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Reparametrization Invariance of Heavy Quark Effective Theory at $\mathcal{O}(1/m_Q^3)$

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Abstract

We extend the investigation of the reparametrization invariance of Heavy Quark Effective Theory to $\mathcal{O}(1/m_Q^3)$. We show that in the presence of radiative corrections reparametrization invariance can only be maintained if the the reparametrization transformation of the fields is renormalized properly.

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1 Introduction

Heavy Quark Effective Field Theory (HQET) [1] has become a well established theoretical tool for the description of hadrons containing one heavy quark [2]. This derives from the fact that it is a systematic expansion in inverse powers of the heavy quark mass m_Q with well defined and calculable coefficients. Furthermore, its realization of the spin and flavor symmetry of the low energy theory is a phenomenologically powerful tool. The $1/m_Q$ expansion has already been applied sucessfully to problems such as the determination of V_{cb}.

Besides its phenomenological application HQET posseses interesting theoretical features which already show up if one studies the HQET lagrangian itself. On one hand it is relevant for power corrections to the $1/m_Q$ expansion of hadron masses and on the other hand allows for a study of reparametrization invariance [3].

This new symmetry of HQET is associated with the fact, that QCD Greens-functions only depend on the full heavy quark momentum $p = m_Q v + k$ independent from its decomposition in on shell momentum mv and residual momentum k in terms of which HQET is defined. This means, that an infinitesimal change of the velocity $v \to v + \delta v$ can be compensated by an appropriate change of the residual momentum $k \to k - m_Q \delta v$ corresponding to a transformation $h_v \to h_v + \delta h_v$ of the heavy quark field. As long as $m\delta v = \mathcal{O}(\Lambda_{QCD})$ this change of the velocity does not violate the condition $k = \mathcal{O}(\Lambda_{QCD}))$ on which the construction of HQET relies.

There is no unique way to define HQET since a redefinition of the heavy quark field leads to an equivalent formulation of HQET due to the equivalence theorem. The equivalent formulations of HQET differ from each other by operators vanishing by the equation of motion (EOM) of the heavy quark. For example in the Foldy–Wouthuysen formulation which is used in NRQCD applications these operators serve to remove all covariant time derivatives from the lagrangian. However, if these unphysical operators are removed consistently, one ends with a unique minimal lagrangian consisting only of physical operators. By definition a physical operator contains no $(ivD)h_v$– or $\bar{h}_v(ivD)$–term. In general the reparametrization transformation of the heavy quark field depends on what specific formulation of HQET has been choosen.

Given the transformation δh_v associated with a certain formulation of HQET at tree level it is easy to verify that the corresponding lowest order lagrangian is reparametrization invariant. However the question arises if the tree level transformation suffices if radiative corrections are included in the lagrangian. From usual symmetries, e.g. gauge symmetries, it is well known that in each order of perturbation theory the BRS–transformations and the Slavnov identities have to be renormalized. Accidentally this seems not to be the case for reparametrization invariance at least if power corrections higher than $\mathcal{O}(1/m_Q^2)$ are excluded. Several authors [4, 5, 7] have verified explicitly that to this order reparametrization invariance holds if the tree level transformation is used even in the presence of radiative corrections. In [6] reparametrization invariance of the NRQCD lagrangian at $\mathcal{O}(1/m_Q^3)$ has been studied. However, a direct comparison with our results is not possible since we are working in different HQET–formulations.

Recently all coefficients of the effective lagrangian at $\mathcal{O}(1/m_Q^3)$ including EOM–operators have become available [8]. It is the subject of this paper to extend the analysis of reparametrization invariance to this order. It will be shown, that the above naive understanding of reparametrization invariance fails and how the emerging inconsistencies can be cured.

This paper is organized as follows: In section 2 we present the effective lagrangian in the formulation introduced by Mannel, Roberts and Ryczak (MRR) [9] and the field redefinition which removes the operators vanishing by the equation of motion. In section 3 we introduce the concept of reparametrization invariance and derive a relation between reparametrization transformations associated with different formulations of HQET. Sections 4 and 5 apply this formalism to the MRR lagrangian and the minimal lagrangian. Finally we present our conclusions in section 6.

2 MRR–HQET up to $\mathcal{O}(1/m_Q^3)$

HQET can be derived from the QCD lagrangian

$$\mathcal{L} = \bar{Q}(i\not{D} - m_Q)Q + \dots \tag{1}$$

by reparametrization of the heavy quark field Q in terms of its particle and antiparticle components:

$$Q = e^{-im_Q vx}(h_v + H_v), \quad \not{v}h_v = h_v, \quad \not{v}H_v = -H_v \tag{2}$$

In order to formulate an effective theory for the particle degree of freedom h_v the heavy degree of freedom H_v has to be integrated out. This yields the nonlocal lagrangian

$$\mathcal{L} = \bar{h}_v(ivD)h_v + \bar{h}_v i\not{D}^\perp \frac{1}{ivD + 2m_Q} i\not{D}^\perp h_v\,. \tag{3}$$

Accordingly the matching condition (2) becomes nonlocal:

$$Q = e^{-im_Q vx}(1 + \frac{1}{ivD + 2m_Q} i\not{D}^\perp)h_v \tag{4}$$

Here $iD^\perp = iD - v(ivD)$ acts on the transverse degrees of freedom. The expansion of (3) in powers of the inverse heavy quark mass m_Q yields the well known tree level lagrangian

$$\mathcal{L} = \bar{h}_v(ivD)h_v + \frac{1}{2m_Q}\bar{h}_v i\not{D}^\perp \sum_{n=0}^{\infty}\left(\frac{-ivD}{2m_Q}\right)^n i\not{D}^\perp h_v\,. \tag{5}$$

However the procedure which leads to the lagrangian (5) is not unique since one can switch to another formulation of HQET by an appropriate field redefinition without affecting matrix elements (see below). What formulation is chosen is more or less a matter of convenience. Therefore we will refer to the formulation of HQET based on the lagrangian (5) as MRR–HQET.

