hep-ph/9808442
TTP98–29
August 28, 1998

One-particle Inclusive Semi-Leptonic B decays

Christopher Balzereit and Thomas Mannel
Institut für Theoretische Teilchenphysik, Universität Karlsruhe,
D – 76128 Karlsruhe, Germany

Abstract

We propose a method for a QCD based calculation of one-particle inclusive decays of the form $B \to \bar{D}X$ or $B \to \bar{D}^*X$. It is based on the heavy mass limit and a short distance expansion of the amplitudes, which yield a power series in the parameter $1/M_X^2$ for the spectra and in $\Lambda_{QCD} m_b/(m_b - m_c)^2$ for the rates. We study the leading term of this expansion for the case of the semi–leptonic decays $B \to \bar{D}X\ell^+\nu$.

1 Introduction

Over the last ten years significant progress has been made in the theoretical description of heavy flavour decays [1]. The application of the $1/m_Q$ expansion (m_Q being the mass of the heavy quark) allows us to perform QCD based calculations, which in some cases yield model independent results. The additional symmetries of the infinite mass limit, the so called heavy quark symmetries [2], reduce the uncertainties due to unknown hadronic matrix elements significantly, and corrections to this infinite mass limit have been studied extensively using the framework of Heavy Quark Effective Theory (HQET) [3].

The heavy mass expansion has been applied to various classes of decays. As far as exclusive decays are concerned the main progress has been achieved for semi–leptonic decays, while exclusive non-leptonic decays still have not simplified through the heavy mass limit.

The other side are the fully inclusive decays, the rates of which can be obtained as a power series in $1/m_Q$ by means of an operator product expansion (OPE) and subsequent application of HQET [4]. Here semi–leptonic as well as the non–leptonic processes may be described, allowing us to compute lifetimes and branching ratios. The pattern is well reproduced by the $1/m_Q$ expansion, although some open problems remain [5].

Up to now no attempt has been made to apply similar methods to one-particle inclusive decays, such as $B \to \bar{D} X \ell^+ \nu$ or $B \to \bar{D} X$ and $B \to \bar{D}^* X$. Obviously the standard method as in the inclusive case does not work in a naive way, since in the final state a $\bar{D}$ or a $\bar{D}^*$ is projected out, and the same set-up as in the inclusive case will not work.

In the present paper we propose a method which allows us to compute one-particle inclusive rates, based on QCD. The main ingredients are similar as in the fully inclusive case. In the next section we shall describe the method and then discuss its application to semi–leptonic decays.

2 Description of the Method

We shall consider first decays of the form $B \to \bar{D} X$ (i.e. a $\bar{b} \to \bar{c}$–transition) and thus study the expression

$$G(M^2) = \sum_X \left| \langle B(p_B) | H_{eff} | \bar{D}(p_{\bar{D}}) X \rangle \right|^2 (2\pi)^4 \delta^4(p_B - p_{\bar{D}} - p_X) \qquad (1)$$

where $|X\rangle$ are momentum eigenstates with momentum p_X and H_{eff} the relevant part of the weak Hamiltonian. The function G depends on the invariant mass $M^2 = (p_B - p_{\bar{D}})^2$ of the state $|X\rangle$ which ranges between

$$0 \leq M^2 \leq (m_B - m_D)^2 \,, \tag{2}$$

where we have neglected the pion mass as well as the lepton masses. This function G is related to the decay rate under consideration by

$$d\Gamma(B \to \bar{D}X) = \frac{1}{2m_B} d\Phi_{\bar{D}} \; G(M^2) \tag{3}$$

where $d\Phi_{\bar{D}}$ is the phase space element of the final state $\bar{D}$ meson.

The region close to $M^2 \approx 0$ is dominated by a few resonances (the π and ρ states in the non-leptonic case), and away from this region one can expect duality to hold. In particular, this should be true in the limit in which m_b, $m_c \to \infty$, since in almost all available phase space we have $M^2 \gg \Lambda_{QCD}$.

In technical terms this means that we are going to set up a short distance expansion for the quantity $G(M^2)$. The procedure is similar as the one for inclusive decays, we write

$$G(M^2) = \sum_X \int d^4x \, \langle B(p_B)|H_{eff}(x)|\bar{D}(p_{\bar{D}})X\rangle\langle\bar{D}(p_{\bar{D}})X|H_{eff}(0)|B(p_B)\rangle \tag{4}$$

and make use of the fact that m_c and m_b are both large scales. We make these scales explicit by redefining the heavy quark fields in H_{eff} by

$$b(x) = b_v(x) e^{-im_b vx} \quad c(x) = c_{v'}(x) e^{-im_c v' x} \tag{5}$$

where the velocities are defined as $p_B = m_B v$ and $p_D = m_D v'$.

Inserting this yields

$$\begin{aligned} G(M^2) = & \sum_X \int d^4x \, e^{-i(m_b v - m_c v')x} \qquad (6) \\ & \qquad \langle B(v)|\tilde{H}_{eff}(x)|\bar{D}(v')X\rangle\langle\bar{D}(v')X|\tilde{H}_{eff}(0)|B(v)\rangle \,, \end{aligned}$$

where $\tilde{H}_{eff}$ is obtained from H_{eff} by the replacements $b \to b_v$ and $c \to c_{v'}$. Equation (6) shows that the large momentum entering the game is $m_b v - m_c v'$.

The next step is a short distance expansion of the matrix element appearing in (6) yielding a power series in inverse powers of the large momentum $M = m_b v - m_c v'$

$$G(M^2) = \sum_{n=0}^{\infty} \sum_i C_i^{(n)}(\mu) \langle B(p_B)|\mathcal{O}_i^{(n)}|B(p_B)\rangle|_\mu \,. \tag{7}$$

The operators $\mathcal{O}_i^{(n)}$ depend on the final state $\bar{D}$–meson and are the analogue of the production operators as they appear in heavy quarkonia production [6] or in one-particle inclusive production in e^+e^- annihilation [7]. They are local and have the generic structure

$$\mathcal{O}_i^{(n)} = \sum_X [\bar{c}_{v'} \Gamma b_v] |\bar{D}(v') X\rangle \langle \bar{D}(v') X| [\bar{b}_v \Gamma' c_{v'}] \,, \tag{8}$$

where $\Gamma^{(\prime)}$ denotes a combination of Dirac matrices and covariant derivatives.

