Electroweak Measurements, QCD and Theory Uncertainties

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Recent results for electroweak radiative corrections will be presented with emphasis on those connecting QCD and electroweak interactions. This includes purely elctroweak corrections at two-loop order and beyond, hadronic effects specific to the $b\bar{b}$ final state in Z decays, the impact of perturbative QCD on the ρ parameter and related observables, and mixed QCD and electroweak vertex corrections. Estimates for the theoretical uncertainties are given.

1 Introduction

During the past years electroweak precision measurements have become one of the central issues in particle physics. The indirect determination of the mass of the top quark through its impact on quantum corrections prior to its observation in proton-antiproton collisions at the TEVATRON can be considered one of the triumphs of the present theoretical framework. Similarly the indirect determination of the mass of the W boson is in perfect agreement with the present measurements at LEP and the TEVATRON, which motivates a combined fit to the parameters of the Standard Model.

Recently the measurements have become sufficiently precise to even give first indications for a relatively small mass of the Higgs boson with an upper limit of order 200 GeV. Significant improvements, in particular for M_W and the left-right asymmetry with polarised beams at the SLC are expected for the next couple of years which might lead to a fairly precise indirect determination of the Higgs mass M_H, thus repeating the success of the top quark mass determination.

To achieve the accuracy of the theoretical predictions needed for this program, not only the two-loop electroweak corrections have to be incorporated — for the dominant terms, those originating from the large top mass, from the running of $\alpha_{\rm QED}$ or from QCD sometimes even three-loop amplitudes have to be investigated. In this contribution a variety of recent results connecting QCD and electroweak physics will be presented. In section 2 purely electroweak effects at two-loop order and beyond will be discussed. Also purely hadronic corrections may affect the determination of precision observables, in particular ratios of various partial decay rates of the Z boson like $\Gamma_b/\Gamma_{\rm had}$. Examples are the so-called singlet contributions and bottom quark mass effects which are both reviewed in section 3. The quadratic top mass dependence of the ρ parameter and related observables is crucial for the indirect m_t determination. More refined studies require a precise definition of the top quark mass and, correspondingly, the evaluation of two- and even three-loop QCD corrections (Sect.4). They are all related to the self-energies of the photon, the W- and the Z-boson or their mixing, are thus "universal" and involve QCD and electroweak interactions simultaneously. Another class of "mixed" QCD and electroweak interactions to be discussed in section 5 is connected to the $Zq\bar{q}$ vertex. They influence decays into up and down type and in particular b quarks differently and, at the same time, affect the determination of α_s from the Z decay rate.

2 Electroweak Results at Two-Loop and Beyond [1]

2.1 Higgs-mass dependence of Δr at two loop

The dependence of a large class of observables on the Higgs mass is of particular interest for the present discussion. Since a complete calculation in two loop approximation is not available in the moment, subtracted quantities of the form

$$\Delta r_{\rm subtr}(M_H) = \Delta r(M_H) - \Delta r(M_H^0) \qquad (1)$$

were considered [2] (and similarly for $\sin^2\theta_{\rm eff}$, Γ_l) where Δr is defined by $M_W^2(1-M_W^2/M_Z^2) = (\pi\alpha)/(\sqrt{2}G_\mu)(1+\Delta r)$. The subtracted quantities indicate the shift in the precision observables caused by varying the Higgs-boson mass between M_H^0 and M_H. Values of M_H in the interval 65 GeV $\le M_H \le$ 1 TeV were considered. Allowing for such a wide range of Higgs-boson mass values may seem somewhat over-conservative regarding the fact that the SM fits favor a light Higgs-boson with a mass smaller than about 300 GeV. However, the form of the fit curve and thus the value of the upper bound is affected also by the SM predictions for larger M_H-values (and also for M_H-values below the current experimental lower bound of M_H).