In the presence of radiative corrections (5) generalizes to

$$\mathcal{L} = \bar{h}_v(ivD)h_v + \sum_{n=1}^{\infty} \frac{1}{(2m_Q)^n}\mathcal{L}^{(n)} \,, \tag{6}$$

where

$$\mathcal{L}^{(n)} = \sum_i C_i^{(n)}(\mu)\mathcal{O}_i^{(n)}(\mu) \tag{7}$$

is a sum of operators multiplied by short distance coefficients.

A redefinition of the field h_v implies a reordering of the $1/m_Q$–expansion and consequently a mixing of the coefficients. This means that in general the decomposition (7) is not unique but depends on what specific formulation of HQET has been chosen.

Under renormalization the set of operators in the tree level lagrangian (5) has to be completed to an operator basis. This basis in general has to include operators which vanish by the lowest order equation of motion $(ivD)h_v = 0$ (EOM), since these operators are needed as counterterms of physical operators. At the highest order $\mathcal{O}(1/m_Q)$ to which the $1/m_Q$–expansion is extended one can forget about EOM–operators completely. However, their presence at a certain lower $\mathcal{O}(1/m_Q)$ affects the coefficients in the orders beyond if the EOM–operators are removed from the lagrangian. There exist several procedures how to remove these operators consistently, each of them leading to the same unique minimal lagrangian [1] which does not depend on the specific formulation of HQET one starts with.

In the present paper this will be done by an appropriate field redefinition. However, first of all in every $\mathcal{O}(1/m_Q)$ a complete off shell operator basis including EOM operators has to be renormalized.

In the present paper we restrict ourselves to the one loop renormalization of the operator bases appearing up to $\mathcal{O}(1/m_Q^3)$. The set of operators up to mass dimension 7 is given by

$$\mathcal{O}_i^{(n)} = \bar{h}_v i\mathcal{D}_i^{(n)} h_v \,, \tag{8}$$

where $i\mathcal{D}_i^{(n)}$ are the combinations of covariant derivatives defined in table 1. To keep things simple we disregard four fermion operators and pure gluonic operators. The lagrangian (6) can be written as

[1] The terminology "minimal" refers to the consistent usage of iD_μ and ivD to construct the operator basis. If instead $iD_\mu^\perp$ and ivD are used EOM–operators are "hidden" in the (ivD)–piece of $iD_\mu^\perp$ and a decomposition into operators vanishing or non–vanishing by the EOM is not obvious.

$$
\begin{aligned}
\mathcal{L} \;&=\; \bar{h}_v \Bigg[(ivD) + \frac{1}{2m_Q} \sum_{i=1}^{3} C_i^{(1)} i\mathcal{D}_i^{(1)} + \frac{1}{(2m_Q)^2} \sum_{i=1}^{7} C_i^{(2)} i\mathcal{D}_i^{(2)} \\
&\qquad\qquad + \frac{1}{(2m_Q)^3} \sum_{i=1}^{24} C_i^{(3)} i\mathcal{D}_i^{(3)} \Bigg] h_v \,. \qquad (9)
\end{aligned}
$$

The Wilson coefficients $C_i^{(n)}$ obey the renormalization group equation

$$
\frac{d}{d\ln\mu} C_i^{(n)}(\mu) + (\frac{\alpha_s(\mu)}{\pi}) \sum_j \hat{\gamma}_{ij}^{(3)\top} C_j^{(3)}(\mu) = 0 \,. \qquad (10)
$$

Since we are working to leading logarithmic accuracy the one loop anomalous dimensions have been already plugged in. The anomalous dimensions are well known up to $\mathcal{O}(1/m_Q^2)$ [10, 4, 11, 7] and have recently been calculated at $\mathcal{O}(1/m_Q^3)$ [8].

The operator basis at $\mathcal{O}(1/m_Q^3)$ is very large which makes an analytic solution of the renormalization group equation (10) too complicated. Therefore we extract from the exact solution

$$
C_i^{(n)}(\mu) = \sum_j \left[\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right)^{\frac{\hat{\gamma}^{(n)\top}}{2\beta^{(0)}}} \right]_{ij} C_j^{(n)}(m_Q) \qquad (11)
$$

the first order logarithmic corrections in the Wilson coefficients:

$$
C_i^{(n)}(\mu) = C_i^{(n)}(m_Q) - \rho \sum_j \hat{\gamma}_{ij}^{(3)\top} C_j^{(n)}(m_Q) + \mathcal{O}((\frac{\alpha_s}{\pi})^2) \qquad (12)
$$

Here $\rho = (\alpha_s(\mu)/\pi)\ln(\mu/m_Q)$ and $C_i^{(n)}(m_Q)$ is the tree level value of the coefficient at the matching scale.

In what follows the coefficients are subject of certain relations. Since the first order logarithmic correction is unique we assume that if the coefficients in the approximation (12) obey the relations so will do the resummed coefficients.