The matrix elements of the $\mathcal{O}_i^{(n)}$ between static B meson states are universal functions of the velocity product $v \cdot v'$. We shall not give a detailed proof of factorization of the matrix elements into long and short distance contributions, rather we are aiming at a phenomenological analysis of the one-particle inclusive semi-leptonic decays. We remark that the method is not as rigorous as in the case of fully inclusive decays, where the heavy mass expansion is derived by an operator product expansion. However, fig.1 makes the argument plausible. For M^2 large enough the large momentum flows through the state $|X\rangle$ and we assume that parton-hadron duality holds for this part of the diagram, and hence we can compute this part in perturbative QCD.

The dimension of $\mathcal{O}_i^{(n)}$ in (7) is $n+6$ and hence $C_i^{(n)}/C_i^{(n+1)}$ is of the order M. The leading term of the expansion involves dimension 6–operators and we shall discuss in the present paper only this contribution. If we consider only this leading term, we may even replace the operators b_v and $c_{c'}$ by static HQET quarks. In the following this replacement is understood.

The corrections to be expected can easily be estimated. Since the full momentum transfer is $Q = M + k$ where k is the sum of the residual momenta of the heavy b and c quark, the corrections to the leading term originate typically from

$$Q^2 = M^2 + 2M \cdot k + \mathcal{O}(\Lambda_{QCD}^2) = M^2 \left(1 + \frac{2M \cdot k}{M^2} + \cdots\right) \tag{9}$$

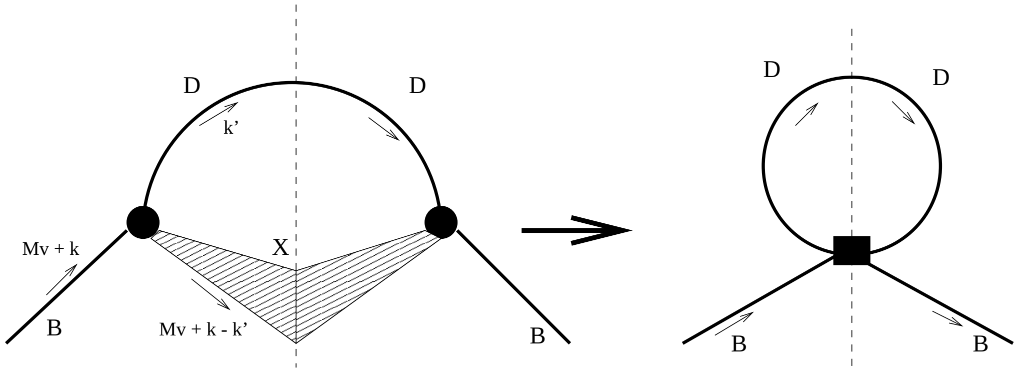

Figure 1: Illustration of the short distance expansion of (6).

and hence the corrections involve typically matrix elements of the form

$$\sum_X \langle B(v)|[\bar{c}_{v'}\Gamma b_v]|\bar{D}(v')X\rangle\langle\bar{D}(v')X|(M\cdot iD)[\bar{b}_v\Gamma^\dagger c_{v'}]|B(v)\rangle \tag{10}$$

where D is the covariant derivative of QCD. This matrix element will be of order $M\cdot v\Lambda_{QCD}$ and consequently the method works as long as

$$\frac{2M\cdot v}{M^2}\Lambda_{QCD} \ll 1\,. \tag{11}$$

M^2 and $M\cdot v$ are not independent variables, since they both can be expressed in terms of the velocity product $v\cdot v'$. Eliminating $M\cdot v$ one obtains

$$\frac{\Lambda_{QCD}}{m_b}\left(\frac{m_b^2-m_c^2}{M^2}+1\right) \ll 1\,, \tag{12}$$

and hence it is obvious that the expansion breaks down for very small M^2. Here again a similar situation occurs as in the inclusive semi–leptonic decays, where the endpoint region may be described in terms of a shape function.

On the other hand one may ask wether the short distance expansion works at all, and thus it is instructive to insert the maximal value for M^2 which is possible in a decay. One finds that the parameter

$$\frac{2\Lambda_{QCD}}{m_b-m_c}$$

should be small compared to unity. Inserting the pessimistic value $\Lambda_{QCD} = 500$ MeV one finds that this parameter is about 1/3, which justifies our approach for the spectra at least close to maximal M^2. In order to get the total rates an integration over the phase space of the $\bar{D}$ meson has to be performed. The details depend on the process under consideration, but the typical size of the corrections can be estimated by computing some arbitrary phase space average. Choosing a pase space measure as

$$d\bar{\Phi} = 2m_b \, d\tilde{p} \, \frac{M^2}{\sqrt{[(m_c + m_b)^2 - M^2][(m_c - m_b)^2 - M^2]}} \tag{13}$$

which yields very simple integrals we find

$$\langle \frac{M \cdot v}{M^2} \rangle = \frac{\int d\bar{\Phi} \frac{M \cdot v}{M^2}}{\int d\bar{\Phi}} = \frac{m_b \Lambda_{QCD}}{(m_b - m_c)^2} \approx \frac{1}{4} \tag{14}$$

justifying the short distance expansion also for the rates.

The second type of corrections are the QCD radiative corrections which can be computed systematically. They will be of the order $\alpha_s(M^2)$ and hence will be small enough to be treated perturbatively. As usual, the logarithms $\alpha_s(M^2) \ln(M^2)$ can be resummed by renormalization group methods; for the leading terms this will be done in section 3.

In the present paper we study only the leading term of the expansion and focus on applications to weak interactions. In this case we need to consider a matrix element of a dimension-six operator involving the left handed currents.

We consider the leptonic case in some detail; inserting the well known effective Hamiltonian for semi–leptonic decays we find

$$\begin{aligned} G(M^2) = {} & \frac{G_F^2}{2} |V_{cb}|^2 P_{\mu\nu}(M) \qquad (15) \\ & \sum_X \langle B(v) | [\bar{c}_{v'} \gamma^\mu (1 - \gamma_5) b_v] | \bar{D}(v') X \rangle \langle \bar{D}(v') X | [\bar{b}_v \gamma^\nu (1 - \gamma_5) c_{v'}] | B(v) \rangle \, , \end{aligned}$$

where $P_{\mu\nu}$ is a tensor originating from contracting the lepton fields in the effective Hamiltonian. This tensor only depends on the vector M and hence has the form

$$P_{\mu\nu}(M) = A(M^2)(M^2 g_{\mu\nu} - M_\mu M_\nu) + B(M^2) M_\mu M_\nu \tag{16}$$