Potentially large M_H-dependent contributions are those associated with the top quark due to the large Yukawa coupling $t\bar{t}H$, and the corrections proportional to $\Delta\alpha$. Further contributions are those from the light fermions (except the corrections already contained in $\Delta\alpha$), and purely bosonic contributions. The latter are expected to be rather small [2]. The subsequent discussion

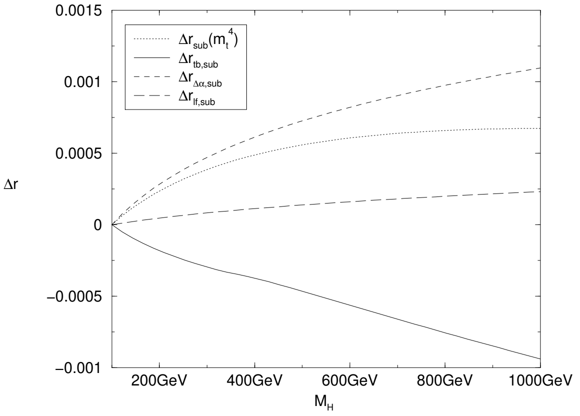


Figure 1: Higgs-mass dependent fermionic contributions to Δr at two-loop order. The different curves show the contribution from the diagrams involving the top/bottom doublet ($\Delta r_{\rm tbh}$), the contribution proportional to $\Delta\alpha$ ($\Delta r_{\Delta\alpha}$), the contribution of the light fermions ($\Delta r_{\rm lfh}$), and the approximation of the top/bottom correction by the leading term proportional to m_t^4 ($\Delta r(m_{\rm t}^4)$). (From [2].)

is therefore focussed on the Higgs-dependent fermionic contributions to the precision observables, for which exact two-loop results are available[2,3].

Fig. 1 shows the Higgs-mass dependence of the two-loop corrections to Δr associated with the top/bottom doublet, with $\Delta\alpha$, and with the light fermions. The dotted line furthermore indicates the Higgs-mass dependence of the leading m_t^4-term in the top/bottom contribution. The top/bottom contribution gives rise to a shift in the W-boson mass of $\Delta M_{\rm W,subtr,(2)}^{\rm top}(M_H = 1000~{\rm GeV}) \sim 16$ MeV, which is about 10% of the one-loop contribution. Its Higgs-mass dependence turns out to be very poorly approximated by the leading m_t^4-term; the contribution of the latter even enters with a different sign. It can be seen from Fig. 1 that the two-loop contributions to a large extent cancel each other. The contribution of the light fermions yields a shift in M_W of up to $\Delta M_{\rm W,subtr,(2)}^{\rm lf}(M_H = 1000~{\rm GeV}) \lesssim 4$ MeV. In total the two-loop contributions lead to a slight increase in the sensitivity of Δr to the Higgs-boson mass compared to the one-loop case.

The Higgs-mass dependence of M_W based on the exact result for the top/bottom doublet has been compared with the results of the expansion in the top-quark mass up to $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ given in Ref.[4]. Besides the difference in the two-loop results also higher-order effects due to differences in the renormalization procedure and in the treatment of non-leading higher-order corrections enter the comparison.

Over the range of Higgs-mass values from 65 GeV to 1 TeV the difference between the results amounts to about 8 MeV, which corresponds to 50% of the two-loop top/bottom contribution, is slightly larger than the theoretical uncertainty quoted in Ref.[5] of 2 MeV from electroweak and 5 MeV from QCD corrections.