In order to remove the EOM–operators from the lagrangian (9) we now redefine the heavy quark field h_v in terms of a "physical" field $h_v^{(p)}$

$$
h_v = \left[1 + \frac{1}{2m_Q} P^{(1)}(v, iD) + \frac{1}{(2m_Q)^2} P^{(2)}(v, iD) + \frac{1}{(2m_Q)^3} P^{(3)}(v, iD) \right] h_v^{(p)} \,, \qquad (13)
$$

where

$$
P^{(1)}(v, iD) = p_1^{(1)}(ivD) \qquad (14)
$$

$$
P^{(2)}(v, iD) = \sum_{i=1}^{3} p_i^{(2)} i\mathcal{D}_i^{(1)} \qquad (15)
$$

i	$i\mathcal{D}_i^{(n)}$	tree level	coefficient of $(\alpha_s(\mu)/\pi)\ln(\mu/m_Q)$
$i\mathcal{D}_1^{(1)}$	$(iD)^2$	1	0
$i\mathcal{D}_2^{(1)}$	$-i\sigma_{\mu\nu}iD^\mu iD^\nu$	1	$1/2C_A$
$i\mathcal{D}_3^{(1)}$	$(ivD)^2$	-1	$-3C_F-3/2\bar{\xi}C_F$
$i\mathcal{D}_1^{(2)}$	$iD_\mu(ivD)iD^\mu$	-1	$2/3C_A+8/3C_F$
$i\mathcal{D}_2^{(2)}$	$i\sigma_{\mu\nu}iD^\mu(ivD)iD^\nu$	1	C_A
$i\mathcal{D}_3^{(2)}$	$(ivD)(iD)^2$	0	$-3/2\bar{\xi}C_F-1/3C_A-13/3C_F$
$i\mathcal{D}_4^{(2)}$	$(iD)^2(ivD)$	0	$-3/2\bar{\xi}C_F-1/3C_A-13/3C_F$
$i\mathcal{D}_5^{(2)}$	$(ivD)^3$	1	$9\bar{\xi}C_F+12C_F$
$i\mathcal{D}_6^{(2)}$	$(ivD)i\sigma_{\mu\nu}iD^\mu iD^\nu$	0	$3/2\bar{\xi}C_F+3C_F$
$i\mathcal{D}_7^{(2)}$	$i\sigma_{\mu\nu}iD^\mu iD^\nu(ivD)$	0	$3/2\bar{\xi}C_F+3C_F$
$i\mathcal{D}_1^{(3)}$	$iD_\mu(ivD)^2iD^\mu$	1	$3/2\bar{\xi}C_F-28/3C_A-23/3C_F$
$i\mathcal{D}_2^{(3)}$	$(iD)^2(iD)^2$	0	$-3/2\bar{\xi}C_F-7/6C_A-17/3C_F$
$i\mathcal{D}_3^{(3)}$	$iD_\mu(iD)^2iD^\mu$	0	$-3/2\bar{\xi}C_F+5C_A-1/3C_F$
$i\mathcal{D}_4^{(3)}$	$iD_\mu iD_\nu iD^\mu iD^\nu$	0	$3/2\bar{\xi}C_F-23/6C_A+3C_F$
$i\mathcal{D}_5^{(3)}$	$i\sigma^{\mu\nu}iD_\mu(ivD)^2iD_\nu$	-1	$-3/2\bar{\xi}C_F+8/3C_A-1/3C_F$
$i\mathcal{D}_6^{(3)}$	$i\sigma^{\mu\nu}iD_\mu iD_\nu(iD)^2$	0	$3/2\bar{\xi}C_F-7/6C_A+13/3C_F$
$i\mathcal{D}_7^{(3)}$	$i\sigma^{\mu\nu}iD_\rho iD_\mu iD_\nu iD^\rho$	0	$3/2\bar{\xi}C_F-19/6C_A+1/3C_F$
$i\mathcal{D}_8^{(3)}$	$i\sigma^{\mu\nu}(iD)^2iD_\mu iD_\nu$	0	$3/2\bar{\xi}C_F-7/6C_A+13/3C_F$
$i\mathcal{D}_9^{(3)}$	$i\sigma^{\mu\nu}iD_\mu(iD)^2iD_\nu$	0	$3/2\bar{\xi}C_F-5C_A+3C_F$
$i\mathcal{D}_{10}^{(3)}$	$i\sigma^{\mu\nu}iD_\mu iD_\rho iD_\nu iD^\rho$	0	$-3/2\bar{\xi}C_F+11/2C_A-3C_F$
$i\mathcal{D}_{11}^{(3)}$	$i\sigma^{\mu\nu}iD_\rho iD_\mu iD^\rho iD_\nu$	0	$-3/2\bar{\xi}C_F+11/2C_A-3C_F$
$i\mathcal{D}_{12}^{(3)}$	$g_s^2F^{a\mu\nu}F^a_{\mu\nu}$	0	$1/12C_A$
$i\mathcal{D}_{13}^{(3)}$	$g_s^2v_\nu v^\rho F^{a\mu\nu}F^a_{\mu\rho}$	0	$-1/3C_A$
$i\mathcal{D}_{14}^{(3)}$	$(ivD)^4$	-1	$-89/2\bar{\xi}C_F-41C_F$
$i\mathcal{D}_{15}^{(3)}$	$(ivD)^2(iD)^2$	0	$-1/2\bar{\xi}C_A+15/2\bar{\xi}C_F+17/6C_A+41/3C_F$
$i\mathcal{D}_{16}^{(3)}$	$(ivD)(iD)^2(ivD)$	0	$-\bar{\xi}C_A+19/2\bar{\xi}C_F-20/3C_A+23/3C_F$
$i\mathcal{D}_{17}^{(3)}$	$(iD)^2(ivD)^2$	0	$-1/2\bar{\xi}C_A+15/2\bar{\xi}C_F+17/6C_A+41/3C_F$
$i\mathcal{D}_{18}^{(3)}$	$iD_\mu(ivD)iD^\mu(ivD)$	0	$\bar{\xi}C_A+31/6C_A+10/3C_F$
$i\mathcal{D}_{19}^{(3)}$	$(ivD)iD_\mu(ivD)iD^\mu$	0	$\bar{\xi}C_A+31/6C_A+10/3C_F$
$i\mathcal{D}_{20}^{(3)}$	$i\sigma^{\mu\nu}iD_\mu iD_\nu(ivD)^2$	0	$-15/2\bar{\xi}C_F-5/6C_A-37/3C_F$
$i\mathcal{D}_{21}^{(3)}$	$i\sigma^{\mu\nu}(ivD)iD_\mu iD_\nu(ivD)$	0	$\bar{\xi}C_A-7/2\bar{\xi}C_F+41/6C_A-11/3C_F$
$i\mathcal{D}_{22}^{(3)}$	$i\sigma^{\mu\nu}(ivD)^2iD_\mu iD_\nu$	0	$-15/2\bar{\xi}C_F-5/6C_A-37/3C_F$
$i\mathcal{D}_{23}^{(3)}$	$i\sigma^{\mu\nu}iD_\mu(ivD)iD_\nu(ivD)$	0	$-37/6C_A-14/3C_F$
$i\mathcal{D}_{24}^{(3)}$	$i\sigma^{\mu\nu}(ivD)iD_\mu(ivD)iD_\nu$	0	$-37/6C_A-14/3C_F$

