TTP98–25
hep-ph/9806524
June 1998

Higher order corrections to Z–decay*

Robert Harlander

Institut für Theoretische Teilchenphysik, Universität Karlsruhe,
D-76128 Karlsruhe, Germany

Abstract

Recent developments in the field of automatic computation of Feynman diagrams using asymptotic expansions are reviewed. The hadronic decay rate of the Z–boson is taken as an example for their physical application.

1 Introduction

The final round of the analysis of data taken at LEP in the runs between 1990 and 1995 at energies around the Z–peak is going to be completed [1]. About 16 millions of Z–bosons have been produced and the resulting accuracy of, e.g., the Z–boson mass and its decay width is impressive (for the latest experimental values of these quantities see [1]). In view of this precision there have been huge efforts in calculating higher order corrections to these observables. This talk is supposed to be not so much a review of theoretical and experimental results but is rather concerned with some of the available tools to perform such calculations.

2 QCD corrections and asymptotic expansions

The decay rate of the Z–boson into quarks constitutes an illuminating example of how to use asymptotic expansions of Feynman diagrams to simplify calculations. In what follows we will only be concerned with the vertex corrections and it will always be assumed that they are computed via the optical theorem by calculating the Z–boson self energy and taking the imaginary part. If one considers, e.g., only QCD-corrections, it certainly is a reasonable lowest order approximation to neglect all quark masses. Only one dimensional quantity is left, the Z–boson mass, and one arrives at massless propagator diagrams for which the so-called *integration-by-parts* algorithm is available and has been explicitely worked out [2] and implemented in a `FORM` [3] package called `MINCER` [4] up to three loops. This immediately gives the answer for the Z–boson decay rate into massless quarks up to $\mathcal{O}(\alpha_s^2)$ [5]. Making additionally use of infrared re-arrangement, it is even possible to extend the result to $\mathcal{O}(\alpha_s^3)$ [6, 7].

As a second step one may want to take effects induced by the quark masses into acccount. Then, however, one is faced with two-scale Feynman integrals. Although the full $\mathcal{O}(\alpha_s)$–result is

*Presented at the Zeuthen Workshop on Elementary Particle Theory “Loops and Legs in Gauge Theories”, Rheinsberg, Germany, April 19–24, 1998.

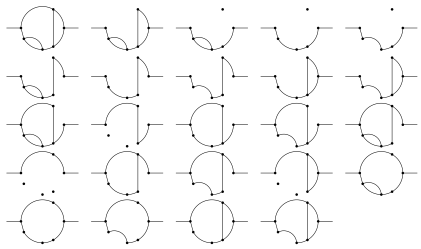


Figure 1: Subdiagrams contributing to the Large Momentum Procedure of a particular diagram.

known [8, 9, 10] — the vector part even for a very long time —, at $\mathcal{O}(\alpha_s^2)$ only certain subclasses of diagrams have been computed analytically (e.g. [11]).

For the remaining part one is forced to consider some approximation procedure to obtain, e.g., an expansion in m_q^2/M_Z^2, with m_q the mass of the quark in the final state. The recipe for the efficient computation of such asymptotic expansions has been worked out in a series of publications and shall not be repeated here (for a review see [12]). The essence is to expand the integrands of certain subgraphs of the initial diagrams, leading to a complete factorization of "small" and "large" quantities. Two special cases may be distinguished: When only masses appear as large quantities, the technique is called the *Hard Mass Procedure*. In contrast, the *Large Momentum Procedure* deals with the case of only large momenta.

Concerning the quark mass effects in the QCD-corrections to the Z–boson decay rate, it is clear that here the appropriate procedure is the second one. There is only one large ($q^2 = M_Z^2$) and one small (m_q^2) mass scale in this problem, so the factorization mentioned above means that products of only single scale diagrams are produced: massless propagator diagrams ($m = 0$ and $q \neq 0$) and massive tadpoles ($m \neq 0$ and $q = 0$). We already mentioned the integration-by-parts algorithm to compute the former ones. The underlying principle may also be applied to the latter ones, as has been done in [13], again up to three loops. The implementation of this procedure has been performed, for example, in a `FORM` package named `MATAD` [14].

