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Spectator Effects in Heavy Quark Effective Theory at $\mathcal{O}(1/m_Q^3)$

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Abstract

We complete the one loop renormalization of the HQET lagrangian at $\mathcal{O}(1/m_Q^3)$ including four fermion operators with two heavy and two light quark fields in the operator basis. It is shown that as a consequence the short distance coefficients of the operators bilinear in the heavy quark field receive nontrivial corrections.

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1 Introduction

Heavy Quark Effective Field Theory (HQET) [1] has become a well established theoretical tool for the description of hadrons containing one heavy quark [2]. This derives from the fact that it is a systematic expansion in inverse powers of the heavy quark mass m_Q with well defined and calculable coefficients. Furthermore, its realization of the spin and flavor symmetry of the low energy theory is a phenomenologically powerful tool. The $1/m_Q$ expansion has already been applied sucessfully to phenomenological problems such as the determination of V_{cb}.

Besides its phenomenological application HQET posseses interesting theoretical features which already show up if one studies the HQET lagrangian itself. On one hand it is relevant for power corrections to the $1/m_Q$ expansion of hadron masses and on the other hand allows for a study of reparametrization invariance [10].

However, the complete construction of HQET has to include radiative corrections. These are computable order by order in perturbation theory by matching HQET to full QCD at a perturbative scale $\mu = m_Q$ followed by the renormalization group running of the respective operators. As indespensable ingredients both the coefficients at the matching scale and the anomalous dimensions of the operators are needed.

In this paper we concentrate on the one loop renormalization of the lagrangian at $\mathcal{O}(1/m_Q^3)$. Results for the terms of $\mathcal{O}(1/m_Q)$ and $\mathcal{O}(1/m_Q^2)$ as well as a matching calculation for the terms of $\mathcal{O}(1/m_Q^3)$ are already known [3, 4, 5, 6, 7, 8, 9] .

In [13] the one loop anomalous dimensions of the effective lagrangian at $\mathcal{O}(1/m_Q^3)$ have been calculated. However, these results were incomplete in the sense that only operators bilinear in the heavy quark field had been included in the operator basis. In the present paper we complete the renormalization of the effective lagrangian extending the operator basis to four fermion operators composed of two heavy and two light fields. At $\mathcal{O}(1/m_Q^2)$ it is well known that the running of the short–distance coefficient of the Darwin operator is modified in the presence of such operators. At higher orders this effect should be expected to cause more drastical consequences because of the large operator bases. In fact, it will be shown that at $\mathcal{O}(1/m_Q^3)$ the coefficients of the bilinear operators receive nontrivial corrections in the presence of heavy–light operators.

A possible phenomenological application of our results is the estimate of $\mathcal{O}(1/m_Q^3)$–corrections in the operator product expansion of the B–meson decay width. It is well known that to this order the width is significantly affected by spectator quark effects induced by heavy–light operators appearing in the OPE. However, there are also time ordered products of the effective lagrangian with the local operators in the OPE. At this stage heavy–light operators in the lagrangian and their short distance corrections which are the subject of this paper induce additional spectator effects.

This note is organized as follows: in section 2 we introduce our operator basis and discuss its reduction to a set of linearly independent operators in section 3.

Our result for the anomalous dimensions is presented in section 4. Finally, we present the logarithmic contributions to the short distance coefficients of the effective lagrangian in section 5 and conclusions in section 6.

2 Operator basis

The lagrangian of HQET can be written as

$$\mathcal{L} = \bar{h}_v(ivD)h_v + \sum_{n=1}^{\infty} \frac{1}{(2m_Q)^n}\mathcal{L}^{(n)} + \mathcal{L}_{\text{light}} \,, \tag{1}$$

where

$$\mathcal{L}_{\text{light}} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{q}(i\not{D} - m_q)q \tag{2}$$

describes the dynamics of the light degrees of freedom. For simplicity we consider only one light quark with mass m_q. Additional light flavor are taken care of by summing over the index q.