Table 1: Operators up to $\mathcal{O}(1/m_Q^3)$ and their coefficients at tree level and $\mathcal{O}(\alpha_s)$. The operators of mass dimensions 5, 6 and 7 are denoted $\mathcal{O}_i^{(1)}$, $\mathcal{O}_i^{(2)}$ and $\mathcal{O}_i^{(3)}$ respectively and the corresponding expressions have to be sandwiched between heavy quark spinors. We use $\bar{\xi}=1-\xi$ as gauge parameter.

$$P^{(3)}(v,iD)=\sum_{i=1}^{7}p_i^{(3)}i\mathcal{D}_i^{(2)} \tag{16}$$

with

$$\begin{aligned}
p_1^{(1)} &= 1/2+\rho(3/4C_F\overline{\xi}+3/2C_F)\\
p_1^{(2)} &= -1/2+\rho(3/4C_F\overline{\xi}+17/6C_F+1/3C_A)\\
p_2^{(2)} &= -1/2+\rho(3/4C_F\overline{\xi}+3/2C_F-1/4C_A)\\
p_3^{(2)} &= -1/8-\rho(27/8C_F\overline{\xi}+15/4C_F)\\
p_1^{(3)} &= 3/2-\rho(3/4C_F\overline{\xi}+37/6C_F+C_A\overline{\xi}+9/2C_A)\\
p_2^{(3)} &= -3/2+\rho(3/4C_F\overline{\xi}+37/6C_F+14/3C_A)\\
p_3^{(3)} &= -1/8+\rho(-27/8C_F\overline{\xi}-85/12C_F+1/2C_A\overline{\xi}-5/2C_A)\\
p_4^{(3)} &= -5/8+\rho(-29/8C_F\overline{\xi}-11/12C_F+1/2C_A\overline{\xi}+3C_A)\\
p_5^{(3)} &= 1/16+\rho(505/32C_F\overline{\xi}+193/16C_F)\\
p_6^{(3)} &= 1/8+\rho(27/8C_F\overline{\xi}+85/12C_F+43/48C_A)\\
p_7^{(3)} &= 5/8+\rho(5/8C_F\overline{\xi}-5/12C_F-1/2C_A\overline{\xi}-137/48C_A)\,.
\end{aligned} \tag{17}$$

In what follows we also need the inverse to (13)

$$h_v^{(p)}=\left[1+\frac{1}{2m_Q}Q^{(1)}(v,iD)+\frac{1}{(2m_Q)^2}Q^{(2)}(v,iD)+\frac{1}{(2m_Q)^3}Q^{(3)}(v,iD)\right]h_v \tag{18}$$

with

$$\begin{aligned}
Q^{(1)}(v,iD) &= -P^{(1)}(v,iD)\\
Q^{(2)}(v,iD) &= -P^{(2)}(v,iD)+(P^{(1)}(v,iD))^2\\
Q^{(3)}(v,iD) &= -P^{(3)}(v,iD)+\{P^{(1)}(v,iD),P^{(2)}(v,iD)\}-(P^{(1)}(v,iD))^3\,.
\end{aligned} \tag{19}$$

In (17) the coefficients $p_i^{(n)}$ are choosen such that under application of (13) up to $\mathcal{O}(1/m_Q^3)$ all operators vanishing by the EOM are removed from the lagrangian (9) and we are left with

$$\begin{aligned}
\mathcal{L}^{(p)} \;=\; \bar{h}_v^{(p)}\Bigg[(ivD)&+\frac{1}{2m_Q}\sum_{i=1}^{2}C_i^{(1)p}i\mathcal{D}_i^{(1)}+\frac{1}{(2m_Q)^2}\sum_{i=1}^{2}C_i^{(2)p}i\mathcal{D}_i^{(2)}\\
&+\frac{1}{(2m_Q)^3}\sum_{i=1}^{13}C_i^{(3)p}i\mathcal{D}_i^{(3)}\Bigg]h_v^{(p)}
\end{aligned} \tag{20}$$

with the transformed coefficients listed in table 2. Comparison with table 1 shows that the coefficients $C_{1/2}^{(1/2)p}$ of the physical operators at $\mathcal{O}(1/m_Q)$ and