There are, of course, certain limitations of this approach both from the technical and the analytical point of view. The former one is connected with the realization of the prescriptions provided by the asymptotic expansions. To three-loop order it becomes a non-trivial task to find and properly expand the contributing subdiagrams. For one particular diagram, all subgraphs contributing to the Large Momentum Procedure are shown in Fig. 1. The solution of this problem is to use the algorithmic nature of the prescriptions and to pass their evaluation to a computer. As far as the Large Momentum Procedure for two-point functions is concerned this has been performed in a `PERL` program called `LMP` [15]. Another program, doing also the Hard Mass Procedure, will be mentioned in Section 4. Therefore, in this sense this technical limitation no longer exists.

The second limitation of asymptotic expansions is more severe. It is clear that a cut series in general contains less information than the full result. In our case of the Z–boson decay, let us, for example, consider the hypothetical case $2m_q < M_Z < 4m_q$. The production of four fermions is then kinematically forbidden. A small m_q–expansion, however, is uncapable of discriminating among any of the cases $M_Z > m_q$, $M_Z > 2m_q$ or $M_Z > 4m_q$, which is why at first sight one cannot expect to obtain reasonable results below the four-quark threshold. However, the

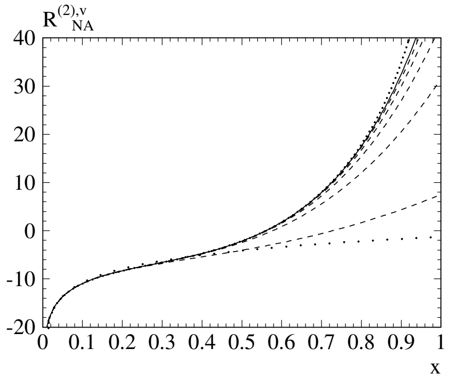


Figure 2: Non-abelian contribution to $R_{\rm had}$ as an example for the application of the Large Momentum Procedure [16], plotted over $x = 2m/\sqrt{s}$. Narrow dots: semi-analytical result from [17], other lines: power corrections up to $(m^2/s)^n$, $n = 0, \ldots, 6$.

situation is not so bad as it seems. The reason is that in general the four-particle channel has a very smooth threshold behaviour due to phase space suppression. In a small-m_q expansion this smoothness is carried over to energies below the four-particle threshold preserving its validity in this region [16].

As an example the contribution to the hadronic R–ratio proportional to the color factor $C_{\rm A}C_{\rm F}$ as a function of $x = 2m_q/\sqrt{s}$, where s is the cms-energy, is shown in Fig. 2. The first threshold is at $\sqrt{s} = 2m_q$ ($x = 1$), the second one at $\sqrt{s} = 4m_q$ ($x = 1/2$). Convergence, however, seems to be warranted at least up to $x = 0.8$.

Anyway, for the Z–boson decay one is above any possible four-quark threshold such that convergence of the expansion to the correct answer is guaranteed. Not only this: The quadratic and quartic mass corrections, which already provide a very good approximation in this case, can also be obtained without really using asymptotic expansions, even up to $\mathcal{O}(\alpha_s^3)$ [18, 15]. As far as the Large Momentum Procedure is concerned, there certainly are more important fields of application for it (see, e.g., [16, 19, 20]). On the other hand, for the Hard Mass Procedure, important contributions to Z–decay have recently been computed where it really proves to be a very useful tool, as we will see in Section 4.

3 Electroweak and mixed QED/QCD-corrections

Let us turn to the electroweak radiative corrections to the Z–boson decay. As far as one only considers QED, the similarity to QCD allows not only to evaluate all pure QED-corrections, but also QED/QCD-corrections of order $\alpha\alpha_s$, $\alpha^2\alpha_s$ and $\alpha\alpha_s^2$ [21] from the knowledge of the diagram-wise results to $\mathcal{O}(\alpha_s^2)$ resp. $\mathcal{O}(\alpha_s^3)$ by altering the color factors.

