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Neutrino mixing and seesaw mechanism

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Abstract

Models of neutrino masses are discussed capable of explaining in a natural way the maximal mixing between ν_μ and ν_τ observed by the Super-Kamiokande collaboration. For three generations of leptons two classes of such models are found implying:
a) ${\Delta m_{23}}^2 \ll {\Delta m_{12}}^2 \approx {\Delta m_{13}}^2$ and a small mixing between ν_e and the other two neutrinos,
b) ${\Delta m_{12}}^2 \ll {\Delta m_{13}}^2 \approx {\Delta m_{23}}^2$ and a nearly maximal mixing for solar neutrino oscillations in vacuum.

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The recent results from Super-Kamiokande [1] indicate a large mixing of ν_μ and ν_τ. In fact the observed mixing angle is nearly maximal

$$\sin^2 2\theta_{\mu\tau} \approx 1 \tag{1}$$

In this note we study a problem if this remarkably large mixing can be explained in a natural way. We assume that the masses of neutrinos are much smaller than the masses of quarks and charged leptons due to the seesaw mechanism [2], see also an excellent review [3]. For three generations of neutrinos the mass matrix is of the form

$$\mathbf{M_6} = \begin{pmatrix} 0 & \mathbf{m_D}^{\mathrm{T}} \\ \mathbf{m_D} & \mathbf{M_R} \end{pmatrix}, \tag{2}$$

where 3×3 matrices $\mathbf{m_D}$ and $\mathbf{M_R}$ describe Dirac and Majorana masses, respectively. For the sake of simplicity we neglect CP violating phases and assume that $\mathbf{m_D}$ and $\mathbf{M_R}$ are real. We investigate a question if eq. (1) can be obtained without imposing large cancellations and correlations between parameters in $\mathbf{M_R}$ and $\mathbf{m_D}$.

After diagonalization of the mass matrix (2) the particle spectrum consists of three light and nearly left-handed neutrinos and three nearly right-handed neutrinos whose masses are of order of a huge mass scale M. It is assumed that non-zero elements of $\mathbf{M_R}$ are of order of M and hence much larger than the matrix elements of $\mathbf{m_D}$. The masses of the light neutrinos ν_L can be calculated by considering Dirac masses as small perturbations. In the second order of perturbation theory the following 3×3 mass matrix is obtained

$$\mathbf{N} = \mathbf{m_D}^{\mathrm{T}} \mathbf{M_R}^{-1} \mathbf{m_D}. \tag{3}$$

The masses of ν_Ls are equal to absolute values of its eigenvalues. Inversion of the matrix $\mathbf{M_R}$ is possible because the condition

$$\det \mathbf{M_R} \neq 0 \tag{4}$$

is assumed, which guarantees that all right-handed neutrinos are heavy.

One might argue that eq.(1) cannot impose any limitation on models because for an arbitrary non–singular matrix $\mathbf{N}$, given the form of Dirac masses $\mathbf{m_D}$, the mass matrix for Majorana masses is equal to

$$\mathbf{M_R} = \mathbf{m_D} \mathbf{N}^{-1} \mathbf{m_D}^{\mathrm{T}}. \tag{5}$$

Thus, one can choose $\mathbf{M_R}$ such that an arbitrary mass and mixing pattern is obtained. However, in general eq.(5) implies large correlations between low mass parameters in $\mathbf{m_D}$ and large mass parameters in $\mathbf{M_R}$. Such correlations, if not removed by a freedom in defining $\mathbf{m_D}$, are unnatural because $\mathbf{m_D}$ and $\mathbf{M_R}$ originate from apparently disconnected mechanism, e.g. from local gauge symmetry breakings for electroweak $SU_2 \times U_1$ and some grand unification group G. Then another rather difficult problem arises if such correlations are stable against radiative corrections. For these reasons we consider only those models which imply eq.(1) without fine tuning between the parameters at low and large mass scales. Let us choose the basis in which the matrix $\mathbf{m_D}$ is diagonal

$$\mathbf{m_D} = \mathrm{diag}(m_1, m_2, m_3) = m_3\, \mathrm{diag}(x^2 y, x, 1). \tag{6}$$

The condition which we impose means that for acceptable models a system of coordinates in generation space exists such that the Majorana mass matrix $\mathbf{M_R}$ does not depend on the parameters x and y. The sector of Majorana masses depends on some other set of parameters $\{\alpha, \beta, \ldots\}$. We do not preclude any mass hierarchy at the high mass scale so some of these parameters may be small. We consider that those models are unnatural whose essential features, like the patterns of their mass spectra, depend in a crucial way on relations between the parameters describing the low and the high mass sectors. Thus, we do not discuss models which assume relations like $\alpha/x \approx 1$ or any other relation implying strong correlations between the parameters in $\mathbf{m_D}$ and $\mathbf{M_R}$.