$C_i^{(n)p}$	tree level	coefficient of ρ
$C_1^{(1)p}$	1	0
$C_2^{(1)p}$	1	$C_A/2$
$C_1^{(2)p}$	-1	$2/3C_A + 8/3C_F$
$C_2^{(2)p}$	1	C_A
$C_1^{(3)p}$	2	$-25/3\,C_A - 32/3\,C_F$
$C_2^{(3)p}$	-1	$-1/2\,C_A$
$C_3^{(3)p}$	-1	$4\,C_A + 8/3\,C_F$
$C_4^{(3)p}$	1	$-17/6\,C_A$
$C_5^{(3)p}$	-2	$5/3\,C_A + 8/3\,C_F$
$C_6^{(3)p}$	1	$-\,C_A$
$C_7^{(3)p}$	1	$-13/6\,C_A - 8/3\,C_F$
$C_8^{(3)p}$	1	$-\,C_A$
$C_9^{(3)p}$	1	$-4\,C_A$
$C_{10}^{(3)p}$	-1	$9/2\,C_A$
$C_{11}^{(3)p}$	-1	$9/2\,C_A$
$C_{12}^{(3)p}$	0	$1/12\,C_A$
$C_{13}^{(3)p}$	0	$-1/3\,C_A$

Table 2: Coefficients of the physical operator basis.

$\mathcal{O}(1/m_Q^2)$ are not affected by the redefinition (13) whereas the coefficients at $\mathcal{O}(1/m_Q^3)$ are modified. In particular new operators appear already at tree level and the one loop contributions are now independent of the gauge parameter, which is a crucial property of coefficients of physical operators. Note that in order to remove the EOM operators appearing at one loop order in the renormalized lagrangian one has to include terms of $\mathcal{O}(\alpha_s)$ in the field redefinition (13). In other words the tree level field redefinition will not suffice to remove all EOM operators from the lagrangian once radiative corrections are included.

Let us shortly comment on other methods which may reduce the MRR–lagrangian (9) to the minimal lagrangian (20).

On the level of matrix elements the reduction effectively takes place if insertions of the EOM–operators in the lagrangian (9) into time ordered products are properly taken care of by contraction identities [8]

$$i\,\mathrm{T}\left[\bar{h}_v F(iD)(ivD)h_v, \bar{h}_v G(iD)h_v\right] = -\bar{h}_v F(iD)G(iD)h_v + \ldots \qquad (21)$$

which allow to remove these unphysical T–products in favour of local operators. This procedure corresponds to a reorganisation of the $1/m_Q$–expansion and is equivalent to a field redefinition.

An alternative method applies the full EOM of the heavy quark, i.e. the

EOM derived from the lagrangian (9) including power corrections

$$(ivD)h_v = \mathcal{O}(1/m_Q)\,, \tag{22}$$

to the EOM operators in the MRR–lagrangian itself. This way these operators are related to operators of higher dimension and the $1/m_Q$–expansion and the coefficients are reshuffeld. We have checked explicitly that all three methods, i.e. field redefinition, the use of contraction identities and application of the full EOM, yield the same minimal lagrangian (20).

3 Reparametrization Invariance and Field Redefinitions

The Greensfunctions of QCD only depend on the full heavy quark momentum independent from its decomposition into on–shell component $m_Q v$ and residual heavy quark momentum in terms of which HQET is defined. This is reflected in HQET by a new symmetry, the reparametrization invariance. A change of the velocity $v \to v + \delta v$, $v \cdot \delta v = 0$, is accompanied by an appropriate change of the heavy quark field $h_v \to h_{v+\delta v} = h_v + \delta h_v$ such that the HQET lagrangian is invariant. In general the field transformation δh_v depends on the specific formulation of HQET. In this section we derive a description how the transformation has to be modified if we switch to a different formulation of HQET by means of a field redefinition.

The MRR–lagrangian (9) is unphysical in the sense, that it contains operators vanishing by the heavy quark EOM which have to be removed carefully to extract the final minimal lagrangian (20). What makes the MRR–lagrangian interesting is that it deduces from the QCD lagrangian in the most direct and simple way. In order to investigate reparametrization invariance which is a genuine property of QCD itself we therefore study first of all the MRR–lagrangian.

Suppose the lagrangian in the MRR formulation $\mathcal{L}(v, h_v, \bar{h}_v)$ is invariant under the reparametrization transformation

$$v \to v + \delta v \qquad h_v \to h_v + \delta h_v \qquad \delta h_v = F(v, \delta v, iD) h_v\,. \tag{23}$$

In general $F(v, \delta v, iD)$ has an expansion in powers $1/m_Q$ and α_s. Its concrete form is irrelevant for the more general consideration in this section and will be specified in section 4.

In terms of the lagrangian reparametrization invariance means

$$\left[\delta v \frac{\partial}{\partial v} + \frac{\delta}{\delta h_v}\delta h_v + \delta \bar{h}_v \frac{\delta}{\delta \bar{h}_v}\right] \mathcal{L}(v, h_v, \bar{h}_v) = 0\,. \tag{24}$$

Given (23) we want to derive an analogous transformation which leaves the minimal lagrangian $\mathcal{L}^{(p)}(v, h_v^{(p)}, \bar{h}_v^{(p)})$ (20) invariant:

$$v \to v + \delta v \tag{25}$$

$$h_v^{(p)} \to h_v^{(p)} + \delta h_v^{(p)} \tag{26}$$

The field redefinition

$$h_v = [1 + P(v, iD)] h_v^{(p)} \leftrightarrow h_v^{(p)} = [1 + Q(v, iD)] h_v \,, \tag{27}$$

where $P(v, iD)$, $Q(v, iD)$ have to be identified with the terms in square brackets in (13,18) connects both formulations of HQET:

$$\mathcal{L}(v, h_v, \bar{h}_v) = \mathcal{L}^{(p)} \left(v, [1 + Q(v, iD)] h_v, \bar{h}_v [1 + \bar{Q}(v, iD)] \right) \tag{28}$$