Neglecting the lepton masses, we obtain at tree level

$$A(M^2) = -\frac{1}{3\pi}\Theta(M^2) \text{ and } B(M^2) = 0\,. \tag{17}$$

Using

$$\bar{b}_v \not{M}(1-\gamma_5)c_{v'} = (m_b - m_c)\bar{b}_v c_{v'} - (m_b + m_c)\bar{b}_v \gamma_5 c_{v'} \tag{18}$$

we can write the leading order contribution as

$$\begin{aligned} G(M^2) = \frac{G_F^2}{6\pi}|V_{cb}|^2 4 m_B m_D \big[&(m_B - m_D)^2 \eta_S(v\cdot v') \\ &+(m_B + m_D)^2 \eta_P(v\cdot v') - M^2(\eta_V(v\cdot v') + \eta_A(v\cdot v'))\big] \end{aligned} \tag{19}$$

where we have defined non-perturbative matrix elements as

$$\begin{aligned} 4 m_B m_D \eta_S(v\cdot v') &= \sum_X \langle B(v)|[\bar{c}_{v'} b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v c_{v'}]|B(v)\rangle \\ -4 m_B m_D \eta_P(v\cdot v') &= \sum_X \langle B(v)|[\bar{c}_{v'}\gamma_5 b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v \gamma_5 c_{v'}]|B(v)\rangle \\ 4 m_B m_D \eta_V(v\cdot v') &= \sum_X \langle B(v)|[\bar{c}_{v'}\gamma^\mu b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v \gamma_\mu c_{v'}]|B(v)\rangle \\ 4 m_B m_D \eta_A(v\cdot v') &= \sum_X \langle B(v)|[\bar{c}_{v'}\gamma^\mu\gamma_5 b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v \gamma_\mu\gamma_5 c_{v'}]|B(v)\rangle. \end{aligned} \tag{20}$$

Once radiative corrections are taken into account, these operators mix with the corresponding operators where the b and the c quark are coupled to a color octett:

$$\begin{aligned} 4 m_B m_D \rho_S(v\cdot v') &= \sum_X \langle B(v)|[\bar{c}_{v'} T^a b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v T^a c_{v'}]|B(v)\rangle \\ -4 m_B m_D \rho_P(v\cdot v') &= \sum_X \langle B(v)|[\bar{c}_{v'}\gamma_5 T^a b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v \gamma_5 T^a c_{v'}]|B(v)\rangle \\ 4 m_B m_D \rho_V(v\cdot v') &= \sum_X \langle B(v)|[\bar{c}_{v'}\gamma^\mu T^a b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v \gamma_\mu T^a c_{v'}]|B(v)\rangle \\ 4 m_B m_D \rho_A(v\cdot v') &= \\ &\sum_X \langle B(v)|[\bar{c}_{v'}\gamma^\mu\gamma_5 T^a b_v]|\bar{D}(v')X\rangle\langle \bar{D}(v')X|[\bar{b}_v \gamma_\mu\gamma_5 T^a c_{v'}]|B(v)\rangle \end{aligned} \tag{21}$$

Note that we are using parton-hadron duality and thus $|X\rangle$ is expressed in terms of QCD degrees of freedom and thus the matrix elements ρ_i are nonvanishing.

3 Renormalization Group Improvement

Under renormalization the matrix elements (20,21) become scale dependend quantities $\eta_i(v\cdot v',\mu)$, $\rho_i(v\cdot v',\mu)$. We chose to construct the short distance expansion at an intermediate scale $\bar{m}=1/2(m_b+m_c)$, where both the b– and c–Quark are treated as static fields described by the HQET. However, the typical scale of the hadronic matrix elements is a low hadronic scale $\mu=\Lambda$. Using the renormalization group the matrix elements can be scaled from the matching scale $\bar{m}$ down to the low scale Λ. In our numerical analysis we use $\alpha_s(\Lambda)=1$, where $\alpha_s(\mu)$ is the one loop expression for the running coupling constant.

To this end one has to renormalize the operators [1]

$$\begin{aligned} \mathcal{O}_i^{(1)} &= \sum_X [\bar{c}_{v'}\Gamma_i b_v]|\bar{D}(v')X\rangle\langle\bar{D}(v')X|[\bar{b}_v\Gamma_i c_{v'}] \\ \mathcal{O}_i^{(8)} &= \sum_X [\bar{c}_{v'}\Gamma_i T^a b_v]|\bar{D}(v')X\rangle\langle\bar{D}(v')X|[\bar{b}_v\Gamma_i T^a c_{v'}] \end{aligned} \tag{23}$$

where $\Gamma_i = 1,\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5$. Because of heavy quark spin symmetry mixing occures only between singlett and octett operators corresponding to one specific Dirac structure Γ_i. That means a basis closing under renormalization is given by $\mathcal{O}_i^{(1)}$ and $\mathcal{O}_i^{(8)}$ for every individual i.

The mixing properties of the operators translate into that of their matrix elements. Therefore we can formulate the renormalization group equation directly in terms of the $\eta_i(v\cdot v',\mu)$ and $\rho_i(v\cdot v',\mu)$ as follows:

$$\begin{aligned} \frac{d}{d\ln\mu}\eta_i(v\cdot v',\mu) &= \gamma_{11}\eta_i(v\cdot v',\mu)+\gamma_{18}\rho_i(v\cdot v',\mu) \\ \frac{d}{d\ln\mu}\rho_i(v\cdot v',\mu) &= \gamma_{81}\eta_i(v\cdot v',\mu)+\gamma_{88}\rho_i(v\cdot v',\mu) \end{aligned} \tag{24}$$

Since we restrict ourselves to the leading logarithmic approximation, it suffices to know the one loop anomalous dimensions. These are given by the

[1] Note that at least to one loop order the renormalization properties of these operators are identical to those of the operators

$$\mathcal{O}_i^{(1)} = [\bar{c}_{v'}\Gamma_i b_v][\bar{b}_v\Gamma_i c_{v'}] \qquad \mathcal{O}_i^{(8)} = [\bar{c}_{v'}\Gamma_i T^a b_v][\bar{b}_v\Gamma_i T^a c_{v'}]\,. \tag{22}$$

since the UV behaviour is independent of the states, including the $\bar{D}$ appearing in the final state.