Assuming mass relations typical for SO_{10} grand unified theories, $\mathbf{m_D}$ should follow the mass pattern of up type quarks whereas $\mathbf{m}_\ell$, the mass matrix for the charged leptons should resemble the corresponding matrix for down type quarks. Moreover, since the Cabibbo–Kobayashi–Maskawa mixing for the second and third generation of quarks is quite small, in the present basis $\mathbf{m}_\ell$ is 'nearly' diagonal. The large mixing (1) between ν_μ and ν_τ originates from the form of the neutrino mass matrix $\mathbf{N}$ because the Dirac mass matrices for all leptons and quarks are nearly diagonal. In the following discussion we take into account that the mass eigenstates of charged leptons can be imperfectly aligned with our coordinates in the generation space. Namely the mass matrix for the charged leptons may have small off-diagonal elements, whose ratios to the diagonal elements would be comparable to the mixing angles in the CKM matrix for quarks. We write

$$\left(\mathbf{M_R}^{-1}\right)_{ij} = \frac{1}{M}\,\mathbf{a}_{ij} \tag{7}$$

where the mass scale M is chosen in such a way that

$$\max\left(\,|a_{ij}|\,\right) = 1. \tag{8}$$

It follows that

$$\mathbf{N} = \frac{{m_3}^2}{M}\begin{pmatrix} x^4y^2a_{11} & x^3ya_{12} & x^2ya_{13} \\ x^3ya_{12} & x^2a_{22} & xa_{23} \\ x^2ya_{13} & xa_{23} & a_{33} \end{pmatrix}. \tag{9}$$

A natural order of magnitude for the parameters x and y is

$$x = O\left(m_c/m_t\right) \sim 10^{-2}, \qquad y = O\left(m_u m_t/m_c^2\right) \sim 10^{-1}. \tag{10}$$

Therefore if $a_{33} = O(1)$ no large mixing is possible between the third and the first two generations. In such a case the mass μ_3 of the heaviest mass eigenstate ν_3 is much larger than μ_2 and μ_1 corresponding to the other two mass eigenstates ν_2 and ν_1. The maximal mixing (1) implies that

$$a_{33} = 0 \tag{11}$$

or at least a_{33} has to be strongly suppressed. If the condition (11) is fulfilled and

$$a_{23} = O(1) \neq 0 \tag{12}$$

the large mixing (1) is obtained. The masses μ_3 and μ_2 are both of the same order of magnitude

$$\mu_3 \approx \mu_2 \approx \frac{{m_3}^2}{M}\,x\,\left|a_{23}\right|. \tag{13}$$

The mass splitting between ν_3 and ν_2 depends on the values of matrix elements a_{22} and a_{13} and is typically of order

$$\frac{{m_2}^2}{M}\,O\left(a_{22}, ya_{13}\right)$$

i.e. suppressed by one power of x with respect to μ_3 and μ_2. The mass of the eigenstate ν_1 is typically much smaller than μ_2 and the mixing angles $\theta_{e\mu}$ and $\theta_{e\tau}$ are also small. As an example of this class of models let us consider $a_{12} = a_{13} = 0$, $|a_{22}/a_{23}| = \alpha$ and $|a_{11}/a_{23}| = \beta$. A large mixing (1) is obtained if $\alpha \ll 1/x$. Then the following mass spectrum of the light neutrinos is derived:

$$\begin{aligned} \mu_3 &= \mu\left(1 + \tfrac{1}{2}\alpha x + \ldots\right) \\ \mu_2 &= \mu\left(1 - \tfrac{1}{2}\alpha x + \ldots\right) \\ \mu_1 &= \mu\, \beta x^3 y^2 \end{aligned} \tag{14}$$

with[1]

$$\mu = m_2 m_3 \left|a_{23}\right| / M. \tag{15}$$

The condition for large mixing (1) does not imply that β must be small. Thus one might argue that contrary to our claims $\mu_1 \approx \mu_2$ if $\beta x^3 y^2 \approx 1$. However in such a case a very strong correlation exists between the parameter β in the Majorana mass sector and the mass ratios x and y in the Dirac mass sector. It is exactly this sort of fine tuning which we consider unnatural and hence unacceptable. It is evident that for $\beta x^3 y^2 \ll 1$

$$\frac{\Delta m_{23}^2}{\Delta m_{12}^2} \approx \frac{\Delta m_{23}^2}{\Delta m_{13}^2} = O(x) \tag{16}$$

where

$$\Delta m_{ij}^2 = \left|\, {\mu_i}^2 - {\mu_j}^2 \,\right|. \tag{17}$$

The mass scale μ can be estimated assuming

$$\Delta m_{23}^2 = 2 \cdot 10^{-3}\, eV^2 \tag{18}$$

as obtained by the Super-Kamiokande collaboration [1] and $x \approx 1/100$

$$\mu \approx \sqrt{\frac{\Delta m_{23}^2}{2\alpha x}} \approx \frac{0.3}{\sqrt{\alpha}}\, eV. \tag{19}$$