2.2 Fermion-loop corrections

The dominant contributions to the electroweak precision observables at one-loop order are the fermion-loop corrections $\Delta\alpha$ and $\Delta\rho = N_C \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}}$, which arise from the renormalization of the electric charge and the weak mixing angle, respectively. One of the main issues in improving the one-loop predictions has therefore been to find a proper resummation of these leading one-loop contributions. In the case of Δr, for instance, the resummation[6]

$$1 + \Delta r \to \frac{1}{(1-\Delta\alpha)(1+\frac{c_W^2}{s_W^2}\Delta\rho) - \Delta r_{\rm rem}} \qquad (2)$$

correctly takes into account the terms of the form $(\Delta\alpha)^2$, $(\Delta\rho)^2$, $(\Delta\alpha\Delta\rho)$, $\Delta\alpha\Delta r_{\rm rem}$ at two-loop order. However, even with the knowledge of the recently evaluated terms[7,4] of $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ different treatments of contributions whose correct resummation is not known (i.e. non-leading two-loop terms and the higher-order terms except the leading term in $\Delta\alpha$) may result in predictions for M_W that differ from each other by several MeV.

Results[1] for the pure fermion-loop contributions to the precision observables, i.e. contributions containing n fermion loops at n-loop order (which in the case of quark loops are therefore proportional to the nth power of the color factor N_C) have been presented[a]. The relevant diagrams in a given loop order are reducible diagrams of vacuum-polarization type, which can easily be taken into account by Dyson summation, and, in addition, graphs with counter-term insertions into fermion-loop diagrams.

Using the on-shell renormalization scheme explicit results for the fermion-loop contributions to the precision observables up to four-loop order have been obtained[1]. The two-loop contribution to Δr when compared to the $\overline{\rm MS}$ result[8] is found to be in very good agreement (within less than 1 MeV) for the M_W prediction. Also the validity of the resummation Eq. 2 for the non-leading two-loop and the higher-order contributions has been investigated. Already the first non-leading two-loop term of $\mathcal{O}(\Delta\rho\log(m_t^2/M_Z^2))$ is not correctly produced by the resummation. The terms of the form $\Delta\rho\Delta r_{\rm top,rem}$ and $(\Delta r_{\rm top,rem})^2$ generated by Eq. 2 give rise to a deviation in M_W of about 7 MeV compared to the exact result for the top/bottom doublet. It turns out, however, that large numerical cancellations occur between the non-leading two-loop terms that are not correctly resummed in Eq. 2. While separately these terms differ from the corresponding terms in the exact result by several MeV, their sum

[a]These contributions obviously constitute an UV-finite and gauge-invariant subset.

approximates the full result remarkably well, up to less than 1 MeV.

In total, due to accidental numerical cancellations, the resummation of the fermion-loop contributions according to Eq. 2 yields a very good numerical approximation of the complete fermion-loop result, valid within about 2 MeV.

3 QCD Corrections, Quark Mass Effects and Uncertainties in the Z Decay Rate [9]

The dominant terms of QCD corrections to the Z decay rate are accumulated in the non-singlet corrections which result from diagrams with the external current coupled to the same quark line. In the limit of massless quarks one obtains the familiar correction factor

$$\frac{\Gamma(Z \to hadrons)}{\Gamma_{\rm Born}} = 1+\frac{\alpha_s}{\pi}+1.409\left(\frac{\alpha_s}{\pi}\right)^2-12.767\left(\frac{\alpha_s}{\pi}\right)^3 \tag{3}$$

which is universal for all quark species. It allows for a theoretically clean and extremely precise determination of α_s from the Z line shape fits. However, at the current level of precision the theoretical and experimental analysis has to include a number of small but nevertheless important effects which are of relevance not only for the total rate, they even modify the partial rates into different quark species differently. This is of particular importance for the partial rate into $b\bar{b}$ and the ratio $\Gamma_b/\Gamma_{\rm had}$, where one hand most of the QCD corrections cancel and which, on the other hand, is particularly sensitive to electroweak effects specific for the tb doublet.