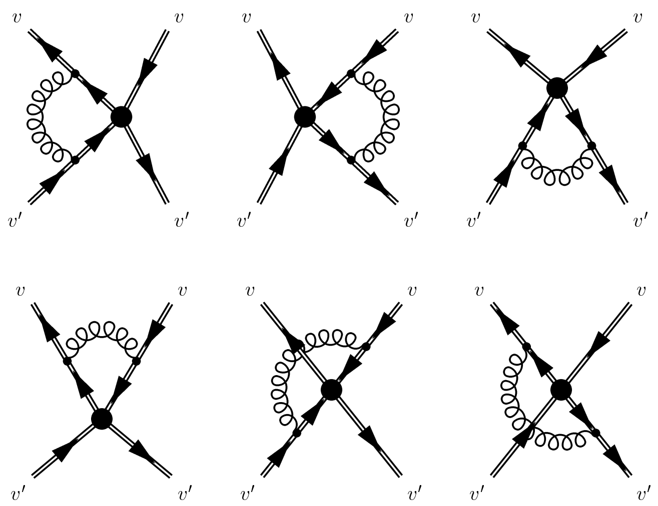

Figure 2: Feynman diagrams contributing to the one loop anomalous dimensions. The blob represents generically the operators corresponding to the η_i and ρ_i.

divergent parts of the Feynman diagrams shown in figure 2 supplemented by wave function renormalization of the heavy quark fields

$$\begin{aligned} \gamma_{11} &= (\frac{\alpha_s}{\pi})(N_c - \frac{1}{N_c})K(v\cdot v') \qquad & \gamma_{18} &= -(\frac{\alpha_s}{\pi})2K(v\cdot v') \\ \gamma_{81} &= (\frac{\alpha_s}{\pi})\frac{1}{2}(\frac{1}{N_c^2} - 1)K(v\cdot v') \qquad & \gamma_{88} &= (\frac{\alpha_s}{\pi})\frac{1}{N_c}K(v\cdot v') \end{aligned} \tag{25}$$

where $N_c = 3$ is the number of colors and

$$K(v\cdot v') = 1 - v\cdot v'\mathrm{Re}[r(v\cdot v')] \qquad r(z) = \frac{\ln(z+\sqrt{z^2-1})}{\sqrt{z^2-1}}\,. \tag{26}$$

The function $r(v\cdot v')$ typically appears in the anomalous dimensions of velocity chanching heavy quark currents [3].

In our case only the real part of $r(v\cdot v')$ shows up in the anomalous dimensions, since the corresponding Feynman amplitudes contribute to the forward matrix element of a hermitian operator which has to be real. Note that individual Feynman diagrams develop imaginary parts which drop out in the sum.

Solving (24) we express the matrix elements η_i, ρ_i at the scale $\bar{m}$ in terms of their value at an arbitrary scale μ:

$$
\begin{aligned}
E_i(v\cdot v') = \eta_i(v\cdot v', \bar{m}) &= C_{11}(v\cdot v',\mu)\eta_i(v\cdot v',\mu) + C_{18}(v\cdot v',\mu)\rho_i(v\cdot v',\mu) \\
R_i(v\cdot v') = \rho_i(v\cdot v', \bar{m}) &= C_{81}(v\cdot v',\mu)\eta_i(v\cdot v',\mu) + C_{88}(v\cdot v',\mu)\rho_i(v\cdot v',\mu)
\end{aligned}
\tag{27}
$$

The coefficient functions $C_{ij}(v\cdot v',\mu)$ are given by

$$
\begin{aligned}
C_{11}(v\cdot v',\mu) &= \frac{1}{N_c^2} + (1-\frac{1}{N_c^2})\zeta(v\cdot v',\mu) \\
C_{18}(v\cdot v',\mu) &= \frac{2}{N_c}\left(1-\zeta(v\cdot v',\mu)\right) \\
C_{81}(v\cdot v',\mu) &= \frac{1}{2N_c}(\frac{1}{N_c^2}-1)\left(\zeta(v\cdot v',\mu)-1\right) \\
C_{88}(v\cdot v',\mu) &= 1+\frac{1}{N_c^2}\left(\zeta(v\cdot v',\mu)-1\right)
\end{aligned}
\tag{28}
$$

where

$$
\zeta(v\cdot v',\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\bar{m})}\right)^{\frac{N_c}{2\beta_0}K(v\cdot v')}
$$

with $\beta_0 = (33-2N_f)/12$ and $N_f = 3$ for three active quark flavours.

Note that in the case of semi–leptonic decays only the functions $E_i(v\cdot v')$ are needed, since in LLA there are no octett contributions at the matching scale.

4 One-particle Inclusive Semi–leptonic Decays

We shall first try to understand the data on the decays $B \to \bar{D}X\ell^+\nu$. In order to do this we need to have some idea about the matrix elements η_i and ρ_i $(i = S, P, V, A)$ which are defined in (20) and (21). We shall work to leading order in the $1/M$ expansion and hence identify $m_c = m_D = m_{D^*}$ and $m_b = m_B$.

We are aiming at the energy spectrum of the $\bar{D}$ meson in the one-particle inclusive decays of the type $B \to \bar{D}X\ell^+\nu$. The rate is obtained by integrating over the phase space of the $\bar{D}$. Taking into account renormalization one gets

$$\begin{aligned}
\frac{d\Gamma}{dy} &= \frac{1}{2m_B} G(M^2) \frac{m_D^2}{4\pi^2} \sqrt{y^2-1} \qquad (29) \\
&= \frac{G_F^2}{12\pi^3} |V_{cb}|^2 m_D^3 \sqrt{y^2-1} \left[(m_B - m_D)^2 E_S(y) \right. \\
&\qquad \left. + (m_B + m_D)^2 E_P(y) - M^2 (E_V(y) + E_A(y)) \right]
\end{aligned}$$

where $y = v \cdot v'$ and the E_i $(i = S, P, V, A)$ are the renormalization group invariant combinations of the η_i and ρ_i

$$E_i(v \cdot v') = C_{11}(v \cdot v', \mu)\eta_i(v \cdot v', \mu) + C_{18}(v \cdot v', \mu)\rho_i(v \cdot v', \mu) \,. \qquad (30)$$

The Wilson coefficients C_{11} and C_{18} have been given in (28).

To get some expression for the functions η_i and ρ_i $(i = S, P, V, A)$ we first observe that at $v \cdot v' = 1$ the inclusive rate is saturated by the exclusive decays into the lowest-lying spin symmetry doublet $\bar{D}$ and $\bar{D}^*$. Furthermore, at this point only the η_i contribute, since $C_{18}(v \cdot v' = 1) = 0$. The $\bar{D}^*$ subsequently decays into $\bar{D}$ mesons and thus at $v \cdot v' = 1$ the sum of the exclusive rates for $B \to \bar{D}\ell^+\nu$ and $B \to \bar{D}^*\ell^+\nu$ is equal to the one-particle inclusive semi–leptonic rate $B \to \bar{D}\ell^+\nu X$, which is again equal to the fully inclusive rate $B \to X_{\bar{c}}\ell^+\nu$. In other words, at this point there are no decays into other charmed hadrons than $\bar{D}$ mesons.

