the correlators in Eq. (3) were extensively studied in the literature (for review and references see [23]). We shall further comment on this point later.

4 Some phenomenological implications

Now, we add a new term proportional to λ^2 to the theoretical side of $\Pi_J(M^2)$, see Eq.(6). It is convenient to consider the effect of $\lambda^2 \neq 0$ channel by channel.

4.1 Constraint on λ^2 from the ρ-channel

Phenomenologically, in the ρ-channel there are severe restrictions [24] on the new term b_ρ:

$$b_\rho \approx - (0.03 - 0.07) \text{ GeV}^2. \tag{20}$$

To find out the implications of this constraint for the gluon mass, we turn to the correlator of two vector currents $J_V^\mu(x) \equiv \bar{\psi}_i \gamma^\mu \psi_j(x)$, viz.

$$\begin{aligned} \Pi_V(Q^2) &\equiv i \int d^4x \, e^{iqx} \langle 0 | \mathcal{T} J_V^\mu(x) \, (J_V^\nu) \dagger (0) | 0 \rangle, \\ &- (g^{\mu\nu} q^2 - q^\mu q^\nu) \Pi_V^{(1)}(Q^2) + q^\mu q^\nu \Pi_V^{(0)}(Q^2). \end{aligned} \tag{21}$$

Here the indexes i, j correspond to quark flavours; m_i is the mass of the quark i. To first order in α_s and expanding in $m_{i,j}$ we obtain: [2]

$$\Pi_V^{(1)} = \Pi_{V,con}^{(1)} + \Pi_{V,NO}^{(1)}, \tag{22}$$

where

$$\Pi_{V,con}^{(1)} = \frac{m_j \langle \bar{\psi}_i \psi_i \rangle + m_i \langle \bar{\psi}_j \psi_j \rangle}{Q Q^2} \tag{23}$$

and

$$\begin{aligned} (16\pi^2) \Pi_{V,NO}^{(1)} = & \\ + & \left[\frac{20}{3} + 6 \frac{m_-^2}{Q^2} - 6 \frac{m_+^2}{Q^2} + 4 l_{\mu Q} + 6 \frac{m_-^2}{Q^2} l_{\mu Q} \right] \\ + & \frac{\alpha_s}{\pi} \left[\frac{55}{3} - 16\,\zeta(3) + \frac{107}{2} \frac{m_-^2}{Q^2} - 24\,\zeta(3) \frac{m_-^2}{Q^2} - 16 \frac{m_+^2}{Q^2} \right. \\ & \left. + 4 l_{\mu Q} + 22 \frac{m_-^2}{Q^2} l_{\mu Q} - 12 \frac{m_+^2}{Q^2} l_{\mu Q} + 6 \frac{m_-^2}{Q^2} l_{\mu Q}^2 \right] \\ + & \frac{\alpha_s}{\pi} \frac{\lambda^2}{Q^2} \left[-\frac{128}{3} + 32\,\zeta(3) - \frac{46}{3} \frac{m_-^2}{Q^2} + 16\,\zeta(3) \frac{m_-^2}{Q^2} - 18 \frac{m_+^2}{Q^2} \right. \\ & \left. + 6 \frac{m_-^2}{Q^2} l_{iQ} - 6 \frac{m_+^2}{Q^2} l_{iQ} + 6 \frac{m_-^2}{Q^2} l_{jQ} - 6 \frac{m_+^2}{Q^2} l_{jQ} \right]. \end{aligned} \tag{24}$$

The above result is in $\overline{\text{MS}}$ scheme and the notations are as follows:

$$m_\pm = m_1 \pm m_2, \quad l_{\mu Q} = \log(\frac{\mu^2}{Q^2}), \quad \text{and} \quad l_{iQ} = \log(\frac{m_i^2}{Q^2}).$$

[2]The results described in Eqs. (24,25,34, 67) below have been obtained with the help of program packages MATAD [25] and MINCER [26] written in FORM [27].

The normal-ordered quark condensates in Eq. (22) do not receive by definition any perturbative corrections and are displayed explictly only to make easier the discussion below. Note that the terms of order λ^2/Q^2 in Eq. (24) are μ independent and, thus, do not depend on the way how the overall UV subtraction of the correlator in Eq. (21) has been fixed. Eq. (24) are written for the case of normal ordered quark condensate, that is the reason why quark mass logs appear there. As is well-known these are of long-distance nature and can (and even should) be absorbed into the quark condensate [28, 29, 30]. To do this we should use normal non-ordered condensates, which means that perturbative theory contributions should *not* be automatically subtracted from the latter. Instead, these contributions are to be minimally renormalized.
In order α_s there are two diagrams (see Fig. 1) leading to nonzero vacuum expectation value of $\langle\bar{\psi}_i\psi_i\rangle$ in perturbation theory. A simple calculation gives

$$\begin{aligned}\langle\bar{\psi}_i\psi_i\rangle = &\quad \frac{3m_i^3}{4\pi^2}\left[1+\ln(\frac{\mu^2}{m_i^2})+2\frac{\alpha_s}{\pi}\left(\ln^2(\frac{\mu^2}{m_i^2})+\frac{5}{3}\ln(\frac{\mu^2}{m_i^2})+\frac{5}{3}\right)\right]\\ &+\quad \frac{m_i\lambda^2}{4\pi^2}\frac{\alpha_s}{\pi}\left(-5+6\ln\frac{\mu^2}{m_i^2}\right).\end{aligned} \tag{25}$$