The parameter α in (19) may be small. In such a case ν_μ and ν_τ are pushed even more towards the maximal mixing. For small α the mass μ may become larger than 1 eV. It is possible in this class of models that ν_μ and ν_τ contribute a non-negligible contribution to the dark matter in the Universe. It is also quite natural to expect neutrino oscillations between ν_e and ν_μ as well as between ν_e and ν_τ characterized by small mixing angles and, at a fixed energy, by much shorter oscillation lengths than for $\nu_\mu \to \nu_\tau$, c.f. eq.(16). In the system of coordinates which we use the mixing angles for quarks are ascribed to weak isospin $I_3 = -\frac{1}{2}$ states. The Dirac mass matrix for the down type quarks, and hence also for the charged leptons is not diagonal. For the mixing between the first and second generation $\theta_{e\mu} \sim \theta_c$ seems quite natural, where θ_c denotes Cabibbo angle.

It is interesting that a small mixing angle $\theta_{e\mu}$ as well as Δm_{12}^2 given by

$$\Delta m_{12}^2 \approx \Delta m_{13}^2 \approx \mu^2 \tag{20}$$

[1] A model for two nearly degenerate neutrino mass eigenstates similar to (14) for $\alpha \approx 2$ has been recently discussed in [4].

are well within the range of parameters allowed by $\nu_\mu \to \nu_e$ oscillations reported by LSND collaboration [5]. A real drawback of these models is, however, that they evidently cannot explain deficits of solar neutrinos by a mixing between ν_e, ν_μ and ν_τ only.

Let us consider now another class of models corresponding to the following choice of the matrix elements in (7):

$$a_{22} = 1, \qquad a_{23} = x, \qquad\qquad a_{33} = x^2. \tag{21}$$

This choice apparently violates the condition that Majorana mass matrix $\mathbf{M_R}$ should not depend on parameters in Dirac mass matrix $\mathbf{m_D}$. However the x-dependence of the elements a_{ij} in eq.(21) can be absorbed into an orthogonal matrix describing a rotation by a small angle x in the $\nu_\mu - \nu_\tau$ plane

$$\mathbf{M_R}^{-1}(x) = \mathbf{O}^{\mathrm{T}}(x)\,\mathbf{M_R}^{-1}(x=0)\,\mathbf{O}(x) \tag{22}$$

where

$$\mathbf{O}(x) \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & x \\ 0 & -x & 1 \end{pmatrix}. \tag{23}$$

Then all x-dependence can be ascribed to a new Dirac mass matrix

$$\mathbf{m'_D} = \mathbf{O}(x)\mathbf{m_D} = m_3 \begin{pmatrix} x^2 y & 0 & 0 \\ 0 & x & x \\ 0 & -x^2 & 1 \end{pmatrix}. \tag{24}$$

When rewritten in this way our model is free from dangerous correlations between the parameters in Dirac and Majorana mass matrices. It does not mean that no small parameters are present. In fact x and y have to be small in a realistic model. Thus we allow also for small parameters, independent of x and y, in the Majorana mass matrix $\mathbf{M_R}^{-1}(x=0)$.
As a specific example we consider

$$\mathbf{M_R}^{-1}(x=0) = \frac{1}{M}\begin{pmatrix} 0 & 0 & \alpha \\ 0 & 1 & 0 \\ \alpha & 0 & 0 \end{pmatrix}. \tag{25}$$

and obtain the characteristic equation for the eigenvalues of $\mathbf{N}M/{m_2}^2$. When small terms of order x^2 are neglected this equation reads:

$$\lambda^3 - 2\lambda^2 - r^2\lambda + r^2 = 0, \tag{26}$$

where

$$r = \alpha y. \tag{27}$$

In the limit of small r the eigenvalues are given by

$$\begin{aligned} \lambda_1 &= \tfrac{1}{\sqrt{2}}r - \tfrac{1}{8}r^2 + \ldots, \\ \lambda_2 &= -\tfrac{1}{\sqrt{2}}r - \tfrac{1}{8}r^2 + \ldots, \\ \lambda_3 &= 2 + \tfrac{1}{4}r^2 + \ldots, \end{aligned} \tag{28}$$

and the corresponding eigenvectors are proportional to

$$v_1 \sim \begin{pmatrix} \sqrt{2}+r/4 \\ -1-r/\sqrt{2} \\ 1 \end{pmatrix} + \ldots, \quad v_2 \sim \begin{pmatrix} -\sqrt{2}+r/4 \\ -1-r/\sqrt{2} \\ 1 \end{pmatrix} + \ldots, \quad v_3 \sim \begin{pmatrix} r/2 \\ 1 \\ 1 \end{pmatrix} + \ldots \tag{29}$$