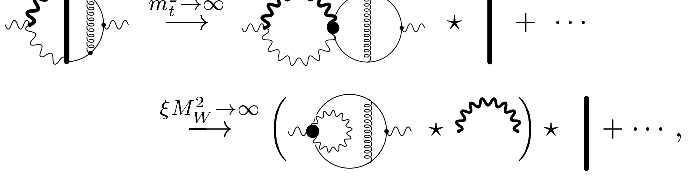
The situation is different when allowing also for Z– and W–boson exchange between the produced quarks, because of their non-negligible masses. As far as the W–boson is concerned, another phenomenon occurs, namely the appearence of the isospin partners of the produced quarks as virtual particles in the loops. In contrast to the decay into u–, d–, s– or c–quarks, where this does not really produce a difference in comparison to virtual Z–exchange because all of them may be considered as massless, the decay into b–quarks is somewhat exceptional because the b–quark is the isospin partner of the t–quark. Neglecting the t–quark mass certainly is not a good approximation, nor can it be set infinite as one knows, for example, from the radiative corrections to the ρ–parameter that appear to be proportional to m_t^2 [22]. Nevertheless, a full result for $\Gamma(Z \to q\bar{q})$ to $\mathcal{O}(\alpha)$ for the decay into $q = u, d, s, c$ [23, 24] as well as for the one into b [25, 24] is available. The leading m_t–behavior for the latter is quadratic like for the ρ–parameter. To $\mathcal{O}(\alpha^2)$ the t–quark enters also the calculation for the decay into u, d, s, c. The leading m_t^4–and the subleading m_t^2–terms to this order are known [26, 27]. For the decay into b–quarks only the leading m_t^4–terms are available [26].

4 Mixed electroweak/QCD-corrections

A part of the mixed $\alpha\alpha_s$–corrections, namely those induced by virtual gluon and photon exchange, has already been mentioned in Section 3. The present section will be concerned with the case of W– or Z–, accompanied by an additional gluon-exchange, i.e., in a sense, QCD-corrections to the $\mathcal{O}(\alpha)$–results described in Section 3. Again it is natural to distinguish between the decay into u, d, s, c and into b. The former case was evaluated in [28], and it is instructive to dwell a bit on the technique which was used in this work.

Again the rate was determined by computing the Z–boson self energy up to the order considered and taking the imaginary part of the result. In the case of virtual Z–boson exchange, one arrives at three-loop on-shell integrals, for W–exchange one gets two-scale diagrams. The idea in [28] was to use the Hard Mass Procedure described in Section 2 to expand the diagrams in M_Z^2/M^2, where M is the mass of the virtual gauge boson, and take the limit $M \to M_Z$ resp. $M \to M_W$ in the final result. Since convergence at these points turned out to be quite slow, a part of the diagrams was also expanded in M^2/M_Z^2. In this way it was possible to obtain a reasonable approximation to the full result.

In the case of $Z \to b\bar{b}$ one faces the problem of an additional mass scale, the t-quark mass. Using the Hard Mass Procedure for $m_t^2 \gg M_Z^2, M_W^2$, one may factor out the m_t–dependence. However, for a part of the diagrams one still is left with two-scale and even three-scale integrals involving M_Z^2 and M_W^2 and $\xi_W M_W^2$, where ξ_W is the electroweak gauge parameter which we want to keep. Although they appear to be only one-loop integrals, their exact evaluation up to $\mathcal{O}(\epsilon)$ produces inconvenient results. Instead, the results of [29] were obtained by applying the Hard Mass Procedure to these kinds of diagrams once more, this time using $\xi_W M_W^2, M_W^2 \gg M_Z^2$. This seemingly unrealistic choice of scales becomes justified by recalling the discussion of Section 2: It is not possible for an expansion to distinguish the inequality $M_W^2 \gg M_Z^2$ from $4M_W^2 \gg M_Z^2$ or $(m_t + M_W)^2 \gg M_Z^2$, the latter ones being perfectly alright. The only matter is to perform the expansion on the appropriate side of all thresholds, and here one is concerned with thresholds at $2M_W$ and at $m_t + M_W$. Therefore, the choice $M_W^2 \gg M_Z^2$ is to be understood purely in this technical sense. Graphically this continued expansion looks as follows:



where only those terms are displayed which are relevant in the discussion above and all others contributing to the Hard Mass Procedure are merged into the ellipse. The thick plain line is the top quark, the thick wavy one a Goldstone boson with mass squared $\xi_W M_W^2$, for example. The thin plain lines are b–quarks, the inner thin wavy lines are W–bosons, the outer ones Z-bosons. The spring-line is a gluon. The mass hierarchy is assumed to be $m_t^2 \gg \xi_W M_W^2 \gg M_W^2 \gg M_Z^2$. The freedom in choosing the magnitude of ξ_W provides a welcome check of the routines and the results.

The outcome of this procedure is a nested series: The coefficients of the M_W/m_t–expansion are in turn series in M_Z/M_W. Note that in contrast to the decay into u, d, s, c there is no threshold at M_W which makes an additional expansion in M_W/M_Z unnecessary.