Now in order to find the polarization operator $\Pi_V^{(1)}$ in the case of normal non-ordered condensates one proceed as follows. $\Pi_{V,NO}^{(1)}$ to the combination $\Pi_{V,NON}^{(1)}+\Pi_{V,con}^{(1)}$ with the condensate values taken from Eq. (25). The result of the treatment indeed does not contain any mass logs and reads

$$\begin{aligned}(16\pi^2)\Pi_{V,NON}^{(1)} &= \left[\frac{20}{3}+6\frac{m_-^2}{Q^2}-6\frac{m_+^2}{Q^2}+4l_{\mu Q}+6\frac{m_-^2}{Q^2}l_{\mu Q}\right]\\ &+ \frac{\alpha_s}{\pi}\left[\frac{55}{3}-16\,\zeta(3)+\frac{107}{2}\frac{m_-^2}{Q^2}-24\,\zeta(3)\frac{m_-^2}{Q^2}-16\frac{m_+^2}{Q^2}\right.\\ &+ \left.4l_{\mu Q}+22\frac{m_-^2}{Q^2}l_{\mu Q}-12\frac{m_+^2}{Q^2}l_{\mu Q}+6\frac{m_-^2}{Q^2}l_{\mu Q}^2\right]\\ &+ \frac{\alpha_s}{\pi}\frac{\lambda^2}{Q^2}\left[-\frac{128}{3}+32\,\zeta(3)-\frac{76}{3}\frac{m_-^2}{Q^2}+16\,\zeta(3)\frac{m_-^2}{Q^2}\right.\\ &- \left.8\frac{m_+^2}{Q^2}+12\frac{m_-^2}{Q^2}l_{\mu Q}-12\frac{m_+^2}{Q^2}l_{\mu Q}\right].\end{aligned} \tag{26}$$

As it follows from Eq. (25) the quark condensate obeys the non-trivial RGE:

$$\mu^2\frac{d}{d\mu^2}m_q\langle\bar{\psi}_{\rm i}\psi_{\rm i}\rangle = \frac{3m_q m_i\lambda^2}{2\pi^2}\frac{\alpha_s}{\pi}, \tag{27}$$

where we have neglected all quartic quark mass terms. For further analysis, we shall choose the subtraction point μ equal to the sum rule scale M, as discussed in [31].

In the light-quark case relevant to the ρ-channels we can neglect the m^2 terms and $\Pi_\rho(M^2)$ simplifies greatly:

$$\Pi_\rho(M^2) = (parton\ model)\cdot\left(1+\left(\frac{\alpha_s}{\pi}\right)\left[1+\frac{\lambda^2}{M^2}\left(-\frac{32}{3}+8\zeta(3)\right)\right]\right), \tag{28}$$

or, in numerical form,

$$\Pi_\rho(M^2) = (parton\ model)\cdot\left(1+\left(\frac{\alpha_s}{\pi}\right)\left[1-1.05\frac{\lambda^2}{M^2}\right]\right), \tag{29}$$

This result was actually obtained earlier [32]. It is interesting that terms of order M^{-2} were looked for first [24] in connection with proposal (see first paper in Ref. [6]) that UV renormalons can be imitated by a nonperturbative quark mass. In terms of the quark mass the correction is substantially larger and is $-6m^2/M^2$ (see Eq. (26)). The conclusion of the analysis [24] was that the data allow only for an *imaginary* quark mass and quite a small one:

$$m_q^2 \simeq -(71-114)^2\ \mathrm{MeV}^2 \tag{30}$$

(further analysis favoured rather lower end of the band [33]).
Now, that we are interpreting possible M^{-2} correction in terms of the gluon mass, the conclusion [24] that a nonperturbative quark mass could only be imaginary fits very well our scheme with a tachyonic gluon mass. Moreover, the value of the quark mass, say, $m_q^2 = -0.01$ GeV^2 is translated into a much larger gluon mass, $\lambda^2 \approx -0.5$ GeV^2.
To summarize our discussion of the ρ-channel, we find an important independent indication that the present data could be interpreted in terms of a tachyonic gluon mass:

$$\lambda^2(1 \text{ GeV}) \approx -(0.2 - 0.5) \text{ GeV}^2 , \tag{31}$$

where we have used $(\alpha_s/\pi)(1 \text{ GeV}) \simeq 0.17 \pm 0.02$. This result indicates that, even relatively large masses , say, $\lambda^2 \approx -0.5$ GeV^2 cannot be ruled out. A further improvement of the (axial-) vector spectral function from e^+e^- or/and τ decay data would improve the present constraint on the eventual existence of such a term.

4.2 Alternative estimate of λ^2 from the π-channel

We shall be concerned with the (pseudo)scalar two-point correlator:

$$\psi_{(5)}(Q^2) \equiv i \int d^4x \ e^{iqx} \ \langle 0|\mathcal{T} J_{(5)}(x) J^\dagger_{(5)}(0)|0\rangle, \tag{32}$$

built from current of the bilinear light quark fields:

$$J_{(5)}(x) = (m_i + (-)m_j)\bar{q}_i(i\gamma_5)q_j, \tag{33}$$

having the quantum numbers of the pseudoscalar (O^{-+}) π or scalar (O^{++}) a^0/δ mesons. In the chiral limit $(m_u \simeq m_d = 0)$, the QCD expression of the absorptive part of the correlator reads:

$$\frac{1}{\pi}\text{Im}\psi_{(5)}(s) \simeq (m_i + (-)m_j)^2 \frac{3}{8\pi^2} s \left[1 + \left(\frac{\alpha_s}{\pi}\right) \left(-2\log\left(\frac{s}{\mu^2}\right) + \frac{17}{3} - 4\frac{\lambda^2}{s} \right) \right] , \tag{34}$$

where one should notice that the coefficient of the λ^2 term:

$$b_\pi \approx 4 b_\rho \ . \tag{35}$$

In order to see the effect of this term, we work with the Borel/Laplace sum rule:

$$\mathcal{L}(\tau) = \int_{t_\leq}^{\infty} dt \ \mathrm{e}^{-t\tau} \ \frac{1}{\pi} \ \text{Im}\psi_{(5)}(t), \tag{36}$$

and the corresponding ratio of moments:

$$\mathcal{R}(\tau) \equiv -\frac{d}{d\tau} \log \mathcal{L}(\tau), \tag{37}$$

where $t_\leq$ is the hadronic threshold. The QCD expression of the sum rule can be written as:

$$\begin{aligned}
\mathcal{L}(\tau) &= \frac{3}{8\pi^2} \Big[\overline{m}_u(\tau) + \overline{m}_d(\tau) \Big]^2 \tau^{-2} \left[1 + \delta_0 + (\delta_2 + \delta_\lambda)\tau + \delta_4\tau^2 + \delta_6\tau^3 \right] \\
-\frac{d\mathcal{L}}{d\tau} &= 2\frac{3}{8\pi^2} \Big[\overline{m}_u(\tau) + \overline{m}_d(\tau) \ Big]^2 \tau^{-3} \left[1 + \delta'_0 + \frac{1}{2}(\delta_2 + \delta_\lambda)\tau - \frac{1}{2}\delta_6\tau^3 \right] \\
\mathcal{R}(\tau) &\equiv -\frac{d\mathcal{L}}{d\tau} \Big/ \mathcal{L} \ ,
\end{aligned} \tag{38}$$

where [34]–[36]:

$$\begin{aligned}
\delta_0 &= 4.821 a_s + 28.953 a_s^2 \\
\delta'_0 &= 6.821 a_s + 57.026 a_s^2
\end{aligned} \tag{39}$$