The mass spectrum of the nearly left-handed neutrinos ν_L is

$$\begin{aligned} \mu_1 &= \mu\, r\left(\tfrac{1}{\sqrt{2}} - \tfrac{1}{8} r + \ldots\right), \\ \mu_2 &= \mu\, r\left(\tfrac{1}{\sqrt{2}} + \tfrac{1}{8} r + \ldots\right), \\ \mu_3 &= \mu\left(2 + \tfrac{1}{4} r^2 + \ldots\right) \end{aligned} \tag{30}$$

with

$$\mu = {m_2}^2/M. \tag{31}$$

Thus two almost degenerate mass eigenstates ν_1 and ν_2 corresponding to the eigenvalues μ_1 and μ_2 are lighter than the third eigenstate ν_3 whose mass is greater by a factor $2\sqrt{2}/r$.

In the leading order of the small parameter r the eigenstates of the weak charged current are expressed in terms of the mass eigenstates in the following way

$$\begin{aligned} |\,\nu_e\,\rangle &= \tfrac{1}{\sqrt{2}}\left[\,|\,\nu_1\,\rangle - |\,\nu_2\,\rangle\,\right] + \ldots, \\ |\,\nu_\mu\,\rangle &= \tfrac{1}{\sqrt{2}}\Big[\,|\,\nu_3\,\rangle - \tfrac{1}{\sqrt{2}}\left(\,|\,\nu_1\,\rangle + |\,\nu_2\,\rangle\,\right)\Big] + \ldots, \\ |\,\nu_\tau\,\rangle &= \tfrac{1}{\sqrt{2}}\Big[\,|\,\nu_3\,\rangle + \tfrac{1}{\sqrt{2}}\left(\,|\,\nu_1\,\rangle + |\,\nu_2\,\rangle\,\right)\Big] + \ldots, \end{aligned} \tag{32}$$

where $\ldots$ denote terms of order r and smaller. The oscillations of ν_μ observed by the Super-Kamiokande collaboration can be understood as an oscillation of the states between $|\,\nu_\mu\,\rangle$ and $|\,\nu_\tau\,\rangle$. Since the eigenmasses μ_1 and μ_2 are degenerate to a high degree there will be no relative phase change between $|\,\nu_1\,\rangle$ and $|\,\nu_2\,\rangle$ while traveling the distance of the Earth size because $\Delta m^2_{12}/E \ll 1/R_{Earth}$. Meanwhile, the splitting between μ_3 and $\mu_1 \approx \mu_2$ allows for a significant phase difference between $|\,\nu_3\,\rangle$ and $[\,|\,\nu_1\,\rangle + |\,\nu_2\,\rangle\,]/\sqrt{2}$ after time $t \sim R_{Earth}$ of neutrino propagation through the Earth. This implies a significant oscillation between ν_μ and ν_τ at the scale of R_{Earth} if $\Delta m^2_{23}/E \sim 1/R_{Earth}$. At such a scale effectively the maximal mixing is observed. As for solar neutrinos the oscillations can be understood as a result of time evolution of the state vector in a two-dimensional vector space spanned by $|\,\nu_1\,\rangle$ and $|\,\nu_2\,\rangle$. As a result of being nearly orthogonal to $|\,\nu_e\,\rangle$ the mass eigenstate $|\,\nu_3\,\rangle$ remains nearly orthogonal to the state vector which during its time evolution oscillates between $|\,\nu_e\,\rangle$ and $[\,|\,\nu_\mu\,\rangle - |\,\nu_\tau\,\rangle\,]/\sqrt{2}$. It is remarkable that a maximal mixing for oscillations of electron neutrinos is predicted by the model, exactly as needed for explaining the solar neutrino problem by vacuum oscillations. Numerical values of μ and r can be derived from the Super-Kamiokande result (18) and the value of Δm^2_{12} obtained from quantitative analyses of solar neutrino oscillations in vacuum [3]. It follows from the mass spectrum (30) that

$$\Delta m^2_{13} \approx \Delta m^2_{23} \approx 4\mu^2 \approx 2 \cdot 10^{-3}\ eV^2, \tag{33}$$

$$\Delta m^2_{12} \approx \frac{\mu^2 r^3}{2\sqrt{2}} \sim 10^{-10}\ eV^2. \tag{34}$$

Taking central values one obtains $r = 0.008$ and $\mu = 0.02$ eV. Assuming that m_2 is equal to the mass of the charm quark the scale of the Majorana mass sector M is between 10^{10} and 10^{11} GeV depending on the scale at which m_c is evaluated.

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