Inserting (28) in (24) and using (27) we derive

$$\left[\delta v \frac{\partial}{\partial v} + \frac{\delta}{\delta h_v^{(p)}} \delta h_v^{(p)} + \delta \bar{h}_v^{(p)} \frac{\delta}{\delta \bar{h}_v^{(p)}} \right] \mathcal{L}^{(p)}(v, h_v^{(p)}, \bar{h}_v^{(p)}) = 0 \tag{29}$$

where now

$$\delta h_v^{(p)} = \left([1 + Q(v, iD)] F(v, \delta v, iD) + [\frac{\partial}{\partial v} Q(v, iD)] \right) [1 + P(v, iD)] h_v^{(p)} \,. \tag{30}$$

Inserting the expressions for $F(v, \delta v, iD)$, $P(v, iD)$ and $Q(v, iD)$ and expanding to the required order $\mathcal{O}(1/m_Q)$ and $\mathcal{O}(\alpha_s)$ yields the reparametrization transformation appropriate to the minimal lagrangian (20). In the next sections we derive this transformation at tree–level and $\mathcal{O}(\alpha_s)$ and show explicitly that the minimal lagrangian (20) is reparametrization invariant.

4 Reparametrization Invariance of the MRR–Lagrangian

From the reparametrization invariance of the tree level matching condition (4)

$$\delta v \frac{d}{dv} Q(x) = 0 \tag{31}$$

we derive the reparametrization transformation at tree level:

$$\delta h_v = \left[i m_Q \delta v x + \frac{\delta \not{v}}{2} + \frac{\delta \not{v}}{2} \frac{i \not{D}^\perp}{2 m_Q} - \frac{\delta \not{v}}{2} \frac{(i v D) i \not{D}^\perp}{(2 m_Q)^2} \right] h_v \tag{32}$$

The second term in the square brackets is irrelevant since $P_v^+ \delta \not{v} P_v^+ = 0$. We have included in the MRR–lagrangian (9) terms up to $\mathcal{O}(1/m_Q^3)$. Since $\delta h_v = \mathcal{O}(m_Q)$ the field reparametrization (32) must be known up to $\mathcal{O}(1/m_Q^2)$. A short calculation shows that the tree level MRR–lagrangian, i.e. (9) with $\rho = 0$ is invariant under the transformation (32), i.e. its variation is $\mathcal{O}(1/m_Q^3)$. The question is now, if (32) is the correct transformation formula if radiative corrections are included in the lagrangian. In fact the tree level transformation (32) leaves the one loop effective lagrangian invariant as long as power corrections not higher than $\mathcal{O}(1/m_Q^2)$ are included. In this case the term of $\mathcal{O}(1/m_Q^2)$ in (32) is irrelevant. However this situation changes drastically if power corrections of $\mathcal{O}(1/m_Q^3)$ or higher are taken into account. The inconsistency shows up if we require reparametrization invariance of the lagrangian (9) under the tree level transformation (32) to all orders in the strong coupling. Then the shortdistance coefficients $C_i^{(n)}$, $n = 1, 2, 3$, must fulfill the following relations:

$$
\begin{aligned}
C^{(1)} &= 1 + \Delta_1 \\
C_2^{(2)} &= 2C_2^{(1)} - 1 + \Delta_2 \\
2C_4^{(2)} + C_1^{(2)} &= 2C_3^{(1)} + 1 + \Delta_3 \\
2C_5^{(2)} + C_1^{(2)} &= 2C_3^{(1)} + 1 + \Delta_4 \\
C_5^{(3)} + C_{24}^{(3)} &= -2C_6^{(2)} - 2C_2^{(2)} - C_2^{(1)} - C_3^{(1)} + 1 + \Delta_5 \\
C_{23}^{(3)} - C_{24}^{(3)} &= 2C_6^{(2)} - 2C_7^{(2)} + \Delta_6 \\
C_5^{(3)} + C_{23}^{(3)} &= -2C_7^{(2)} - 2C_2^{(2)} - C_2^{(1)} - C_3^{(1)} + 1 + \Delta_7 \\
C_{11}^{(3)} + 2C_8^{(3)} + C_7^{(3)} &= 2C_6^{(2)} + \Delta_8 \\
C_{11}^{(3)} + C_{10}^{(3)} + 2C_9^{(3)} &= 2C_2^{(2)} - 2C_2^{(1)} + \Delta_9 \\
C_{10}^{(3)} + C_7^{(3)} + 2C_6^{(3)} &= 2C_7^{(2)} + \Delta_{10} \\
C_{18}^{(3)} + 2C_{17}^{(3)} + C_1^{(3)} &= 2C_5^{(2)} + C_3^{(1)} + C_2^{(1)} - 1 + \Delta_{11} \\
C_{19}^{(3)} + C_{18}^{(3)} + 2C_{16}^{(3)} &= 2C_5^{(2)} - 2C_2^{(1)} + \Delta_{12} \\
C_{19}^{(3)} + 2C_{15}^{(3)} + C_1^{(3)} &= 2C_5^{(2)} + C_3^{(1)} + C_2^{(1)} - 1 + \Delta_{13} \\
C_4^{(3)} + C_3^{(3)} + 2C_2^{(3)} &= 2C_3^{(2)} + C_1^{(1)} - C_2^{(1)} + \Delta_{14} \\
C_4^{(3)} + C_3^{(3)} &= C_1^{(2)} + C_2^{(1)} + \Delta_{15} \\
C_4^{(3)} + C_3^{(3)} + 2C_2^{(3)} &= 2C_4^{(2)} + C_1^{(1)} - C_2^{(1)} + \Delta_{16} \\
C_{10}^{(3)} + C_9^{(3)} &= C_2^{(1)} - C_1^{(1)} + \Delta_{17} \\
C_{11}^{(3)} - C_{10}^{(3)} &= \Delta_{18} \\
C_{11}^{(3)} + C_9^{(3)} &= C_2^{(1)} - C_1^{(1)} + \Delta_{19}
\end{aligned}
\tag{33}
$$