Off this point things become more complicated. However, as far as the total rates are concerned, still the exclusive decays $B \to \bar{D}\ell^+\nu$ and $B \to \bar{D}^*\ell^+\nu$ saturate the fully inclusive rate $B \to X_{\bar{c}}\ell^+\nu$ at a level of about 70%. Since the $\bar{D}^*$ decay all into $\bar{D}$ mesons, it is certainly a good starting point to approximate the η_i by something one obtains from the sum of the exclusive decays. In other words, we shall express the η_i in terms of the Isgur-Wise function [2].

The approximation we are going to use corresponds to some kind of factorization assumption formulated for $G(M^2)$. The functions η_i are defined by the matrix elements (20) and we shall approximate these matrix elements. However, as with the usual factorization, our approximation is not a scale invariant concept, and hence we have to define, at which scale it should hold.

At a small hadronic scale Λ we replace in (20)

$$\begin{aligned}
&\sum_X \langle B(v)|[\bar{c}_{v'}\Gamma_i b_v]|\bar{D}(v')X\rangle\langle\bar{D}(v')X|[\bar{b}_v\Gamma_i c_{v'}]|B(v)\rangle|_{\mu=\Lambda} \longrightarrow \qquad (31)\\
&\langle B(v)|\bar{c}_{v'}\Gamma_i b_v|\bar{D}(v')\rangle|_{\mu=\Lambda}\langle\bar{D}(v')|\bar{b}_v\Gamma_i c_{v'}|B(v)\rangle|_{\mu=\Lambda}\\
&+\sum_{Y(\bar{D}^*)}\langle B(v)|\bar{c}_{v'}\Gamma_i b_v|\bar{D}(v')Y(\bar{D}^*)\rangle|_{\mu=\Lambda}\langle\bar{D}(v')Y(\bar{D}^*)|\bar{b}_v\Gamma_i c_{v'}|B(v)\rangle|_{\mu=\Lambda}
\end{aligned}$$

where $Y(\bar{D}^*)$ is defined by $\bar{D}^* \to \bar{D}Y(\bar{D}^*)$, i.e. $Y(\bar{D}^*)$ is either a pion or a photon originating from a $\bar{D}^*$ decay. In the following we shall call this replacement factorization, since it is closely related to the factorization assumption known from non-leptonic decays. We get, again schematically

$$\begin{aligned}
&\sum_X \langle B(v)|[\bar{c}_{v'}\Gamma_i b_v]|\bar{D}(v')X\rangle\langle\bar{D}(v')X|[\bar{b}_v\Gamma_i c_{v'}]|B(v)\rangle|_{\mu=\Lambda} \longrightarrow \qquad (32)\\
&\langle B(v)|\bar{c}_{v'}\Gamma_i b_v|\bar{D}(v')\rangle|_{\mu=\Lambda}\langle\bar{D}(v')|\bar{b}_v\Gamma_i c_{v'}|B(v)\rangle|_{\mu=\Lambda}\\
&+\sum_{Pol}\langle B(v)|\bar{c}_{v'}\Gamma_i b_v|\bar{D}^*(v',\epsilon)\rangle|_{\mu=\Lambda}\langle\bar{D}^*(v',\epsilon)|\bar{b}_v\Gamma_i c_{v'}|B(v)\rangle|_{\mu=\Lambda}\\
&\qquad\cdot\mathrm{Br}(\bar{D}^* \to \bar{D}Y(\bar{D}^*))
\end{aligned}$$

where the sum runs over the polarization states of the D^*. In (32) we have used the narrow width approximation for the $\bar{D}^*$ in the intermediate state.

The matrix elements appearing in the factorized expression (32) can all be expressed in terms of the Isgur–Wise function:

$$\begin{aligned}
\langle B(v)|\bar{c}_{v'}\Gamma_i b_v|\bar{D}(v')\rangle|_\mu &= \frac{1}{4}\sqrt{m_B m_D}\,\mathrm{Tr}\{\gamma_5(1+\not{v})\Gamma_i(1+\not{v}')\gamma_5\}\xi(v\cdot v',\mu)\\
\langle B(v)|\bar{c}_{v'}\Gamma_i b_v|\bar{D}^*(v',\epsilon)\rangle|_\mu &= \frac{1}{4}\sqrt{m_B m_{D^*}}\,\mathrm{Tr}\{\gamma_5(1+\not{v})\Gamma_i(1+\not{v}')\not{\epsilon}\}\xi(v\cdot v',\mu)
\end{aligned} \qquad (33)$$

From this we get

$$\eta_i(v\cdot v',\mu) = \frac{|X(v\cdot v')|^2}{C_3^2(v\cdot v',\mu)}\left[c_i(v\cdot v') + c_i^*(v\cdot v')\mathrm{Br}(\bar{D}^* \to \bar{D}X(\bar{D}^*))\right] \qquad (34)$$

where

$$\begin{aligned}
c_i(v\cdot v') &= \frac{1}{16}|\mathrm{Tr}\{\gamma_5(1+\not{v})\Gamma_i(1+\not{v}')\gamma_5\}|^2\\
c_i^*(v\cdot v') &= \frac{1}{16}\sum_{Pol}|\mathrm{Tr}\{\gamma_5(1+\not{v})\Gamma_i(1+\not{v}')\not{\epsilon}\}|^2
\end{aligned}$$

and $X(v \cdot v')$ is the renormalization group invariant combination

$$X(v \cdot v') = C_3(v \cdot v', \mu)\xi(v \cdot v', \mu)\,. \tag{35}$$

The Wilson coefficient $C_3(v \cdot v', \mu)$ renormalizing the $b_v \to c_{v'}$ current is known to two loops, but since we computed C_{11} and C_{18} only to one loop, it is sufficient to insert the one loop result

$$C_3(v \cdot v', \mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\bar{m})} \right)^{\frac{1}{2\beta_0}\gamma_{hh}(v \cdot v')} \tag{36}$$

where

$$\gamma_{hh}(v \cdot v') = \frac{1}{2}(N_c - \frac{1}{N_c})\left(1 - v \cdot v' \, r(v \cdot v')\right). \tag{37}$$

The factorization assumption yields expressions for the matrix elements η_i at the small scale Λ, but it does not tell us anything about the color octett contributions ρ_i. It is well known that factorization should hold in the limit $N_c \to \infty$. This fact is indeed reflected in the N_c–dependence of the Wilson coefficients, since

$$\begin{aligned} &\lim_{N_c \to \infty} C_{18} = \lim_{N_c \to \infty} C_{81} = 0 \\ &\lim_{N_c \to \infty} C_{88} = 1 \\ &\lim_{N_c \to \infty} C_{11} = |C_3|^2 \end{aligned} \tag{38}$$

and thus the dimension 6–operators renormalize as products of dimension 3–currents and factorization becomes scale independent. This does still not tell us much about the ρ_i, but a natural assumption is that they are of the order $1/N_C$ and hence we shall take ρ_i to be constant with $\rho_i(v \cdot v', \mu) = 1/N_C$. This simple ansatz, ignoring a possible dependence on $v \cdot v'$, does not introduce large uncertainties for the one-particle inclusive semi–leptonic decays, since the ρ_i are only induced through radiative corrections.