Table 1: Ratio of moments $\mathcal{R}(\tau)$ and value of λ^2

$\tau \equiv 1/M^2[\text{GeV}^{-2}]$	$\mathcal{L}_{exp}/(2f_\pi^2 m_\pi^4)$	R_{exp}	$R_{QCD}^{\lambda^2=0}$	$-(\alpha_s/\pi)\lambda^2[\text{GeV}]^2$
1.4	1.20	0.27 ± 0.06	$0.66^{+0.50}_{-0.31}$	$0.09^{+0.08}_{-0.06}$
1.2	1.29	0.36 ± 0.07	$0.79^{+0.42}_{-0.28}$	$0.10^{+0.09}_{-0.05}$
1.0	1.42	0.46 ± 0.10	$0.94^{+0.34}_{-0.25}$	$0.11^{+0.07}_{-0.06}$
0.8	1.61	0.58 ± 0.12	$1.10^{+0.26}_{-0.21}$	0.12 ± 0.06
0.6	1.89	0.73 ± 0.15	$1.24^{+0.20}_{-0.17}$	0.12 ± 0.06
0.4	2.29	0.89 ± 0.16	$1.33^{+0.15}_{-0.13}$	0.10 ± 0.06

while:

$$\delta_\lambda = -4a_s\lambda^2 \ . \tag{40}$$

By keeping the leading linear and quadratic mass terms, one has [1, 2]:

$$\begin{aligned}
\delta_2 &= -2(\overline{m}_u^2 + \overline{m}_d^2 - \overline{m}_u\overline{m}_d)\tau, \\
\delta_4 &= \frac{8\pi^2}{3}\left\{\frac{1}{8\pi}\langle\alpha_s G^2\rangle - \left(m_d - \frac{m_u}{2}\right)\langle\bar{u}u\rangle + (u \leftrightarrow d)\right\}, \\
\delta_6 &= k\frac{896}{27}\pi^3\alpha_s\langle\bar{u}u\rangle^2.
\end{aligned} \tag{41}$$

$\overline{m}_i(\tau)$ and $a_s \equiv \overline{\alpha}_s(\tau)/\pi$ are respectively the running quark masses and QCD coupling. The expression of the running coupling to two-loop accuracy can be parametrized as ($\nu^2 \equiv \tau^{-1}$):

$$a_s(\nu) \equiv \left(\frac{\bar{\alpha}_s}{\pi}\right) = a_s^{(0)}\left\{1 - a_s^{(0)}\frac{\beta_2}{\beta_1}\log\log\frac{\nu^2}{\Lambda^2} + \mathcal{O}(a_s^2)\right\}, \tag{42}$$

with:

$$a_s^{(0)} \equiv \frac{1}{-\beta_1\log(\nu/\Lambda)} \ , \tag{43}$$

and β_i are the $\mathcal{O}(a_s^i)$ coefficients of the β function in the $\overline{\text{MS}}$-scheme for n_f flavours, which, for three flavours, read:

$$\beta_1 = -9/2 \ , \qquad \beta_2 = -8 \ . \tag{44}$$

Λ is a renormalization group invariant scale but is renormalization scheme dependent. The expression of the running quark mass in terms of the invariant mass $\hat{m}_i$ is [2]:

$$\overline{m}_i(\nu) \quad = \quad \hat{m}_i\left(-\beta_1 a_s(\nu)\right)^{-\gamma_1/\beta_1}\left\{1 + \frac{\beta_2}{\beta_1}\left(\frac{\gamma_1}{\beta_1} - \frac{\gamma_2}{\beta_2}\right)a_s(\nu) + \mathcal{O}(a_s^2)\right\} \tag{45}$$

where γ_i are the $\mathcal{O}(a_s^i)$ coefficients of the quark-mass anomalous dimension. For three flavours, we have:

$$\gamma_1 = 2 \ , \qquad \gamma_2 = 91/12 \ . \tag{46}$$

We shall use in our numerical analysis, the QCD parameters compiled in [2]:

$$\begin{aligned}\Lambda &= (375\pm 50)\ \mathrm{MeV}\\ \langle\alpha_s G^2\rangle &= (0.07\pm 0.01)\ \mathrm{GeV}^4\\ \langle\bar{u}u\rangle^{1/3}(1\ \mathrm{GeV}) &= -(229\pm 9)\ \mathrm{MeV}\\ k &\simeq 2-3\\ M_0^2 &= (0.8\pm 0.1)\ \mathrm{GeV}^2\end{aligned}\tag{47}$$

where k indicates the deviation from the vacuum saturation assumption of the four-quark condensates, while M_0^2 parametrize the mixed quark-gluon condensate:

$$\langle\bar{u}G^a_{\mu\nu}\frac{\lambda_a}{2}u\rangle = M_0^2\langle\bar{u}u\rangle\ .\tag{48}$$

One can notice that the ratio of moments $\mathcal{R}(\tau)$ is not affected by the quark masses to leading order of chiral symmetry breaking terms. Therefore, we shall use it for extracting λ^2. We parametrize the pseudoscalar spectral function as usual by the pion pole plus the 3π contribution.The 3π threshold has been calculated using lowest order chiral perturbation theory, while two π' resonances are introduced with the following widths and masses in units of MeV [37]:

$$M_1 = 1300\pm 100\quad \Gamma_1 = 400\pm 200\quad \text{and}\quad \mathrm{M}_2 = 1770\pm 30\quad \Gamma_2 = 310\pm 50\tag{49}$$