If the tree level reparametrization transformation (32) is the correct transformation even in the presents of radiative corrections all Δ_i have to vanish. A deviation form this value would signal reparametrization invariance breaking. Insertion of the coefficients in table 1 shows that 5 of the relations involving coefficients of operators at $\mathcal{O}(1/m_Q^3)$ are violated at $\mathcal{O}(\alpha_s)$:

$$
\begin{aligned}
\Delta_5 = \Delta_7 = -\Delta_{11} = -\Delta_{13} &= -\rho(C_A + 2C_F) \\
\Delta_{12} &= -\rho(2C_A + 2C_F - \overline{\xi}C_F) \\
\Delta_i &= 0 \quad i \neq 5, 7, 11, 12, 13
\end{aligned}
\tag{34}
$$

Obviously the naive understanding of reparametrization invariance that the tree level transformation remains correct even if radiative corrections are included in the lagrangian fails beyond $\mathcal{O}(1/m_Q^2)$. However, the violation of reparametrization invariance can be cured if the reparametrization transformation itself is renormalized, i.e. is supplemented by $\mathcal{O}(\alpha_s)$–corrections. Since the tree level transformation works well up to $\mathcal{O}(1/m_Q^2)$ the term of $\mathcal{O}(1/m_Q^2)$ in (32) which is responsible for the relations between $\mathcal{O}(1/m_Q^3)$ and the lower orders has to be corrected at $\mathcal{O}(\alpha_s)$. It comes out that (32) generalizes to

$$\delta h_v = \left[i m_Q \delta v x + \frac{1}{2m_Q} F_1 + \frac{1}{(2m_Q)^2} F_2 \right] h_v \tag{35}$$

where

$$F_1 = c_1 \frac{\delta\!\!\!/ v}{2} i D\!\!\!\!/ \tag{36}$$

$$F_2 = c_{21} \frac{\delta\!\!\!/ v}{2} (ivD) i D\!\!\!\!/ + c_{22} \frac{\delta\!\!\!/ v}{2} i D\!\!\!\!/ (ivD) + c_{23} (i\delta v D)(ivD) + c_{24} (ivD)(i\delta v D) \tag{37}$$

with

$$\begin{aligned} c_1 &= 1 \\ c_{21} &= -1 + \rho(C_A + 2C_F) \\ c_{22} &= \rho(-C_A + \frac{1}{2}\bar{\xi} C_F - C_F) \\ c_{23} &= c_{24} = 0\,. \end{aligned}$$

In (35) the expression in square brackets coincides with the operator $F(v, \delta v, iD)$ in (23). One may argue that one can always find a transformation formular similar to (35) which leaves the lagrangian invariant whatever values the coefficients $C_i^{(n)}$ take. However this argument fails, since in the transformation formula at $\mathcal{O}(1/m_Q^2)$ there are in general only 4 terms which can be modified to correct all possible violated relations in (33). For example in our case the correction of only two terms in the transformation formula suffices to recover reparametrization invariance.

5 Switching to the Minimal Operator Basis

Now we are in the position to combine the results of the previous sections to calculate the reparametrization transformation of the "physical" heavy quark field $h_v^{(p)}$. Inserting (13,18) and (35) into (30) and keeping only terms up to $\mathcal{O}(1/m_Q^2)$ and $\mathcal{O}(\alpha_s)$ we get

$$\delta h_v^{(p)} = \left[i m_Q \delta v x + \frac{1}{2m_Q} F_1^{(p)} + \frac{1}{(2m_Q)^2} F_2^{(p)} \right] h_v^{(p)} \tag{38}$$

where

$$F_1^{(p)} = c_{11}^{(p)} \frac{\delta\not{v}}{2} i\not{D} + c_{12}^{(p)} i\delta v D \tag{39}$$

$$F_2^{(p)} = c_{21}^{(p)} \frac{\delta\not{v}}{2}(ivD) i\not{D} + c_{22}^{(p)} \frac{\delta\not{v}}{2} i\not{D}(ivD) + c_{23}^{(p)}(i\delta v D)(ivD) + c_{24}^{(p)}(ivD)(i\delta v D) \tag{40}$$

with

$$\begin{aligned}
c_{11}^{(p)} &= 1 \\
c_{12}^{(p)} &= -1 + \frac{1}{3}\rho(C_A + 4C_F) \\
c_{21}^{(p)} &= -1 - \rho(\frac{25}{6}C_A + \frac{8}{3}C_F) \\
c_{22}^{(p)} &= \rho(\frac{1}{2}\overline{\xi}C_F + \frac{25}{16}C_A + \frac{11}{3}C_F) \\
c_{23}^{(p)} &= \frac{1}{2} - \rho(\frac{1}{4}\overline{\xi}C_F + \frac{11}{6}C_A + \frac{11}{6}C_F) \\
c_{24}^{(p)} &= 1 - \rho(\frac{7}{3}C_A + 4C_F)\,.
\end{aligned} \tag{41}$$

Note that the gauge parameter dependent terms in (41) cancels if the transformation formula is applied to the lagrangian. We have checked that the minimal lagrangian (20) is indeed invariant under (38).