In the following we shall consider the decays of the B^+ and the B^0, both of which contain a $\bar{b}$ quark undergoing a semi–leptonic transition $\bar{b} \to \bar{c}\ell^+\nu$. In the heavy mass limit for the c quark the final states involving a c quark (D^0 or D^+ states) are suppressed, since this would involve a $c\bar{c}$ pair creation. To leading order in $1/m_c$ the possible decays are thus

$$\begin{aligned} B^+ &\to \overline{D}^0 \ell^+ \nu X \qquad & B^+ &\to D^- \ell^+ \nu X \\ B^0 &\to \overline{D}^0 \ell^+ \nu X \qquad & B^0 &\to D^- \ell^+ \nu X\,. \end{aligned} \tag{39}$$

Since many of the $\bar{D}$ mesons originate from $\bar{D}^*$ decays we have to take into account the relevant branching ratios of the $\bar{D}^*$ mesons into the $\bar{D}$ mesons of different charge. We use [11]

$$\begin{array}{ll} \mathrm{Br}(D^{*-} \to \overline{D}^0 X) \approx 68\% & \mathrm{Br}(D^{*-} \to D^- X) \approx 32\% \\ \mathrm{Br}(\bar{D}^{*0} \to \bar{D}^0 X) \approx 100\% & \end{array} \quad (40)$$

and hence we can have the following decay chains

$$\begin{array}{ccc} B^+ \to \overline{D}^{*0} \ell^+ \nu & \stackrel{\mathrm{Br}(\bar{D}^{*0} \to \bar{D}^0 X)}{\longrightarrow} & B^+ \to \overline{D}^0 \ell^+ \nu X \\ B^0 \to D^{*-} \ell^+ \nu & \stackrel{\mathrm{Br}(D^{*-} \to D^- X)}{\longrightarrow} & B^0 \to D^- \ell^+ \nu X \\ B^0 \to D^{*-} \ell^+ \nu & \stackrel{\mathrm{Br}(D^{*-} \to \overline{D}^0 X)}{\longrightarrow} & B^0 \to \overline{D}^0 \ell^+ \nu X \, , \end{array} \quad (41)$$

where the arrow indicates that – in addition to the direct decay channel $B \to \bar{D} \ell^+ \nu$ – the exclusive mode on the l.h.s. contributes to the one-particle inclusive rate on the r.h.s. weighted with the branching ratios (40).

We shall label the E_i for the different decay modes (39) with a superscript indicating the initial B and the final $\bar{D}$ meson. Taking into account the $\bar{D}^*$ branching ratios (40) we arrive at the following expressions for the E_i involved in the B^+ decays:

$$\begin{aligned} E_S^{B^+\overline{D}^0}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} \frac{1}{4}(y+1)^2 |X(y)|^2 + C_{18}(y,\Lambda) \frac{1}{N_C} \\ E_P^{B^+\overline{D}^0}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} \frac{1}{4}(y^2-1) |X(y)|^2 + C_{18}(y,\Lambda) \frac{1}{N_C} \\ E_V^{B^+\overline{D}^0}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} \frac{1}{2} y(y+1) |X(y)|^2 + C_{18}(y,\Lambda) \frac{1}{N_C} \\ E_A^{B^+\overline{D}^0}(y) &= -\frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} \frac{1}{2}(y+2)(y+1) |X(y)|^2 + C_{18}(y,\Lambda) \frac{1}{N_C} \end{aligned} \quad (42)$$

In the B^0 decays we have to take into account the D^{*-} branching ratios as

$$
\begin{aligned}
E_S^{B^0\overline{D}^0}(y) &= C_{18}(y,\Lambda)\frac{1}{N_C} \\
E_P^{B^0\overline{D}^0}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} Br(D^{*-}\to \overline{D}^0 X)\frac{1}{4}(y^2-1)|X(y)|^2 + C_{18}(y,\Lambda)\frac{1}{N_C} \\
E_V^{B^0\overline{D}^0}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} Br(D^{*-}\to \overline{D}^0 X)\frac{1}{2}(y^2-1)|X(y)|^2 + C_{18}(y,\Lambda)\frac{1}{N_C} \\
E_A^{B^0\overline{D}^0}(y) &= -\frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} Br(D^{*-}\to \overline{D}^0 X)\frac{1}{2}(y+2)(y+1)|X(y)|^2 \\
&\qquad +C_{18}(y,\Lambda)\frac{1}{N_C} \qquad (43)
\end{aligned}
$$

and

$$
\begin{aligned}
E_S^{B^0 D^-}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)}\frac{1}{4}(y+1)^2|X(y)|^2 + C_{18}(y,\Lambda)\frac{1}{N_C} \qquad (44) \\
E_P^{B^0 D^-}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} Br(D^{*-}\to D^- X)\frac{1}{4}(y^2-1)|X(y)|^2 + C_{18}(y,\Lambda)\frac{1}{N_C} \\
E_V^{B^0 D^-}(y) &= \frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} \left(\frac{1}{2}(y+1) + Br(D^{*-}\to D^- X)\frac{1}{2}(y^2-1)\right)|X(y)|^2 \\
&\qquad +C_{18}(y,\Lambda)\frac{1}{N_C} \\
E_A^{B^0 D^-}(y) &= -\frac{C_{11}(y,\Lambda)}{C_3^2(y,\Lambda)} Br(D^{*-}\to D^- X)\frac{1}{2}(y+2)(y+1)|X(y)|^2 \\
&\qquad +C_{18}(y,\Lambda)\frac{1}{N_C}\,.
\end{aligned}
$$

Note that at $v\cdot v' = 1$ we have simply the sum of the exclusive channels $B\to \bar{D}\ell^+\nu$ and $B\to \bar{D}^*\ell^+\nu$, where the $\bar{D}^*$ component is weighted with the appropriate $\bar{D}^*$ branching ratios, since here $C_{11} = C_3 = 1$ and $C_{18} = 0$. Off the point $v\cdot v' = 1$ we still have $C_{11}/(C_3)^2 \approx 1$ but there is also an additional contribution from the octett contributions ρ_i. As we shall see, this additional contributions are consistent with the data, despite of our crude estimate.