Including finite width corrections, one obtains:

$$\frac{1}{\pi}\mathrm{Im}\psi_5(s) = 2f_\pi^2 m_\pi^4\left[\delta(s-m_\pi^2)+\theta(s-9m_\pi^2)\frac{1}{(16\pi^2 f_\pi^2)^2}\frac{s}{18}\rho_{had.}(s)\right]\tag{50}$$

where $\rho_{had.}(t)$ is given in [37]. We use, in our analysis, the largest range of predictions given by the two different parametrisations of the hadronic spectral function proposed in [37], which correspond to the two values $\xi = -0.23 + i\ 0.65$ (best duality with QCD) and $\xi = 0.23 + i\ 0.1$ (best fit to the experimental curve for the observed $\pi'(1770)$ in hadronic reactions) of the phenomenological interference complex parameter ξ between the two π' states. The result of this analysis corresponding to $\mathcal{R}_{exp}(\tau)$ is summarized in Table 1, where we have also used $t_c \simeq 2.5$ GeV^2 as fixed from the duality analysis in [37]. Therefore, we can deduce to a good approximation, at the $\tau-$stability region ($\tau \approx 0.8$ GeV^{-2}):

$$(\alpha_s/\pi)\lambda^2 \simeq \frac{1}{4}\left[\mathcal{R}_{exp} - \mathcal{R}_{QCD}^{\lambda^2=0}\right] \simeq -(0.12\pm 0.06)\ \mathrm{GeV}^2.\tag{51}$$

One can notice, by comparing this result with the one from the ρ-channel, that, to leading order in λ^2, the two estimates lead to the consistent value:

$$(\alpha_s/\pi)\lambda^2 \simeq -(0.06-0.07)\ \mathrm{GeV}^2 \quad\Longrightarrow\quad \lambda^2(\tau\approx 0.8\ \mathrm{GeV}^{-2}) \simeq -(0.43\pm 0.09)\ \mathrm{GeV}^2\ .\tag{52}$$

One can also notice in Table 1 that the introduction of λ has enlarged the region of QCD duality [37] or τ stability [2, 38] to a lower value of $M^2 \equiv 1/\tau$ of about M_ρ.
To summarize, the pseudoscalar channel can be considered as a success for the phenomenology with $\lambda^2 \neq 0$.

4.3 Effects of λ^2 on $\overline{m}_{u,d}$ and $\overline{m}_s$ from the (pseudo)scalar channels

Re-analysing the $\mathcal{L}$ sum rule by the inclusion of this new λ term, one can also notice that the presence of such a term tends to decrease only slightly the prediction for the sum $(\overline{m}_u+\overline{m}_d)$ of the running light quark masses by about:

$$\delta_{ud} \approx -5.6\%\tag{53}$$

Applying this effect to the recent estimates in [37, 38], one obtains:

$$(\overline{m}_u+\overline{m}_d)(1\ \mathrm{GeV}) = (11.3\pm 2.4)\ \mathrm{MeV} \quad\Longrightarrow\quad (\overline{m}_u+\overline{m}_d)(2\ \mathrm{GeV}) = (8.6\pm 2.1)\ \mathrm{MeV}\ .\tag{54}$$

For the estimate of the strange quark mass from the kaon channel, one also obtains a decrease of:

$$\delta_{us} \approx -5\%\tag{55}$$

which is slightly lower than the one of the pion channel due to the partial cancellation of the λ effect by the m_s^2 one.

4.4 λ^2 and the value of $\alpha_s(M_\tau)$ from tau decays

A natural extension of the analysis is to test the effect of λ^2 on the value of α_s extracted from tau-decays. For this purpose, we re-do the analysis from R_τ given for the first in [39], by adding the new λ^2 term. The modified expression of the tau hadronic width is [39]:

$$R_\tau = 3\left((|V_{ud}|^2 + |V_{us}|^2\right) S^\tau_{EW}\left\{1 + \delta^\tau_{EW} + \delta^\tau_{PT} + \delta^\tau_2 + \delta^\tau_\lambda + \delta^\tau_{NP}\right\}. \tag{56}$$

where: $|V_{ud}| = 0.9751 \pm 0.0006$, $|V_{us}| = 0.221 \pm 0.003$ are the weak mixing angles; $S^\tau_{EW} = 1.0194$ and $\delta^\tau_{EW} = 0.0010$ are the electroweak corrections; δ^τ_2 is a small correction from the running light quark masses; δ^τ_{NP} are non-perturbative effects due to operators of dimension ≥ 4. The correction due to

Table 2: Reduction of $\alpha_s(M_\tau)$ for different values of λ^2

$-\lambda^2$ [GeV^2]	$-\delta^{\alpha_s}_\lambda$[%]
0.16	4 ± 1
0.25	7 ± 1
0.34	9 ± 1
0.43	11 ± 2
0.52	14 ± 2
0.61	16 ± 3

the new dimension-2 term is:

$$\delta^\tau_\lambda = 2\frac{b_\rho}{M^2_\tau}. \tag{57}$$

where, from Eq. (29), $b_\rho = -1.05a_s\lambda^2$ and λ^2 is given in Eq. (52). At fixed order of perturbation theory, one has [40, 39]:

$$\delta^\tau_{PT} = a_s + 5.2023a_s^2 + 26.366a_s^3 + \mathcal{O}(a_s^4)\ , \tag{58}$$

where we have truncated the series at the complete computed coefficients. We use the values of the QCD non-perturbative parameters given in [33] and the experimental value [17] $R_\tau = 3.484 \pm 0.024$. By comparing the extracted value of $\alpha_s(M_\tau)$ obtained using $\lambda^2 = 0$ and $\lambda^2 \neq 0$, we deduce in Table 2, using the previous approximations, a reduction of the $\alpha_s(M_\tau)$-value for different values of λ^2. The error given in Table 2 reflects the uncertainty on the eventual scale-dependence of the quantity $a_s\lambda^2$ or λ^2. The quoted central value comes from the average between the result corresponding to the freedom by taking one of these two quantities as the input number in the analysis.

• If we use the value of λ^2 as given in Eq. (52), then, we get to leading order in λ^2:

$$\delta^{\alpha_s}_\lambda \simeq -(11 \pm 3)\%. \tag{59}$$

On the other hand, a comparison, at the τ-mass, of the value of α_s from the Z or/and the world average [41] with the one from τ decays, where the latter includes α_s^3 corrections, gives a difference of about [42]:

$$\delta^{\alpha_s}_\lambda \simeq -(5 \pm 10)\%, \tag{60}$$

which is of the same sign and compatible with the correction in Eq. (59). One should also have in mind that there are obvious limitations on the accuracy of the estimate in Eq. (59), due to the no control so far over terms proportional to λ^2 and higher powers of $\alpha_s(M^2_\tau)$, including the anomalous dimension of the tachyon mass. These unknown effects might be taken into account by enlarging the error in Eq. (59) by about a factor 2.