The reparametrization transformation (38) of the physical heavy quark field receives one loop corrections already at $\mathcal{O}(1/m_Q)$ which is not the case for the transformation (35) of the MRR–lagrangian. From this we expect, that to one loop order reparametrization invariance of the minimal lagrangian is already broken at $\mathcal{O}(1/m_Q^2)$. Let us therefore in analogy to section 4 derive the relations among the coefficients $C_i^{(n)p}$ which have to hold if we require the minimal lagrangian (20) to be invariant under the tree level reparametrization transformation (i.e. (38) with $\rho = 0$) to all orders in perturbation theory:

$$\begin{aligned}
C_1^{(1)p} &= 1 + \Delta_1^{(p)} \\
C_2^{(2)p} &= 2C_2^{(1)p} - 1 + \Delta_2^{(p)} \\
C_1^{(2)p} &= -1 + \Delta_3^{(p)} \\
C_5^{(3)p} &= -2C_2^{(2)p} - C_2^{(1)p} + 1 + \Delta_4^{(p)} \\
C_{11}^{(3)p} + 2C_8^{(3)p} + C_7^{(3)p} &= 2C_2^{(1)p} + \Delta_5^{(p)} \\
C_{11}^{(3)p} + C_{10}^{(3)p} + 2C_9^{(3)p} &= 2C_2^{(2)p} - 2C_2^{(1)p} + \Delta_6^{(p)} \\
C_{10}^{(3)p} + C_7^{(3)p} + 2C_6^{(3)p} &= 2C_2^{(1)p} + \Delta_7^{(p)} \\
C_1^{(3)p} &= C_2^{(1)p} + 1 + \Delta_8^{(p)} \\
0 &= C_2^{(1)p} - 1 + \Delta_9^{(p)}
\end{aligned} \tag{42}$$

$$
\begin{aligned}
C_4^{(3)p} + C_3^{(3)p} + 2C_2^{(3)p} &= -C_1^{(1)p} - C_2^{(1)p} + \Delta_{10}^{(p)} \\
C_4^{(3)p} + C_3^{(3)p} &= C_1^{(2)p} + C_2^{(1)p} + \Delta_{11}^{(p)} \\
C_{10}^{(3)p} + C_9^{(3)p} &= C_2^{(1)p} - C_1^{(1)p} + \Delta_{12}^{(p)} \\
C_{11}^{(3)p} - C_{10}^{(3)p} &= \Delta_{13}^{(p)} \\
C_{11}^{(3)p} + C_9^{(3)p} &= C_2^{(1)p} - C_1^{(1)p} + \Delta_{14}^{(p)}
\end{aligned}
$$

Invariance under the tree level reparametrization transformation requires $\Delta_i^{(p)} = 0$. Inserting the coefficents $C_i^{(n)p}$ from table 2 shows that all relations hold at tree level but some are violated at $\mathcal{O}(\alpha_s)$:

$$
\begin{aligned}
\Delta_4 &= \rho(25/6C_A + 8/3C_F) \\
\Delta_3 = -\Delta_5 = -\Delta_7 = \Delta_{10} &= \rho(2/3C_A + 8/3C_F) \\
\Delta_8 &= -\rho(53/6C_A + 32/3C_F) \\
\Delta_9 &= -\rho C_A/2 \\
\Delta_i &= 0 \quad i \neq 3,4,5,7,8,9,10
\end{aligned}
\tag{43}
$$

Again the naive understanding of reparametrization invariance fails and in order to maintain reparametrization invariance one has to add corrections of $\mathcal{O}(\alpha_s)$ to the tree level transformation. We have explicitly checked that (38) leaves the minimal lagrangian invariant if the one loop corrections in (41) are included in the transformation.

In case of the MRR–lagrangian the naive application of reparametrization invariance accidentally works well up to $\mathcal{O}(1/m_Q^2)$, while in case of the minimal lagrangian it leads to inconsistencies already appearing at lower orders. Namely the third and the ninth relation in (42) would imply non renormalization of the operator $\mathcal{O}_1^{(2)}$ (Darwin operator) and the chromomagnetic operator $\mathcal{O}_2^{(1)}$. However, the usual renormalization group approach yields non vanishing radiative corrections for the coefficients of these operators.

6 Conclusions

We have extended the analysis of the reparametrization invariance of HQET to $\mathcal{O}(1/m_Q^3)$. We have shown explicitly that the reparametrization transformation depends on the specific formulation of HQET and that the transformation has to be modified if one switches to another formulation. As examples we have studied the MRR–formulation and the minimal lagrangian.

The main subject of this paper, however, was the question if and in what sense reparametrization invariance is maintained under renormalization. We have demonstrated explicitly that the naive understanding of reparametrization invariance – application of the tree level transformation to the renormalized lagrangian – fails if higher orders $1/m_Q$ as well as radiative corrections are included. In the case of the MRR–lagrangian accidentally the tree–level transformation works well up to $\mathcal{O}(1/m_Q^2)$ but fails at $\mathcal{O}(1/m_Q^3)$. By contrast, in case of the physical lagrangian an inconsistency already appears at $\mathcal{O}(1/m_Q^2)$.

In order to recover reparametrization invariance of both the MRR–lagrangian (9) and the physical lagrangian (20) in the presence of radiative corrections the corresponding field transformations δh_v and $\delta h_v^{(p)}$ have to be renormalized properly. However, given the correct transformation formula in one formulation of HQET the corresponding transformation in any other formulation related to the first by means of a field redefiniton is fixed by (30).

To summarize, the application of reparametrization invariance at higher orders of the $1/m_Q$–expansion in order to extract short distance coefficients from lower order calculations has to be treated with care, since the transformation prescription depends on what specific formulation of HQET has been chosen and requires renormalization if radiative corrections are included in the effective lagrangian.

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