Inserting these lenghty expressions into the master formula (29) one obtains expressions for the one-particle inclusive energy spectra of the $\bar{D}$ mesons. To leading order in $1/m_c$ the energy of the $\bar{D}^*$ meson is equal to the energy of the $\bar{D}$ meson originating from the decay $\bar{D}^*\to \bar{D}X$ since X is soft of the order $1/m_c$.

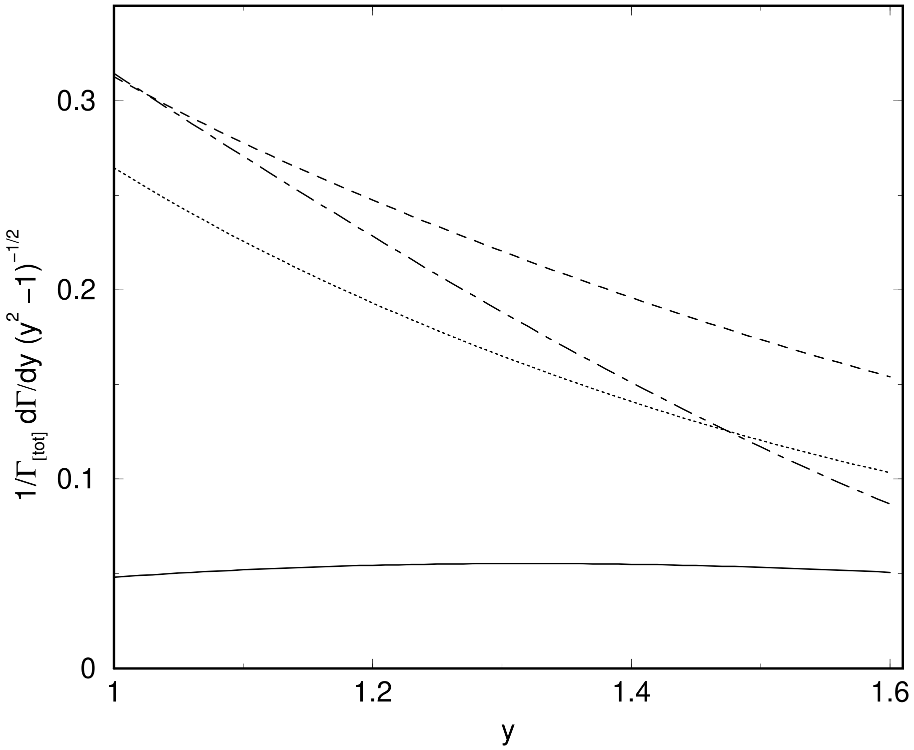

Figure 3: Decay spectra of the one-particle inclusive decays. Solid line: $B \to D^- \ell^+ \nu X$, dotted line: $B \to \overline{D}^0 \ell^+ \nu X$, dashed line: $B \to (D^- + \overline{D}^0)\ell^+ \nu X$, dashed-dotted line: $B^0 \to (D^- + D^{*-})\ell^+ \nu$.

In order to actually obtain numbers one needs the Isgur Wise function as an input. A good fit to the experimental data is obtained already with a linear function, which is fitted to the renormalization group invariant $X(y)$

$$X(y) = 1 - a(y-1) \quad \text{with} \quad a = 0.84 \quad [12]\,. \tag{45}$$

In figure 3 we plot the spectra of the D meson for the combined rates

$$\begin{aligned}
\frac{d\Gamma}{dy}(B \to D^- \ell^+ \nu X) &= \frac{1}{2}\left(\frac{d\Gamma}{dy}(B^+ \to D^- \ell^+ \nu X) + \frac{d\Gamma}{dy}(B^0 \to D^- \ell^+ \nu X)\right) \\
\frac{d\Gamma}{dy}(B \to \overline{D}^0 \ell^+ \nu X) &= \frac{1}{2}\left(\frac{d\Gamma}{dy}(B^+ \to \overline{D}^0 \ell^+ \nu X) + \frac{d\Gamma}{dy}(B^0 \to \overline{D}^0 \ell^+ \nu X)\right)
\end{aligned}$$

Mode	Br (theory)	Br (data from [11])
$B \to D^- \ell^+ \nu X$	2.3%	$(2.7 \pm 0.8)\%$
$B \to \overline{D}^0 \ell^+ \nu X$	6.9%	$(7.0 \pm 1.4)\%$
$B^0 \to D^- \ell^+ \nu$		$(1.5 \pm 0.5)\%$
$B^0 \to D^{-*} \ell^+ \nu$		$(4.68 \pm 0.25)\%$
$B \to \bar{D}^{**} \ell \nu$	$(3.5 \pm 0.6)\%$	$(2.7 \pm 0.7)\%$
$B \to$ non-$\bar{D} \ell^+ \nu$	$(0.6 \pm 0.4)\%$	

Table 1: Comparison of our results with data. To get branching ratios, we used $\tau_{B^+} = \tau_{B^0} = 1.55$ ps. Here $\bar{D}^{**}$ denotes any final state with a $\bar{D}$ meson which does not come from the exclusive decays listed in row three and four. The last two rows are commented in the text.

and compare it to the sum of the exclusive decays $B^0 \to D^- \ell^+ \nu$ and $B^0 \to D^{*-} \ell^+ \nu$.

One may also integrate the spectra to obtain a total rate for the one-particle inclusive semi–leptonic processes. In table 1 we compare the rates we obtain from our approach with the experimental data from [11].