• Inversely, one can improve the previous determinations of λ^2 from the ρ and π channels, either by adding, in the different experimental fits [17] of the moments of the τ-decays [39, 45], the new λ^2 term with the extra (compared with previous experimental fits) constraint that the sign of its contribution is unambiguously fixed, or by attributing the eventual small deviation of the values of α_s from τ and from e.g. $Z-$decays as being due to this new term. If one proceeds in the later way, and use the result in Eq. (60), one would obtain from Table 2:

$$\lambda^2(M_\tau) \simeq -(0.2 \pm 0.4)\ \mathrm{GeV}^2\ , \tag{61}$$

which is less conclusive than the result in Eq. (52).

• Since the introduction of the gluon mass still presumably gives a better control over $1/M_\tau^2$ terms which have been introduced so far ad hoc and are the major sources of theoretical uncertainties [43]–[46], one expects, within the present approach, an improvement of the theoretical errors in the determination of $\alpha_s(M_\tau)$ by about a factor 2.

• Furthermore, by comparing our previous results with some other approaches which resum the higher order terms of the perturbative series (see e.g. [45, 46]), the effect of the $1/M^2$ term becomes manifest when one extends the analysis of the tau hadronic widths to a lower hypothetical τ-lepton mass [39, 33, 17]. We have therefore checked that until $M_\tau \approx 1.3$ GeV, the agreement of the prediction with the recent data [17] remains quite good.

• Finally, one should notice that a naïve redefinition of α_s by a coupling which contains implicitly a λ^2/M^2 term, in the expression of the physical hadronic width R_τ, but not on the level of the propagator in Eq. (11), will lead to inconsistencies.

4.5 Effects of λ^2 on the value of $\overline{m}_s$ from e^+e^- and τ decays

In the determinations of m_s based on the ratio [38] or on the difference [47] of the non-strange and strange quark channels, the leading terms which are flavour independent will not contribute. Taking the leading term in $m_s^2\lambda^2$, which can be obtained from the previous expression of $\Pi_V^{(1)}$, one can deduce that the presence of the λ term implies a slight decrease of the value of m_s of about (1–3)%, which is, however, smaller than the quoted errors (15%) in its determination.
In the case of the determinations of m_s from the alone $\Delta S = 1$ part of the inclusive τ decay rate [48], one can check, after using the resummed series given there, and the expressions of the different corrections in the massless case given in the previous subsection, that the presence of λ tends to increase by about (8–9)% the value of m_s obtained from this method. Again, this effect is still smaller than the large uncertainties (about 30%) of this determination from this method.
However, the effect of λ tend to improve the agreement betwen the two different determinations.

4.6 The quest of the scalar a^0/δ and K_0^*-channels

Consideration of the correlator of the scalar J_δ current (or of its strange analogue $J_{K_0^*}$), where

$$J_\delta = \frac{1}{\sqrt{2}}\left(\bar{u}u - \bar{d}d\right) \tag{62}$$

brings most interesting problems and challenges to the phenomenology with $\lambda^2 \neq 0$.
Indeed, in the limit of massless quarks, the (ordinary) QCD expressions of the pion and scalar sum rules only differ for the contributions of the dimension-six (or more) condensates, where here:

$$\delta_6 = -k\frac{1408}{81}\pi^3\alpha_s\langle\bar{u}u\rangle^2, \tag{63}$$

such that the strength of M_{crit}^2 or the optimization scale of about 2 GeV^2 [2] is obviously the same in both channels, i.e. much larger than the one of the ρ-channel. However, one should also notice

that, in this case of ordinary OPE ($\lambda^2 = 0$ and neglect of the instanton effects), the scalar sum rules can reproduce within a good accuracy the experimental mass of the a^0/δ meson, while the predicted decay constant leads to a width consistent with the present data [2]. In the same way, the estimate of the running light quark masses from the scalar sum rules [49, 2, 35, 36] within the ordinary OPE gives a prediction consistent with the analysis from other methods [41] or from the e^+e^- [38], τ decay data [48] or a global sum rule extraction of the light quark condensate [50].
It is also obvious that M^2_{crit} associated with $\lambda^2 \neq 0$ is the same in the pion and δ-channels:

$$M^2_{crit}(\pi - channel) \;\approx\; M^2_{crit}(\delta - channel). \tag{64}$$

What may be even a more drastic prediction is that the sign of the leading M^{-2} correction is the same in the two channels. Moreover, a detailed analysis of the a_0/δ sum rule in the case $\lambda^2 \neq 0$ gives acceptable phenomenological results, and moves the scale of optimization to lower values of M^2_{crit} like the case of the pion.
On the other hand, direct instantons predict the same M^2_{crit} in the both channels but the opposite signs for the leading correction [23]. Thus, the scalar channel might be the right place to discuss in more details the choice between the direct instantons and the M^{-2} corrections.
On pure theoretical grounds, both direct instantons and nonperturbative M^{-2} corrections seem to be indispensable parts of the QCD phenomenology. Indeed, instantons are necessary to resolve the $\eta^{'}$-problem. The nonperturbative M^{-2} terms are necessary to match the M^{-2} uncertainty of the perturbative series due to the UV renormalons.
All these qualitative arguments cannot fix, however, which correction (if any of the two) becomes important first at large M^2. The signatures of the direct instantons and of the tachyonic mass are different in the π- and δ-channels. Namely, the tachyonic mass gives the same-sign correction while direct instantons result in opposite signs for the deviations from the asymptotic freedom. The existing lattice data [18] indicate, to our mind, that it is rather a mixture of the two mechanisms which works. Indeed, the corrections in the two channels are rather of opposite signs but the correction in the pion channel is substantially stronger. In the δ-channel the correction is much smaller than it should be if the instanton-model parameters are normalized to the π-channel data.
Further data with better accuracy could be helpful. For example, it is not ruled out that in the δ-channel the correction is first positive because of the gluon mass and becomes negative at lower M^2 because of the effect of the direct instantons.