To lowest order in the heavy mass expansion only one operator, $\bar{h}_v(ivD)h_v$, shows up which has the celebrated spin– and flavor–symmety properties.

However there are power corrections of $\mathcal{O}(1/m_Q)$ to the heavy–quark limit which break these symmetries. They are given by a sum of operators of appropriate canonical dimension multiplied by short distance coefficients:

$$\mathcal{L}^{(n)} = \sum_i C_i^{(n)}(\mu)\mathcal{O}_i^{(n)}(\mu) \tag{3}$$

At given order $\mathcal{O}(1/m_Q^n)$ all operators of the respective canonical dimension allowed by the symmetries of the effective theory may contribute.

First of all there are the operators bilinear in the heavy quark fields already appearing in the tree level lagrangian (denoted H–operators).

Operators containing two heavy and an even number of additional light quark fields appear if one matches QCD amplitudes with heavy and light external fields onto HQET to one loop. Their inclusion accounts for spectator quark effects inside hadronic states, between which these operators are sandwiched when calculating physical matrix elements. However, since their massdimension must be larger or equal to 6 these operators show up at $\mathcal{O}(1/m_Q^2)$ for the first time. To this order they can be constructed by two heavy and two light quark fields. At $\mathcal{O}(1/m_Q^3)$ a covariant derivative or a factor m_q increases the mass dimension. In what follows these operators are referred to as HL–operators.

Since we remain in the one particle sector of HQET operators with four or more heavy quark fields are ommited.

To complete the operator basis we should also consider operators constructed of four light quark fields and the corresponding penguin operator. However, to one loop order these operators only mix with the HL–operators. Since we are focusing on the corrections to the H–operators we therefore neglect such operators.

Before we proceed to $\mathcal{O}(1/m_Q^3)$ let us recall the operators appearing at lower orders.

At $\mathcal{O}(1/m_Q)$ we choose the conventional basis

$$\begin{aligned} \mathcal{O}_1^{(1)} &= \bar{h}_v (iD)^2 h_v \qquad \mathcal{O}_2^{(1)} = \frac{g}{2} \bar{h}_v \sigma^{\mu\lambda} F_{\mu\lambda} h_v \\ \mathcal{O}_3^{(1)} &= \bar{h}_v (ivD)^2 h_v \, , \end{aligned} \tag{4}$$

and at $\mathcal{O}(1/m_Q^2)$

$$\begin{aligned} \mathcal{O}_1^{(2)} &= \bar{h}_v iD_\mu (ivD) iD^\mu h_v & \mathcal{O}_2^{(2)} &= \bar{h}_v i\sigma^{\mu\lambda} iD_\mu (ivD) iD_\lambda h_v \\ \mathcal{O}_3^{(2)} &= \bar{h}_v (ivD)(iD)^2 h_v & \mathcal{O}_4^{(2)} &= \bar{h}_v (iD)^2 (ivD) h_v \\ \mathcal{O}_5^{(2)} &= \bar{h}_v (ivD)^3 h_v & \mathcal{O}_6^{(2)} &= \bar{h}_v (ivD) i\sigma^{\mu\lambda} iD_\mu iD_\lambda h_v \\ \mathcal{O}_7^{(2)} &= \bar{h}_v i\sigma^{\mu\lambda} iD_\mu iD_\lambda (ivD) h_v \, . \end{aligned} \tag{5}$$

The definition of the covariant derivative $iD = i\partial + g_s T^a A^a$ and field strength tensor $F_{\mu\nu}^a T^a = -i/g_s [iD_\mu, iD_\nu]$ follows usual conventions.