The peculiarities of Γ_b are corrections due to the relatively large bottom mass, $4m_b^2/M_Z^2 \approx 10^{-2}$, and the singlet contributions of the axial correlator, with gluons as intermediate states only. Quark mass effects are conveniently evaluated through an expansion in m_b^2/M_Z^2. For the quadratic terms which are sufficient at the scale of M_Z the logarithms can be absorbed in the running quark mass and the perturbative series has been evaluated [10,11] to order α_s^3 (apart from the tiny m_b dependent singlet terms which are available [12] in order α_s^2 only). These quark mass effects lead to a reduction of Γ_b by 1.53 MeV with the dominant uncertainty from the error in the quark mass, $m_b = 4.7 \pm 0.2$ MeV.

Singlet diagrams are mainly relevant for the axial part of Γ_b. They result from the incomplete cancellation between top and bottom quark loops in "anomalous" triangle diagrams and have been calculated [13,14,15] in order α_s^2 and α_s^3. The former is still plagued by a considerable scale uncertainty (M_Z vs. m_t!) which is largely removed by the latter. The singlet piece thus leads to a further reduction $\Delta\Gamma_{\rm singlet} = -1.82$ MeV and the remaining uncertainty of $\Delta\Gamma_{\rm singlet} = 0.1$ MeV is entirely due to the parametric uncertainty in α_s.

In total one finds

$$\Delta R_b \equiv \frac{\Delta\Gamma_{\rm mass} + \Delta\Gamma_{\rm singlet}}{\Gamma_{\rm had}} = -0.2\%, \tag{4}$$

a shift comparable to the current experimental precision of 0.34%. The remaining uncertainty $\delta R_b = 1 - 2 \cdot 10^{-4}$ can safely be neglected. It is also worth mentioning that the reduction of the total width by 3.3 MeV also affects the determination of α_s, leading to a shift in α_s of 0.006, i.e. twice the current error.

4 Gauge boson self-energies, the mass of the top quark and QCD

The indirect determination of the top quark through quantum corrections prior to its observation in hadronic collisions can be considered one of the triumphs of the Standard Model. The experimental precision of the key observables, the masses of the top quark and the W boson together with the weak mixing angle as determined by asymmetry measurements has increased during the past years and this process will continue in the foreseeable future. In order to control the influence of the top quark at an adequate level the inclusion of QCD corrections in the top and bottom induced self energies is mandatory. The dominant terms are characterised by the ρ parameter which, in lowest order, is given by [16]

$$\Delta\rho = 3\sqrt{2}\frac{G_F m_t^2}{16\pi^2}. \tag{5}$$

In view of the large difference between pole and running mass at scale m_t

$$m_t(pole) = \bar{m}_t(m_t)\left(1+\frac{4}{3}\frac{\alpha_s}{\pi}+\cdots\right) \tag{6}$$

inclusion of two and even three loop QCD corrections to $\Delta\rho$ is mandatory.

Analytic results are available in two loop approximation not only for the leading term [17] in $\Delta\rho$ but also for all self energies, with arbitrary top and bottom masses [18]. The resulting shift in the prediction for M_W for fixed G_F, $\alpha_{\rm QED}$, M_Z and $m_t = 175$ GeV amounts to 68 MeV, well comparable to the present precision and significantly larger than the anticipated accuracy of roughly 30 MeV. To arrive at a precise prediction for the central value and to control the theoretical uncertainty, three-loop QCD contributions to $\Delta\rho$ as well as to Δr are required. The ρ parameter [19] can again be expressed through diagrams with vanishing external momentum (vacuum or tadpole diagrams), the remaining quantities involve two point functions at non-vanishing external momentum and can be obtained [20] through an expansion in the small mass ratio M_Z^2/m_t^2. The leading three-loop term corresponds to a shift of -10.9 MeV. The first three terms, amounting to -13.7 MeV, are adequate for a prediction with an accuracy better than 1 MeV (Table 1). Conversely, this combined shift is equivalent to a reduction of the effective top mass by about 1.6 GeV.