Table 1 and also figure 3 exhibit a few interesting features. First of all the experimental data are well reproduced. Furthermore, although we have used the assumption (32) our result is not simply the sum of the inclusive decays $B \to \bar{D} \ell^+ \nu$ and $B \to \bar{D}^* \ell^+ \nu$, since (32) is a scale dependent statement. We assume that (32) holds at the small scale Λ; running up to the matching scale $\overline{m}$ yields a significant contribution from gluon exchanges. We interpret these contributions as $B \to \bar{D}^{**} \ell^+ \nu$ where $\bar{D}^{**}$ now stands for all $\bar{D}$-meson final states, which do not originate from $B \to \bar{D} \ell^+ \nu$ or $B \to \bar{D}^* \ell^+ \nu$. Although the ansatz for the octett matrix elements ρ_i is extremely simple, we obtain a reasonable number, namely $\mathrm{Br}(B \to \bar{D}^{**} \ell^+ \nu) \approx 32\% \times \mathrm{Br}(B \to X_{\bar{c}} \ell^+ \nu)$ where we use $Br(B \to X_{\bar{c}} \ell^+ \nu) = (10.4 \pm 0.4)\%$ from [11]. From figure 3 it is obvious that the $\bar{D}^{**}$–contribution vanishes at $v \cdot v' = 1$ as required by the heavy quark limit.

The last row of table 1 gives the branching ratio for decays which do not have a $\bar{D}$ meson in the final state, rather some other charmed hadron. The only other ground state hadron is a $\bar{\Lambda}_c$ so this should be the branching ratio for $B \to \bar{\Lambda}_c X \ell^+ \nu$ for which we obtain $\mathrm{Br}(B \to \bar{\Lambda}_c X \ell^+ \nu) = 6\% \times \mathrm{Br}(B \to$

$X_{\bar{c}}\ell^+\nu$). This is what one would expect on the basis of the naive reasoning that a heavy quark hadronizes into a baryon with a branching ratio of about ten percent.

5 Conclusions

Exclusive semi–leptonic as well as fully inclusive decays of heavy hadrons have a well established basis in QCD. While in the former case it is the heavy mass limit of QCD, formulated as an effective theory (HQET), in the latter case it is the heavy mass limit combined with parton-hadron duality, formulated as an operator-product expansion.

On the other side there are the exclusive non–leptonic decays, where no theoretically solid basis for a calculation of branching ratios exists. However, these decays are of prime interest with respect to CP violation and the determination of the CKM matrix. In this field the heavy mass limit has not brought any significant progress.

In this work we have set up a QCD based description for one-particle inclusive decays. The basic ingredients are the heavy mass limit and a short distance expansion. We obtain operators similar to the ones describing heavy quarkonia production or one particle inclusive processes. We have formulated this method for decays of the type $B \to \bar{D}X$ where X in principle can be any state.

We have applied this method to one-particle inclusive semi–leptonic decays of the form $B \to \bar{D}X\ell^+\nu$, studying the leading order in the operator-product expansion. Higher order terms are suppressed by inverse powers of a large scale related to the heavy quark masses. To leading order, all these decays are parametrized in terms of eight functions η_i and ρ_i which depend on the velocities of the B and the $\bar{D}$ meson.

The main problem is to obtain these non-perturbative functions η_i and ρ_i and we employed the fact that the inclusive semi–leptonic decays are dominated by the two channels $B \to \bar{D}\ell^+\nu$ and $B \to \bar{D}^*\ell^+\nu$. Using this as a starting point we may obtain four of the unknown functions (the η_i) in terms of the Isgur-Wise function with a well motivated factorization–ansatz. The remaining four (the ρ_i) are suppressed by powers of α_s as well as by factors $1/N_C$.

Estimating the four functions ρ_i to be of the order $1/N_C$ we are able to describe the features of the one-particle inclusive semi–leptonic decays.

In particular, QCD radiative corrections induce a relatively large amount of decays which originate not from the exclusive modes $B \to \bar{D}\ell^+\nu$ and $B \to \bar{D}^*\ell^+\nu$. This is in accordance with the experimental data giving us some confidence in our method.

The approach suggested in the present paper opens the door to a QCD based description of one-particle inclusive processes; it is not limited to semi–leptonic decays. In particular, the non-perturbative functions η_i and ρ_i are universal and should also describe other one-particle inclusive processes.

Acknowledgements

We are grateful to Xavier Calmet for producing the plot and for performing the numerical analysis. This work was supported by the "Graduiertenkolleg: Elementarteilchenphysik and Beschleunigern" and the "Forschergruppe: Quantenfeldtheorie, Computeralgebra und Monte Carlo Simulationen" of the Deutsche Forschungsgemeinschaft.

References

[1] A subjective selection of recent reviews is:
N. Isgur and M. Wise: contribution to *Heavy Flavors*, A. Buras and M. Lindner (eds.) (World Scientific, Singapore, 1992);
M. Neubert, Phys. Rept. **245** (1994) 259;
I. Bigi, M. Shifman and N. Uraltsev, TPI-MINN-97-02-T, (1997), hep-ph/9703290;
T. Mannel, J. Phys. **G 21** (1995) 1007; Rep. Prog. Phys. **60** (1997) 1113.

[2] N. Isgur and M. Wise, Phys. Lett. **B232** (1989) 113 and **B237** (1990) 527; M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. **45** (1987) 292 and **47** (1988) 511.

[3] B. Grinstein, Nucl. Phys. **B339** (1990) 253;
H. Georgi, Phys. Lett. **B240** (1990) 447;
A. Falk, H. Georgi, B. Grinstein and M. Wise, Nucl. Phys. **B343** (1990) 1.

[4] I. Bigi, N. Uraltsev and A. Vainshtein, Phys. Lett. **B293** (1992) 430;
I. Bigi et al., Minnesota TPI-MINN-92/67-T (1992) and Phys. Rev. Lett. **71** (1993) 496;
J. Chay, H. Georgi and B. Grinstein, Phys. Lett. **B247** (1990) 399;
A. Manohar and M. Wise, Phys. Rev. **D49** (1994) 1310;
T. Mannel, Nucl. Phys. **B423** (1994) 396.

[5] M. Neubert, Talk given at *2nd International Conference on B Physics and CP Violation*, Honolulu, HI, 24-28 Mar 1997, e-Print Archive: hep-ph/9707217.

[6] G. Bodwin, E. Braaten and P. Lepage Phys. Rev. **D51** (1995) 1125; Erratum-ibid. **D55** (1997) 5853.

[7] For a review see e.g.: A. Mueller, Phys. Rept. **73** (1981) 237.

[8] B. Grinstein et al., Nucl. Phys. **B363** (1991) 19.

[9] W. Kilian, T. Mannel and T. Ohl, Phys. Lett. **B304** (1993) 311.

[10] A. Falk, M. Luke and M. Wise, Phys. Lett. **B299** (1993) 123.

[11] C. Caso et al, (Particle Data Group), The European Physical Journal **C3** (1998) 1.

[12] B. Barish et al, (CLEO Collaboration), Phys. Rev. **D51** (1995) 1014.