We choose the HL–operators at $\mathcal{O}(1/m_Q^2)$ as

$$\begin{aligned} \mathcal{M}_1^{(2)s/o} &= g_s^2 [\bar{q} \mathcal{C}_{s/o}^a q][\bar{h}_v \mathcal{C}_{s/o}^a h_v] \\ \mathcal{M}_2^{(2)s/o} &= g_s^2 [\bar{q} \not{v} \mathcal{C}_{s/o}^a q][\bar{h}_v \mathcal{C}_{s/o}^a h_v] \\ \mathcal{M}_3^{(2)s/o} &= g_s^2 [\bar{q} \gamma_5 \gamma^\mu \mathcal{C}_{s/o}^a q][\bar{h}_v \gamma_5 \gamma_\mu \mathcal{C}_{s/o}^a h_v] \\ \mathcal{M}_4^{(2)s/o} &= g_s^2 [\bar{q} i\sigma^{\mu\nu} \mathcal{C}_{s/o}^a q][\bar{h}_v i\sigma_{\mu\nu} \mathcal{C}_{s/o}^a h_v] \end{aligned} \tag{6}$$

where $\mathcal{C}_s^a = 1$ in the color singlett and $\mathcal{C}_o^a = T^a$ in the color octett case.

At $\mathcal{O}(1/m_Q^3)$ we find 13 local H–operators contributing to physical matrix-elements:

$$\begin{aligned} \mathcal{O}_1^{(3)} &= \bar{h} iD_\mu (ivD)^2 iD^\mu h & \mathcal{O}_2^{(3)} &= \bar{h} (iD)^2 (iD)^2 h \\ \mathcal{O}_3^{(3)} &= \bar{h} iD_\mu (iD)^2 iD^\mu h & \mathcal{O}_4^{(3)} &= \bar{h} iD_\mu iD_\nu iD^\mu iD^\nu h \\ \mathcal{O}_5^{(3)} &= \bar{h} i\sigma^{\mu\nu} iD_\mu (ivD)^2 iD_\nu h & \mathcal{O}_6^{(3)} &= \bar{h} i\sigma^{\mu\nu} iD_\mu iD_\nu (iD)^2 h \\ \mathcal{O}_7^{(3)} &= \bar{h} i\sigma^{\mu\nu} iD_\rho iD_\mu iD_\nu iD^\rho h & \mathcal{O}_8^{(3)} &= \bar{h} i\sigma^{\mu\nu} (iD)^2 iD_\mu iD_\nu h \\ \mathcal{O}_9^{(3)} &= \bar{h} i\sigma^{\mu\nu} iD_\mu (iD)^2 iD_\nu h & \mathcal{O}_{10}^{(3)} &= \bar{h} i\sigma^{\mu\nu} iD_\mu iD_\rho iD_\nu iD^\rho h \\ \mathcal{O}_{11}^{(3)} &= \bar{h} i\sigma^{\mu\nu} iD_\rho iD_\mu iD^\rho iD_\nu h & \mathcal{O}_{12}^{(3)} &= g_s^2 \bar{h} F^{a\mu\nu} F_{\mu\nu}^a h \\ \mathcal{O}_{13}^{(3)} &= g_s^2 \bar{h} v_\nu v^\rho F^{a\mu\nu} F_{\mu\rho}^a h \end{aligned} \tag{7}$$

Note that, in contrast to the case of the lower order operators, we omit operators vanishing by the heavy quark equation of motion (EOM)

$$(ivD) h_v = 0 \tag{8}$$

from the very beginning. This is justified since we are not going to insert such operators in time ordered products of order $\mathcal{O}(1/m_Q^4)$ or higher. We may not remove such operators from the operatorbases at $\mathcal{O}(1/m_Q)$ and $\mathcal{O}(1/m_Q^2)$ since time ordered products of these operators at $\mathcal{O}(1/m_Q^3)$ may be identical to local physical operators and contribute to their anomalous dimensions (see section 3.2). However, the following operators are needed as parts of the gluon EOM and though being irrelevant we will keep them in the basis:

$$
\begin{aligned}
\mathcal{O}_{14}^{(3)} &= \bar{h}_v (ivD)^2 (iD)^2 h_v & \qquad \mathcal{O}_{15}^{(3)} &= \bar{h}_v (ivD)(iD)^2(ivD) h_v \\
\mathcal{O}_{16}^{(3)} &= \bar{h}_v (iD)^2 (ivD)^2 h_v & \qquad \mathcal{O}_{17}^{(3)} &= \bar{h}_v iD_\mu (ivD) iD^\mu (ivD) h_v \\
\mathcal{O}_{18}^{(3)} &= \bar{h}_v (ivD) iD_\mu (ivD) iD^\mu h_v
\end{aligned}
$$