δM_W in MeV	α_s^0	α_s^1	α_s^2
M_t^2	611.9	-61.3	-10.9
const.	136.6	-6.0	-2.6
$1/M_t^2$	-9.0	-1.0	-0.2

Table 1: The change in M_W separated according to powers in α_s and M_t in the on-shell scheme (From [20].)

To exploit the precision expected from a future linear collider which will pin down m_t to better than 200 or perhaps even 100 MeV, the inclusion of α_s^2 terms is evidently mandatory.

Similar considerations [20] are valid for the effective weak mixing angle which can be deduced from the left right asymmetry or the forward backward asymmetry in a straightforward way.

An important issue in this connection is the size of uncertainties, arising either from uncalculated higher orders, or from the "parametric" uncertainties in α_s and m_t. Shifts of $\delta\alpha_s = 0.003$ and $\delta m_t = 5$ GeV lead to changes in M_W of -2.4 MeV and 35 MeV respectively. The uncertainty from uncalculated higher orders is completely negligible. A W-mass determination with a precision in the 10 MeV range should therefore be accompanied by a top mass measurement with a precision around or better than 1 GeV.

5 Mixed QCD and electroweak vertex corrections

As stated above, gauge boson self energies, in particular those induced by fermion loops, give rise to the dominant radiative corrections for electroweak precision observables. Nevertheless, for a complete treatment of $\mathcal{O}(\alpha\alpha_s)$ the inclusion of irreducible vertex corrections is necessary. These have to be distinguished from the reducible ones which are easily incorporated, if the electroweak result, including one-loop weak terms, is simply multiplied by the QCD correction factor $(1+\alpha_s/\pi+\cdots)$.

The physics and the techniques of calculation are markedly different for vertices leading to light (u, d, s and c) quark pairs on one hand [21] and for decays into $b\bar{b}$ on the other hand [22], a consequence of the presence of top quarks with their enhanced contribution to the vertex proportional m_t^2/M_W^2. The irreducible one-loop vertex diagrams are shown in Fig. 5.

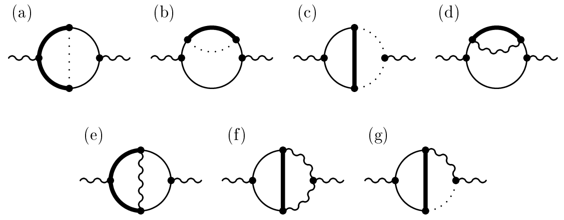


Figure 2: Diagrams contributing to $\delta\Gamma_b^W$ in $\mathcal{O}(\alpha)$. Thin lines correspond to bottom quarks, thick lines to top quarks, dotted lines to Goldstone bosons and inner wavy lines represent W bosons.

To obtain all one-particle irreducible vertex diagrams in two-loop approximation , these have to be dressed with gluons in all conceivable ways. The resulting amplitudes are first studied for arbitrary q^2 by considering expansions in the ratio $x_Z = q^2/M_Z^2$ and $x_W = q^2/m_W^2$ (or for some diagrams, in $1/x_{Z,W}$). Even for the limiting values $x_Z = 1$ and $x_W = M_Z^2/M_W^2$ the exact results are well approximated, once sufficiently many terms are included. Using $\alpha_s = 0.12$, $\alpha = 1/129$, $\sin\theta_W = 0.223$ $M_Z = 91.19$ GeV, it is found [21] that the net effect of the nonfactorizable corrections is

$$\Gamma^{(2\text{ loop EW/QCD})} - \frac{\alpha_s}{\pi}\Gamma^{(1\text{ loop EW})} = \begin{cases} -1.13(4)\times 10^{-4}\text{ GeV} & \text{for } Z\to\bar{u}u \\ -1.60(6)\times 10^{-4}\text{ GeV} & \text{for } Z\to\bar{d}d \end{cases} \quad (7)$$