4.7 The gluonia channels

We shall first be concerned with the two-point correlator:

$$\Pi_G(Q^2) \;\equiv\; i\int d^4x \; e^{iqx} \; \langle 0|\mathcal{T} J_G(x)\,(J_G(0))^\dagger\,|0\rangle \tag{65}$$

associated to the scalar gluonium current:

$$J_G = \frac{\beta(\alpha_s)}{\alpha_s}(G^a_{\mu\nu})^2. \tag{66}$$

Its evaluation leads to:

$$\Pi_G(M^2) = (parton\ model)\left(1 - \frac{3\lambda^2}{M^2} + ...\right). \tag{67}$$

Thus, one can expect that the λ^2 correction in this channel is relatively much larger since it is not proportional to an extra power of α_s. With $\lambda^2 \approx -0.5$ GeV^2 we obtain, by using the same 10 per cent convention as in section 3:

$$M^2_{crit}(0^+ - gluonium) \;\approx\; 15 \text{ GeV}^2 \tag{68}$$

in amusing agreement with the independent estimate in Eq. (19).

Exactly the same phenomenon is observed for the case of the two point correlator of the pseudoscalar gluonium currents: the relative strength of the λ^2/M^2 term added to the parton result coincides with that for the scalar gluonium displayed in Eq. (67).

However, one should notice that a more quantitative analysis based on τ-stability of the corresponding (pseudo)scalar sum rules [19]–[21] leads to a lower value of $M^2_{crit} \approx (3-5)$ GeV^2, but still much larger than the scale of the ρ meson.

Let us consider now the case of the tensor gluonium with the correlator:

$$
\begin{aligned}
&\psi^T_{\mu\nu\rho\sigma}(q) \equiv i \int d^4x \; e^{iqx} \langle 0|\mathcal{T}\theta^g_{\mu\nu}(x)\theta^g_{\rho\sigma}(0)^\dagger|0\rangle \\
= \;& \psi^T_4 \left(q_\mu q_\nu q_\rho q_\sigma - \frac{q^2}{4}(q_\mu q_\nu g_{\rho\sigma} + q_\rho q_\sigma g_{\mu\nu}) + \frac{q^4}{16}(g_{\mu\nu}g_{\rho\sigma}) \right) \\
+ \;& \psi^T_2 \left(\frac{q^2}{4} g_{\mu\nu}g_{\rho\sigma} - q_\mu q_\nu g_{\rho\sigma} - q_\rho q_\sigma g_{\mu\nu} + q_\mu q_\sigma g_{\nu\rho} + q_\nu q_\sigma g_{\mu\rho} + q_\mu q_\rho g_{\nu\sigma} + q_\nu q_\rho g_{\mu\sigma} \right) \\
+ \;& \psi^T_0 \left(g_{\mu\sigma}g_{\nu\rho} + g_{\mu\rho}g_{\nu\sigma} - \frac{1}{2} g_{\mu\nu}g_{\rho\sigma} \right) , \qquad (69)
\end{aligned}
$$

where

$$\theta^g_{\mu\nu} = -G^\alpha_\mu G_{\nu\alpha} + \frac{1}{4} g_{\mu\nu} G_{\alpha_\beta} G^{\alpha\beta} \, . \qquad (70)$$

A direct calculation gives the following results for the structure functions ψ^T_i and their respective Borel/Laplace transforms:

$$
\begin{aligned}
\pi^2\psi^T_4 &= \frac{l_{\mu Q}}{15} + \frac{17}{450} - \lambda^2 \frac{1}{3Q^2} \Longrightarrow \frac{1}{15}\left(1 - 5\frac{\lambda^2}{M^2}\right) , && (71) \\
\pi^2\psi^T_2 &= \frac{Q^2 l_{\mu Q}}{20} + \frac{9Q^2}{200} + \lambda^2 \left(\frac{l_{\mu Q}}{6} - \frac{2}{9}\right) \Longrightarrow -\frac{M^2}{20}\left(1 - \frac{10}{3}\frac{\lambda^2}{M^2}\right) , && (72) \\
\pi^2\psi^T_0 &= \frac{Q^4 l_{\mu Q}}{20} + \frac{9Q^4}{200} + \lambda^2 Q^2 \left(\frac{l_{\mu Q}}{4} - \frac{1}{12}\right) \Longrightarrow \frac{M^4}{20}\left(1 - \frac{5}{2}\frac{\lambda^2}{M^2}\right) . && (73)
\end{aligned}
$$

If, instead of considering $\theta^g_{\mu\nu}$, we would introduce the total energy-momentum tensor of interacting quarks and gluons $\theta_{\mu\nu}$, then various functions components of $\psi_{\mu\nu\rho\sigma}$ are related to each other because of the energy-momentum conservation. Indeed, requiring that

$$\psi^T_{\mu\nu\rho\sigma} q_\mu \equiv 0$$

we immediately obtain:

$$\psi^T_2 = \frac{3}{4} Q^2 \psi^T_4 \quad \text{and} \quad \psi^T_0 = \frac{3}{4} Q^4 \psi^T_4 \, ,$$

and, a consequence, the following representation of the function in Eq. (69):

$$\psi^T_{\mu\nu\rho\sigma}(q) = \left(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma} \right) \psi^T(Q^2), \qquad (74)$$

where:

$$\psi^T(Q^2) \equiv Q^4 \frac{3}{4} \psi^T_4(Q^2), \quad \eta_{\mu\nu} \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \qquad (75)$$

To compare the sum rules based on $\lambda^2 \neq 0$ with the existing ones with $\lambda^2 = 0$ [19], one should have in mind that the results in [19] come from the sum rules corresponding to the correlator ψ^T defined in Eq. (75). Literally, these sum rules are not affected by the new λ^2 terms because, in terms of the imaginary parts of the structure functions, the λ^2 correction exhibited in Eq. (71) is proportional to $\lambda^2\delta(s)$ which vanishes once multiplied by an extra power of s. However, further analysis of once more subtracted sum rules as well as of the $\tau-$stability [19] would reveal now a large mass scale.
To summarize, we conclude, from Eqs (71–75) above, that the Borel/Laplace transforms of the functions $\psi^T_i (i = 0, 2, 4)$ as well as ψ^T will have a scale analogue to the one of the (pseudo)scalar gluonium channels. This feature might reveal that all gluonia channels have an universal scale , in distinction from the instanton model [23] where the tensor-gluonium channel is expected to be insensitive to the largest scale exhibited in the spin-0 gluonium channel. Measurements of the corresponding correlators could provide therefore an interesting test of the theory.