To construct a basis of HL–operators at $\mathcal{O}(1/m_Q^3)$ we let a covariant derivative act either on a light or a heavy quark field. A further classification has to be taken according to the direction in which the covariant derivative acts. We define

$$
iD_\mu^+ = i\overrightarrow{\partial}_\mu + g_s A_\mu^a T^a \qquad iD_\mu^- = i\overleftarrow{\partial}_\mu - g_s A_\mu^a T^a \tag{9}
$$

and it is understood that in the octett case the covariant derivative stands left/right of the color matrix when acting to the left/right. With these definitions the HL–operators of mass dimension 7 are choosen as:

$$
\begin{aligned}
\mathcal{M}_{1\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q}\mathcal{C}_{s/o}^a q][\bar{h}_v \mathcal{C}_{s/o}^a (ivD^\pm) h_v] \\
\mathcal{M}_{2\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q}\not{v}\mathcal{C}_{s/o}^a q][\bar{h}_v \mathcal{C}_{s/o}^a (ivD^\pm) h_v] \\
\mathcal{M}_{3\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q}\gamma_\mu \mathcal{C}_{s/o}^a q][\bar{h}_v i\sigma^{\mu\nu} \mathcal{C}_{s/o}^a iD_\nu^\pm h_v] \\
\mathcal{M}_{4\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q}\gamma^\mu \mathcal{C}_{s/o}^a q][\bar{h}_v \mathcal{C}_{s/o}^a iD_\mu^\pm h_v] \\
\mathcal{M}_{5\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q} i\sigma_{\mu\lambda} v^\lambda \mathcal{C}_{s/o}^a q][\bar{h}_v i\sigma^{\mu\nu} \mathcal{C}_{s/o}^a iD_\nu^\pm h_v] \\
\mathcal{M}_{6\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q} i\sigma^{\mu\lambda} v_\lambda \mathcal{C}_{s/o}^a q][\bar{h}_v \mathcal{C}_{s/o}^a iD_\mu^\pm h_v] \\
\mathcal{M}_{7\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q}\gamma_5 \not{v} \mathcal{C}_{s/o}^a q][\bar{h}_v \gamma_5 \mathcal{C}_{s/o}^a i\not{D}^\pm h_v] \\
\mathcal{M}_{8\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q}\gamma_5 \gamma^\mu \mathcal{C}_{s/o}^a q][\bar{h}_v \gamma_5 \gamma_\mu \mathcal{C}_{s/o}^a (ivD^\pm) h_v] \\