The total change in the partial width $\Gamma(Z\to hadrons)$ is obtained by summing over 2 down-type and 2 up-type quarks:

$$\Delta\Gamma(Z\to u,d,s,c) = -0.55(3)\text{ MeV} \quad (8)$$

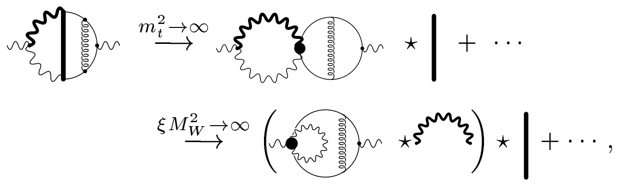
which translates into the change of the strong coupling constant determined at LEP 1 equal to

$$\Delta\alpha_s = -\pi\frac{\Delta\Gamma(Z\to hadrons)}{\Gamma(Z\to hadrons)} = \pi\frac{0.55}{1741} \approx 0.001 \quad (9)$$

This shift is somewhat smaller but still of the same order of magnitude as the experimental accuracy and should to be taken into account in the final analysis of LEP 1 data. Electroweak parameters extracted from Z decays are not affected by this correction.

The two-loop corrections to the $Zb\bar{b}$ vertex are dominated by terms quadratic in m_t. The QCD corrections to these have been evaluated already some time ago[23]. The sub-leading terms $\propto \ln m_t^2$ were obtained in [24] and are of comparable magnitude, thus indicating relatively slow convergence of the series. The complete evaluation is thus mandatory and has been recently been performed[25]. In contrast to the previous case one is confronted with three different scales, the masses of top, the W and the Z boson.

Using the Hard Mass Procedure for $m_t^2 \gg M_Z^2, M_W^2$, one may factor out the m_t–dependence. However, for a part of the diagrams one still is left with two-scale and even three-scale integrals involving M_Z^2 and M_W^2 and $\xi_W M_W^2$, where ξ_W is the electroweak gauge parameter which we want to keep. Although they appear to be only one-loop integrals, their exact evaluation up to $\mathcal{O}(\epsilon)$ produces inconvenient results. Instead, the results [25] were obtained by applying the Hard Mass Procedure to these kinds of diagrams once more, this time using $\xi_W M_W^2, M_W^2 \gg M_Z^2$. This seemingly unrealistic choice of scales can be well justified: It is not possible for an expansion to distinguish the inequality $M_W^2 \gg M_Z^2$ from $4M_W^2 \gg M_Z^2$ or $(m_t + M_W)^2 \gg M_Z^2$, the latter ones being perfectly alright. The only matter is to perform the expansion on the appropriate side of all thresholds, and here one is concerned with thresholds at $2M_W$ and at $m_t + M_W$. Therefore, the choice $M_W^2 \gg M_Z^2$ is to be understood purely in this technical sense. Graphically this continued expansion looks as follows:



where only those terms are displayed which are relevant in the discussion above and all others contributing to the Hard Mass Procedure are merged into the ellipse. The thick plain line is the top quark, the thick wavy one a Goldstone boson with mass squared $\xi_W M_W^2$, for example. The thin plain lines are b–quarks, the inner thin wavy lines are W–bosons, the outer ones Z-bosons. The spring-line is a gluon. The mass hierarchy is assumed to be $m_t^2 \gg \xi_W M_W^2 \gg M_W^2 \gg M_Z^2$. The freedom in choosing the magnitude of ξ_W provides a welcome check of the routines and the results.

The outcome of this procedure is a nested series: The coefficients of the M_W/m_t–expansion are in turn series in M_Z/M_W. Note that in contrast to the decay into u, d, s, c there is no threshold at M_W which makes an additional expansion in M_W/M_Z unnecessary.

In view of this calculation the procedure of successive application of the Hard Mass Procedure resp. the Large Momentum Procedure has been implemented in a Fortran 90 program named `EXP` [26]. Therefore, given an arbitrary hierarchy of mass scales, the computation of a three-loop two-point function can now be done fully automatically. Even more, the link to the Feynman diagram generator `QGRAF` [27] in a common environment called `GEFICOM` [28] allows to obtain the result of a whole physical process without any human interference except for specification of the process and final renormalization.