In view of this calculation the procedure of successive application of the Hard Mass Procedure resp. the Large Momentum Procedure has been implemented in a Fortran 90 program named `EXP` [30]. Therefore, the computation of a three-loop two-point function can now be done fully automatically given some arbitrary hierarchy of mass scales. Even more, the link to the Feynman diagram generator `QGRAF` [31] in a common environment called `GEFICOM` [32] allows to obtain the result of a whole physical process without any human interference except for specification of the process and final renormalization.

Finally, let us present the result for the W–induced corrections to the Z–decay rate $\delta\Gamma^W(Z \to b\bar{b})$ in the form of the renormalization scheme independent difference to the decay rate into $d\bar{d}$. Inserting the on-shell top mass $m_t = 175$ GeV, the Z–mass $M_Z = 91.91$ GeV and $\sin^2\theta_W = 0.223$ gives

$$
\begin{aligned}
\delta\Gamma^W(Z \to b\bar{b}) - \delta\Gamma^W(Z \to d\bar{d}) &= \Gamma^0 \frac{1}{\sin^2\theta_W}\frac{\alpha}{\pi}\bigg\{ -0.50 \\
&+ (0.71 - 0.48) + (0.08 - 0.29) + (-0.01 - 0.07) + (-0.007 - 0.006) \\
&+ \frac{\alpha_s}{\pi}\bigg[1.16 + (1.21 - 0.49) + (0.30 - 0.65) + (0.02 - 0.21 + 0.01) \\
&+ (-0.01 - 0.04 + 0.004)\bigg]\bigg\} = \\
&= \Gamma^0 \frac{1}{\sin^2\theta_W}\frac{\alpha}{\pi}\bigg\{ -0.50 - 0.07 + \frac{\alpha_s}{\pi}\bigg[1.16 + 0.13\bigg]\bigg\}, \qquad (1)
\end{aligned}
$$

where the factor $\Gamma^0\alpha/(\pi\sin^2\theta_W)$ with $\Gamma^0 = M_Z\alpha/(4\sin^2\theta_W\cos^2\theta_W)$ has been pulled out for convenience. The numbers after the first equality sign correspond to successively increasing orders in $1/m_t^2$, where the brackets collect the corresponding constant, $\log m_t$ and, if present, $\log^2 m_t$–terms. The numbers after the second equality sign represent the leading m_t^2–term and the sum of the subleading ones. The $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha\alpha_s)$–results are displayed separately. Comparison of this expansion of the one-loop terms to the exact result of [24] shows agreement up to 0.01% which gives quite some confidence in the $\alpha\alpha_s$–contribution. One can see that although the m_t^2–, m_t^0– and $m_t^0\log m_t$–terms are of the same order of magnitude, the final result is surprisingly well represented by the m_t^2–term, since the subleading terms largely cancel among each other.

Acknowledgements: I would like to thank K.G. Chetyrkin, J.H. Kühn, M. Steinhauser and T. Seidensticker for fruitful collaboration, and the organizers and participants at the workshop for the enjoyable atmosphere. This work was supported by the "Landesgraduiertenförderung" and the "Graduiertenkolleg Elementarteilchenphysik" at the University of Karlsruhe.

References

[1] R. Clare, *LEP Physics Results*, these proceedings.

[2] F.V. Tkachov, *Phys. Lett.* **B 100** (1981) 65;
K.G. Chetyrkin and F.V. Tkachov, *Nucl. Phys.* **B 192** (1981) 159.

[3] J.A.M. Vermaseren, *Symbolic Manipulation with FORM* (Computer Algebra Netherlands, Amsterdam, 1991).

[4] S.A. Larin, F.V. Tkachov and J.A.M. Vermaseren, Rep. No. NIKHEF-H/91-18 (Amsterdam, 1991).

[5] K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, *Phys. Lett.* **B 85** (1979) 277;
M. Dine and J. Sapirstein, *Phys. Rev. Lett.* **43** (1979) 668;
W. Celmaster and R.J. Gonsalves, *Phys. Rev. Lett.* **44** (1980) 560.

[6] S.G. Gorishny, A.L. Kataev and S.A. Larin, *Phys. Lett.* **B 259** (1991) 144;
L.R. Surguladze and M.A. Samuel, *Phys. Rev. Lett.* **66** (1991) 560; (E) ibid., 2416.

[7] K.G. Chetyrkin, *Phys. Lett.* **B 404** (1997) 161.