\mathcal{M}_{9\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q}\gamma_5 \mathcal{C}_{s/o}^a q][\bar{h}_v \gamma_5 \mathcal{C}_{s/o}^a i\not{D}^\pm h_v] \\
\mathcal{M}_{10\pm}^{(3h)s/o} &= \pm g_s^2 [\bar{q} i\sigma^{\mu\nu} \mathcal{C}_{s/o}^a q][\bar{h}_v i\sigma_{\mu\nu} \mathcal{C}_{s/o}^a (ivD^\pm) h_v] \\
\mathcal{M}_{1\pm}^{(3l)s/o} &= \pm g_s^2 [\bar{q}\mathcal{C}_{s/o}^a i\not{D}^\pm q][\bar{h}_v \mathcal{C}_{s/o}^a h_v] \\
\mathcal{M}_{2\pm}^{(3l)s/o} &= \pm g_s^2 [\bar{q}\mathcal{C}_{s/o}^a (ivD^\pm) q][\bar{h}_v \mathcal{C}_{s/o}^a h_v] \\
\mathcal{M}_{3\pm}^{(3l)s/o} &= \pm g_s^2 [\bar{q}\not{v}\mathcal{C}_{s/o}^a (ivD^\pm) q][\bar{h}_v \mathcal{C}_{s/o}^a h_v] \\
\mathcal{M}_{4\pm}^{(3l)s/o} &= \pm g_s^2 [\bar{q} i\sigma^{\lambda\nu} v_\lambda \mathcal{C}_{s/o}^a iD_\nu^\pm q][\bar{h}_v \mathcal{C}_{s/o}^a h_v] \\
\mathcal{M}_{5\pm}^{(3l)s/o} &= \pm g_s^2 [\bar{q} i\sigma^{\mu\nu} \mathcal{C}_{s/o}^a (ivD^\pm) q][\bar{h}_v i\sigma_{\mu\nu} \mathcal{C}_{s/o}^a h_v]
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
\mathcal{M}^{(3l)s/o}_{6\pm} &= \pm g_s^2[\bar{q}\gamma_5 \not{v}\mathcal{C}^a_{s/o} iD^\pm_\mu q][\bar{h}_v\gamma_5\gamma^\mu\mathcal{C}^a_{s/o}h_v] \\
\mathcal{M}^{(3l)s/o}_{7\pm} &= \pm g_s^2[\bar{q}\gamma_5\gamma^\mu\mathcal{C}^a_{s/o}(ivD^\pm)q][\bar{h}_v\gamma_5\gamma_\mu\mathcal{C}^a_{s/o}h_v] \\
\mathcal{M}^{(3l)s/o}_{8\pm} &= \pm g_s^2[\bar{q}\gamma_5\mathcal{C}^a_{s/o} iD^\pm_\mu q][\bar{h}_v\gamma_5\gamma^\mu\mathcal{C}^a_{s/o}h_v] \\
\mathcal{M}^{(3l)s/o}_{9\pm} &= \pm g_s^2[\bar{i}\sigma_{\lambda\nu} q\mathcal{C}^a_{s/o} iD^\pm_\mu q][\bar{h}_v i\sigma^{\mu\nu}\mathcal{C}^a_{s/o}h_v] \\
\mathcal{M}^{(3l)s/o}_{10\pm} &= \pm g_s^2[\bar{q}\gamma_\nu\mathcal{C}^a_{s/o} iD^\pm_\mu q][\bar{h}_v i\sigma^{\mu\nu}\mathcal{C}^a_{s/o}h_v]
\end{aligned}
$$