4.8 Heavy quarks: $\bar{Q}Q$ and $\bar{Q}q$-channels.

Introduction of the gluon mass would result in a substantial change in the QCD phenomenology of heavy quark interactions. However, we expect that it would be rather a reshuffle of the parameters than a decisive test. Indeed, there are successful description of the quarkonia states with the linear potential in Eq. (9) extrapolated down to $r = 0$, see, e.g., [52]. We plan to come back to these points in a future publication.

5 Summary and conclusions

To summarize our discussions of the various channels, the introduction of the tachyonic gluon mass allows to explain in a very simple and unified way the variety of scales of the violation of the asymptotic freedom at moderate Q^2 in various channels. More specifically, the phenomenology based on the introduction of $\lambda^2 \simeq -0.5$ GeV^2 leads to the following successful predictions:

• The correct sign and order of magnitude of the M^{-2} correction in the ρ-channel.

• The new M^{-2} correction in the pion channel breaks the asymptotic freedom in this channel at the mass scale M^2_{crit} which is about factor of 4 higher than in the ρ-meson channel. This scale falls close to the value of M^2_{crit} determined independently from the values of f_π and quark masses. The sign of the correction due to the tachyonic mass is also the one which is needed to bring QCD in agreement with phenomenology.

• A natural explanation of M^2_{crit} in the scalar-gluonium channel (Eq. (68)) which is much larger than M^2_{crit} in the ρ-channel and which was found independently from a low energy theorem (Eq. (19)).

• The account for the M^{-2} correction lowers by $(11 \pm 3)\%$ the value of $\alpha_s(M_\tau)$ determined from the τ-decays, and may improve the accuracy of its determination by a factor of about 2. It also decreases by about 5% the value of the u, d and s running quark masses from the pseudoscalar channels. The presence of this term slightly decreases by about (1–3) % the value of m_s obtained from e^+e^- [38] and τ decay data [47]. It increases by about (8–9)% the results in the in [48] from the individual $\Delta S = 1$ inclusive τ decay channel. This effect, then, improves the agreement between the two different determinations.

To overview the logic of our analysis, we were motivated to introduce a tachyonic gluon mass by the data on the $Q\bar{Q}$ potential at short distances (see section 2). In this case $\lambda^2 \neq 0$ brings in a linear term kr at short distances which is, however, only a small correction to the Coulombic potential. Any precise measurement of this correction would require a subtraction of large perturbative contributions and as a result there are no error bars available on the value of k at short distances. Thus, the idea was to embed this (hypothetical) linear term in the potential into a relativistic framework and consider further consequences from this extension. Hence, the introduction of the tachyonic gluon mass. This extension allowed to analyse the current correlators. As a result, the bounds on the λ^2 narrow substantially. In particular, $\lambda^2 \approx -1$ GeV^2 which would naively correspond to the known value of k does not pass phenomenological hurdles, which is not so surprising. What is much more amusing, the value $\lambda^2 \approx -0.5$ GeV^2 fits the data well (see above).
Most remarkably, the introduction of $\lambda^2 \neq 0$ brought large qualitative effects, namely a variety of the mass scales M^2_{crit} where the asymptotic freedom is broken at various channels. Various M^2_{crit} may differ by a factor of about 20. Further crucial checks of the phenomenology based on $\lambda^2 \neq 0$ could be provided by measurements of the correlators on the lattice. The a^0/δ-scalar channel and gluonia channels appear most interesting from this point of view.
However, the extension of the standard QCD sum rules by introducing $\lambda^2 \neq 0$ cannot resolve all the phenomenological problems. Namely, the η'-problem cannot be solved in this way and asks for introduction of instantons or instanton-like configurations. The existing data on lattice simulations might indicate that in the a^0/δ-scalar channels, direct instantons become important at about the same M^2 as the effects of the gluon mass do. Further measurements of the current correlators on the lattice could provide checks of the phenomenology explored in the present paper.

Acknowledgements

The work by K.G. Chetyrkin has been supported by DFG under Contract Ku 502/8-1 and INTAS under Contract INTAS-93-744-ext.
S. Narison wishes to thank M. Davier and S. Menke for useful communications, and A. Pich for comments.
V.I. Zakharov gratefully acknowledges hospitality of Centre National de la Recherche Scientifique (CNRS) during his stay at the University of Montpellier where part of this work was done. He is also thankful to R. Akhoury, G. Bali, L. Stodolsky for valuable discussions.

References

[1] M.A.Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448.

[2] For a review, see e.g. S. Narison, "*QCD spectral sum rules*", (1989), World Scientific, Singapore; ibid. "*Recent progress in QCD spectral sum rules*", book in preparation.

[3] G. Grunberg, hep-ph/9705290,hep-ph/ 9711481, hep-ph/9807494.
R. Akhoury, V.I. Zakharov, hep-ph/9705318.

[4] M. Beneke, hep-ph/9807443;
R. Akhoury and V.I. Zakharov, hep-ph/9610492, hep-ph/9710257.

[5] M. Beneke , V.I. Zakharov, *Phys. Rev. Lett.* **69** (1992) 2472;
A.H. Mueller, *Phys. Lett.* **B308** (1993) 355.

[6] V.I. Zakharov, Nucl. Phys. **B385** (1992) 452;
S. Narison, Phys.Lett. B300 (1993) 293, **B361** (1995) 121.
C.A. Dominguez, *Phys.Lett.* **B345** (1995) 291.
A.I. Vainshtein, V.I. Zakharov, *Phys. Rev. Lett.* **73** (1994) 1207, *Phys. Rev.* **D54** (1996) 4039;
G Altarelli, P. Nason, G. Ridolfi,*Z. Phys.* **C68** (1995) 257.
K. Yamawaki, V.I. Zakharov, hep-ph/9406399, hep-ph/9406373;
S. Peris, E. de Rafael, *Nucl. Phys.***B500** (1997) 325;
M. Beneke, V.M. Braun, N. Kivel, *Phys. Lett.* **B404** (1997) 315.