[8] G. Källen and A. Sabry, *K. Dan. Videnk. Selsk. Mat.-Fys. Medd.* **29** (1955) No. 17.

[9] R. Barbieri and E. Remiddi, *Nuovo Cim.* **13A** (1973) 99.

[10] J. Jersák, E. Laermann and P. Zerwas, *Phys. Rev.* **D 25** (1982) 1218; (E) ibid. **D 36** (1987) 310.

[11] A.H. Hoang, M. Jeżabek, J.H. Kühn and T. Teubner, *Phys. Lett.* **B 338** (1994) 330;
A.H. Hoang, J.H. Kühn and T. Teubner, *Nucl. Phys.* **B 452** (1995) 173;
A.H. Hoang and T. Teubner, Rep. Nos. DTP/97/68, UCSD/PHT 97-16 (Durham, San Diego, 1997), hep-ph/9707496.

[12] V.A. Smirnov, *Renormalization and Asymptotic Expansion* (Birkhäuser, Basel, 1991); *Mod. Phys. Lett.* **A 10** (1995) 1485.

[13] D.J. Broadhurst, *Z. Phys.* **C 54** (1992) 54.

[14] M. Steinhauser, Dissertation (Shaker Verlag, Aachen, 1996).

[15] R. Harlander, Dissertation at the University of Karlsruhe, 1998, to be published.

[16] K.G. Chetyrkin, R. Harlander, J.H. Kühn and M. Steinhauser, *Nucl. Phys.* **B 503** (1997) 339.

[17] K.G. Chetyrkin, J.H. Kühn and M. Steinhauser, *Phys. Lett.* **B 371** (1996) 93; *Nucl. Phys.* **B 482** (1996) 213.

[18] K.G. Chetyrkin and J.H. Kühn, *Nucl. Phys.* **B 432** (1994) 337; *Phys. Lett.* **B 248** (1990) 359; *Phys. Lett.* **B 406** (1997) 102.

[19] R. Harlander and M. Steinhauser, *Phys. Rev.* **D 56** (1997) 3980.

[20] R. Harlander and M. Steinhauser, *Eur. Phys. J.* **C 2** (1998) 151.

[21] A.L. Kataev, *Phys. Lett.* **B 287** (1992) 209;
L.R. Surguladze, Rep. No. UAHEP-969 (Alabama, 1998), hep-ph/9803211.

[22] D.A. Ross and M. Veltman, *Nucl. Phys.* **B 95** (1975) 135;
M. Veltman, *Nucl. Phys.* **B 123** (1977) 89.

[23] B. Grzadkowski, J.H. Kühn, P. Krawczyk and R.G. Stuart, *Nucl. Phys.* **B 281** (1987) 18.

[24] W. Beenakker and W. Hollik, *Z. Phys.* **C 40** (1988) 141.

[25] A. Akhundov, D. Bardin and T. Riemann, *Nucl. Phys.* **B 276** (1986) 1;
J. Bernabeu, A. Pich and A. Santamaria, *Phys. Lett.* **B 200** (1988) 569;
B.W. Lynn and R.G. Stuart, *Phys. Lett.* **B 252** (1990) 676.

[26] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci and A. Vicere, *Phys. Lett.* **B 288** (1992) 95; (E) ibid. **B 312** (511) ; *Nucl. Phys.* **B 409** (1993) 105;
J. Fleischer, O.V. Tarasov and F. Jegerlehner, *Phys. Lett.* **B 319** (1993) 249.

[27] G. Degrassi, S. Fanchiotti and A. Sirlin, *Nucl. Phys.* **B 351** (1991) 49;
G. Degrassi and A. Sirlin, *Nucl. Phys.* **B 352** (1991) 342;
G. Degrassi, P. Gambino and A. Sirlin, *Phys. Lett.* **B 394** (1997) 188;
G. Degrassi, P. Gambino and A. Vicini, *Phys. Lett.* **B 383** (1996) 219.

[28] A. Czarnecki and J.H. Kühn, *Phys. Rev. Lett.* **77** (1996) 3955; hep-ph/9608366(v2).

[29] R. Harlander, T. Seidensticker and M. Steinhauser, *Phys. Lett.* **B 426** (1998) 125.

[30] T. Seidensticker, Diploma Thesis (Karlsruhe, 1998), unpublished.

[31] P. Nogueira, *J. Comp. Phys.* **105** (1993) 279.

[32] K.G. Chetyrkin and M. Steinhauser, unpublished.