To complete the set of local operators we also consider operators of lower dimension multiplied by an appropriate power of the light quark mass:

$$
m_q\mathcal{O}^{(1)}_i, m_q^2\mathcal{O}^{(1)}_i, m_q\mathcal{O}^{(2)}_i, m_q\mathcal{M}^{(2)s/o}_1 \tag{11}
$$

These operators are needed as counterterms during renormalization as long as the light quark is kept massive. However the mass dependent HL–operator can be removed with help of the light–quark EOM. This is not the case for the mass dependent H–operators which are induced by penguin type diagrams. They cause corrections proportional to powers of m_q/m_Q to the short distance coefficients of the respective operators. Note that the operators $\mathcal{M}^{(3h)s/o}_i$, $i = 1, 2, 8, 10$ vanish by the heavy quark EOM. Instead of disregarding these operators from the very beginnig we will keep them in the basis since they show up in several operator identities.

In addition to the local operators there are the time ordered products of dimension 7 composed of lower dimension operators:

$$
\begin{aligned}
\mathcal{T}^{(12)}_{ij} &= i\,\mathrm{T}\left[\mathcal{O}^{(1)}_i, \mathcal{O}^{(2)}_j\right] && i = 1,2,3 \quad j = 1,\ldots,7 \\
\mathcal{T}^{(111)}_{ijk} &= -\mathcal{S}_{ijk}\,\mathrm{T}\left[\mathcal{O}^{(1)}_i, \mathcal{O}^{(1)}_j, \mathcal{O}^{(1)}_k\right] && i,j,k = 1,2,3 \quad i \le j \le k \\
\mathcal{T}^{(12hl)s/o}_{ij} &= i\,\mathrm{T}\left[\mathcal{O}^{(1)}_i, \mathcal{M}^{(2)s/o}_j\right] && i = 1,2,3 \quad j = 1,\ldots,4
\end{aligned} \tag{12}
$$

The symmetry factor $\mathcal{S}_{ijk}$ equals $1, 1/2$ or $1/6$, if no, two or all inserted operators are identical. Again a mass dependent T–product

$$
m_q(1 - \frac{1}{2}\delta_{ij})i\,\mathrm{T}\left[\mathcal{O}^{(1)}_i, \mathcal{O}^{(1)}_j\right] \tag{13}
$$

contributes. Although the short distance coefficients of time ordered products are the products of the coefficents of their operator components, local operators are required for their renormalization. This in turn modifies the running of the coefficents of the local operators.

3 Reduction of the operator basis

The operator basis presented in the previous section is overcomplete, i.e. there exist several interdependencies between the operators. To arrive at a physical basis of linearely independent operators all redundant operators have to be removed, otherwise artificial gauge dependencies may show up in the corresponding short distance coefficients [13].

3.1 Momentum conservation

Since all operators are integrated over their argument, momentum conservation is manifest.

In case of the H–operators the only effect is that the direction in which the covariant derivative acts is irrelevant.

The partial derivative of a HL–operator of mass dimension 6 vanishes. Performing the differentiation explicitly yields relations among the HL–operators of dimension 7:

$$\begin{aligned}
0 &= \partial_\mu[\bar{q}\Gamma^{\mu\nu}q][\bar{h}_v\Gamma_\nu h_v] \\
&= [\bar{q}iD_\mu^-\Gamma^{\mu\nu}q][\bar{h}_v\Gamma_\nu h_v] + [\bar{q}\Gamma^{\mu\nu}iD_\mu^+q][\bar{h}_v\Gamma_\nu h_v] \\
&\quad + [\bar{q}\Gamma^{\mu\nu}q][\bar{h}_v iD_\mu^-\Gamma_\nu h_v] + [\bar{q}\Gamma^{\mu\nu}q][\bar{h}_v\Gamma_\nu iD_\mu^+h_v]
\end{aligned} \tag{14}$$

$$\begin{aligned}
0 &= \partial_\mu[\bar{q}\Gamma_\nu q][\bar{h}_v\Gamma^{\mu\nu}h_v] \\
&= [\bar{q}iD_\mu^-\Gamma_\nu q][\bar{h}_v\Gamma^{\mu\nu}h_v] + [\bar{q}\Gamma_\nu q][\bar{h}_v\Gamma^{\mu\nu}iD_\mu^+h_v] \\
&\quad + [\bar{q}\Gamma_\nu q][\bar{h}_v iD_\mu^-\Gamma^{\mu\nu}h_v] + [\bar{q}\Gamma_\nu q][\bar{h}_v\Gamma^{\mu\nu}iD_\mu^+h_v]
\end{aligned} \tag{15}$$