[7] Ya.Ya. Balitskii, *Nucl. Phys.* **B254** (1983) 166.

[8] R. Akhoury, V.I. Zakharov, hep-ph/9710487.

[9] D.B. Leinweber, J.I. Skullerud, A.G. Williams and C. Parrinello, hep-lat/9811027.

[10] K. Buttner and M.R. Pennington, Phys. Rev. **D52** (1995) 5220.

[11] G. S. Bali, K. Schilling, A. Wachter, hep-lat/9506017;
G. S. Bali, K. Schilling, A. Wachter, *Phys. Rev.* **D55** (1997) 5309.

[12] V.I. Zakharov, hep-ph/9802416.

[13] R. Anishetty, hep-ph/9804204.

[14] F.V. Gubarev, M.I. Polikarpov, V.I. Zakharov, hep-th/9805175.

[15] M. Beneke, V.M. Braun, V.I. Zakharov, *Phys. Rev. Lett.* **73** (1994) 3058.

[16] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *Nucl. Phys.* **B191** (1981) 301.

[17] K.Ackerstaff et al (the OPAL Collaboration), hep-ex/9808019;
R. Barate et al. (the ALEPH Collaboration), *Euro. Phys. Journ.* **C4** (1998) 409.

[18] M.C. Chu, J.M. Grandy, S. Huang, J.W. Negele, *Phys. Rev.* **D48** (1993) 3340.

[19] S. Narison, *Nucl. Phys.* **B509** (1998) 312; *Nucl. Phys. (Proc. Suppl.)* **B64** (1998) 210.

[20] S. Narison, G. Veneziano, *Int. J. Mod. Phys.* **A4** (1989) 2751.

[21] S. Narison, G. Shore and G. Veneziano, *Nucl. Phys.* **B433** (1995) 209 and work in preparation.

[22] G. 't Hooft, *Phys.Rev.* **D14** (1976) 3432.

[23] T. Schafer, E.V. Shuryak, *Rev. Mod. Phys.* **70** (1998) 323;
E.V. Shuryak Rev. Mod. Phys. **65** (1993) 1,

[24] S. Narison, *Phys. Lett.* **B300** (1993) 293.

[25] M. Steinhauser, Ph.D. thesis, Karlsruhe University (Shaker Verlag, Aachen, 1996).

[26] S.A. Larin, F.V. Tkachev and J.A.M. Vermaseren, Report No. NIKHEF–H/91–18 (September 1991).

[27] J.A.M. Vermaseren, Symbolic Manipulation with FORM, (Computer Algebra Netherlands, Amsterdam, 1991).

[28] D.J. Broadhurst and S.C. Generalis, Open University report No. OUT-4102-12 (1984) (unpublished).

[29] V.P. Spiridonov, K.G. Chetyrkin , Sov. J. Nucl. Phys. **47** (1988) 3.

[30] M. Jamin and M. Munz, Z. Phys. **C60** (1993) 569.

[31] K.G. Chetyrkin, A. Kwiatkowski, *Z. Phys.* **C59** (1993) 525 and hep-ph/9805232.

[32] P. Ball, M. Beneke, V.M. Braun, *Nucl. Phys.* **B452** (1995) 563.

[33] S. Narison, *Phys. Lett.* **B361** (1995) 121; **B216** (1989) 191.

[34] C. Becchi, S. Narison, F.J. Yndurain and E. de Rafael, *Z. Phys.* **C8** (1981) 335.

[35] M. Jamin, M. Münz, Z. Phys. **C66** (1995) 633.

[36] K.G. Chetyrkin, C.A. Dominguez, D. Pirjol, K. Schilcher, *Phys. Rev.* **D51** (1995) 5090.

[37] J. Bijnens, J. Prades, E. de Rafael, *Phys. Lett.* **B348** (1995) 226.

[38] S. Narison, *Phys. Lett.* **B358** (1995) 113.

[39] E. Braaten, S. Narison, A. Pich, *Nucl. Phys.* **B373** (1992) 581.

[40] S.G. Gorishny, A.L. Kataev, S.A. Larin, Phys. Lett. **B259** (1991) 144.

[41] R.M. Barnett et al., *Phys. Rev.* **D54** (1996) 1;
C. Caso et al. *Euro. Phys. Journ.* **C3** (1998) 1.

[42] Private communications from M. Davier and S. Menke.

[43] S. Narison, *Nucl. Phys. (Proc. Suppl.)* **B40** (1995) 47.

[44] F. Le Diberder, *Nucl. Phys. (Proc. Suppl.)* **B,C39** (1995) 318;
ibid, A. Kataev, V. Starshenko **B,C39** (1995) 312.

[45] F. Le Diberder, A. Pich, *Phys. Lett.* **B286** (1992) 147 and **B289** (1992) 165.

[46] C.J. Maxwell, *Nucl. Phys. (Proc. Suppl.)* **B64** (1998) 360 and references therein;
M. Neubert, *Nucl. Phys.* **B463** (1996) 511;
P.A. Raczka and A. Szymacha, *Z. Phys.* **C70** (1996) 125;
A. A. Pivovarov, *Z. Phys.* **C53** (1992) 461;
E. Braaten, *Tau 97* (Colorado-September 1997).

[47] S. Chen (the Aleph collaboration), *Nucl. Phys. (Proc. Suppl.)* **B64** (1998);
M. Davier (the Aleph collaboration), *Tau 98* (Santander-September 1998);

[48] A. Pich and J. Prades, JHEP **06** (1998) 013;
K.G. Chetyrkin, J.H. Kühn and A.A. Pivovarov, *Nucl. Phys.* **B533** (1998) 473.

[49] S. Narison, N. Paver, E. de Rafael, D. Treleani, *Nucl. Phys.* **B212** (1983) 365.

[50] H.G. Dosch, S. Narison, *Phys. Lett.* **B417** (1998) 173.

[51] E. Bagan, S. Narison, *Phys. Lett.* **B214** (1988) 451.

[52] J. Pantaleone, S.-H. Tye, Y.J. Ng, *Phys. Rev.* **D33** (1986) 777;
F. Halzen et. al., *Phys. Rev.* **D47** (1993) 3013.