Finally, let us present the result for the W–induced corrections to the Z–decay rate $\delta\Gamma^W(Z \to b\bar{b})$ in the form of the renormalization scheme independent difference to the decay rate into $d\bar{d}$. Inserting the on-shell top mass $m_t = 175$ GeV, the Z–mass $M_Z = 91.91$ GeV and $\sin^2\theta_W = 0.223$ gives [25]

$$
\begin{aligned}
&\delta\Gamma^W(Z \to b\bar{b}) - \delta\Gamma^W(Z \to d\bar{d}) = \\
&= \Gamma^0 \frac{1}{\sin^2\theta_W}\frac{\alpha}{\pi}\Big\{ -0.50 + (0.71 - 0.48) \\
&+ (0.08 - 0.29) + (-0.01 - 0.07) + (-0.007 - 0.006) \\
&+ \frac{\alpha_s}{\pi}\Big[1.16 + (1.21 - 0.49) + (0.30 - 0.65) \\
&+ (0.02 - 0.21 + 0.01) + (-0.01 - 0.04 + 0.004)\Big]\Big\} = \\
&= \Gamma^0 \frac{1}{\sin^2\theta_W}\frac{\alpha}{\pi}\Big\{ -0.50 - 0.07 + \frac{\alpha_s}{\pi}\Big[1.16 + 0.13\Big]\Big\},
\end{aligned}
\tag{10}
$$

where the factor $\Gamma^0\alpha/(\pi\sin^2\theta_W)$ with $\Gamma^0 = M_Z\alpha/(4\sin^2\theta_W\cos^2\theta_W)$ has been pulled out for convenience. The numbers after the first equality sign correspond to successively increasing orders in $1/m_t^2$, where the brackets collect the corresponding constant, $\log m_t$ and, if present, $\log^2 m_t$–terms. The numbers after the second equality sign represent the leading m_t^2–term and the sum of the sub-leading ones. The $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$–results are displayed separately. Comparison of this expansion of the one-loop terms to the exact result [29] shows agreement up to 0.01% which gives quite some confidence in the $\alpha\alpha_s$–contribution. One can see that although the m_t^2–, m_t^0– and $m_t^0\log m_t$–terms are of the same order of magnitude, the final result is surprisingly well represented by the m_t^2–term, since the sub-leading terms largely cancel among each other.

The uncertainty from uncalculated higher order QCD terms is far below the foreseeable experimental precision, and the parametric uncertainty from α_s and m_t dominates. An important lesson to be learned from the $Zb\bar{b}$ vertex in one and two loop approximation concerns the interplay between "dominant" and "sub-leading" pieces: whenever leading and sub-leading terms are of comparable magnitude, inclusion of a sizeable number of terms in the expansion is required. The estimate of the final result or of theoretical uncertainties based on the first two terms of the series may lead to a wrong or misleading result.

Summary: The interplay between electroweak interactions and QCD is crucial for the evaluation and interpretation of precision measurements. Terms of order $\alpha\alpha_s$ are under complete control, the top contribution to the ρ parameter and related observables are even available in order $\alpha\alpha_s^2$. These results are important for the interpretation of the current data and measurements to be performed in the near future. The remaining theoretical uncertainties from the usage of expansion techniques and uncalculated higher orders are far below the anticipated experimental precision. In fact even the parametric uncertainties from α_s and m_t which dominate the current error can be ignored in the foreseeable future.

6 Acknowledgments:

I would like to thank K. Chetyrkin, R. Harlander, Th. Seidensticker, M. Steinhauser, and G. Weiglein for providing important input material and for instructive discussions. TEXnical help by R. Harlander is gratefully acknowledged.

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