Choosing appropriate Dirac matrices $\Gamma_{\mu\nu}$ and Γ_μ this allows us to remove the following operators from the basis (in favour of the combination on the r.h.s):

$$\begin{aligned}
\mathcal{M}_{1+}^{(3l)s/o} &= \mathcal{M}_{1-}^{(3l)s/o} - \mathcal{M}_{4+}^{(3h)s/o} + \mathcal{M}_{4-}^{(3h)s/o} \\
\mathcal{M}_{3+}^{(3l)s/o} &= \mathcal{M}_{3-}^{(3l)s/o} - \mathcal{M}_{2+}^{(3h)s/o} + \mathcal{M}_{2-}^{(3h)s/o} \\
\mathcal{M}_{5+}^{(3l)s/o} &= \mathcal{M}_{5-}^{(3l)s/o} - \mathcal{M}_{10+}^{(3h)s/o} + \mathcal{M}_{10-}^{(3h)s/o} \\
\mathcal{M}_{6+}^{(3l)s/o} &= \mathcal{M}_{6-}^{(3l)s/o} - \mathcal{M}_{7+}^{(3h)s/o} + \mathcal{M}_{7-}^{(3h)s/o} \\
\mathcal{M}_{7+}^{(3l)s/o} &= \mathcal{M}_{7-}^{(3l)s/o} - \mathcal{M}_{8+}^{(3h)s/o} + \mathcal{M}_{8-}^{(3h)s/o} \\
\mathcal{M}_{10-}^{(3l)s/o} &= -\mathcal{M}_{10+}^{(3l)s/o} + \mathcal{M}_{3+}^{(3h)s/o} - \mathcal{M}_{3-}^{(3h)s/o}
\end{aligned} \tag{16}$$

$$\tag{17}$$

3.2 Contraction Identities

Time ordered products, in which a $(ivD)h_v$–term acts on an internal line, are identical to a combination of local operators:

$$i\,\mathrm{T}\left[\bar{h}_vF(iD)(ivD)h_v, \bar{h}_vG(iD)h_v\right] = -\bar{h}_vF(iD)G(iD)h_v + \ldots \tag{18}$$

If only H–operators are involved such *contraction identities* have been studied in [13], where it has been shown that all time ordered products of this type are redundant and can be removed in favour of local H–operators. This way the anomalous dimensions of the local H–operators receive corrections that are crucial for the gauge independence of their short distance coefficients.

In the presence of HL–operators additional contraction identies arise:

$$
\begin{aligned}
i\,\mathrm{T}\left[[\bar{h}_v(ivD)^2 h_v], g_s^2[\bar{q}\Gamma^\mu q][\bar{h}_v\Gamma'_\mu h_v]\right] &= -g_s^2[\bar{q}\Gamma^\mu q][\bar{h}_v(ivD^-)\Gamma'_\mu h_v] \\
&\quad -g_s^2[\bar{q}\Gamma^\mu q][\bar{h}_v\Gamma'_\mu(ivD^+)h_v] \qquad (19)
\end{aligned}
$$

Since the T–product is only related to HL–operators vanishing by the heavy quark EOM the additional contraction identities are irrelevant.

3.3 Light Quark EOM

Using the equation of motion for the light quark

$$
i\not{D}^+ q = m_q q \qquad -\bar{q}i\not{D}^- = m_q\bar{q} \qquad (20)
$$

we can remove the mass dependent HL–operators in (11) from the operator basis:

$$
\begin{aligned}
m_q[\bar{q}\Gamma^\mu q][\bar{h}_v\Gamma'_\mu h_v] &= [\bar{q}\Gamma^\mu i\not{D}^+ q][\bar{h}_v\Gamma'_\mu h_v] \qquad (21) \\
&= -[\bar{q}i\not{D}^-\Gamma^\mu q][\bar{h}_v\Gamma'_\mu h_v]
\end{aligned}
$$

Taking a symmetrical combination of both equations (21) and choosing the dirac matrices Γ^μ and Γ'_μ appropriately we derive the relations

$$
\begin{aligned}
m_q\mathcal{M}_1^{(2)s/o} &= -\frac{1}{2}\mathcal{M}_{4+}^{(3h)s/o} + \mathcal{M}_{1-}^{(3l)s/o} + \frac{1}{2}\mathcal{M}_{4-}^{(3h)s/o} \\
m_q\mathcal{M}_2^{(2)s/o} &= \frac{1}{2}\mathcal{M}_{2+}^{(3l)s/o} - \frac{1}{2}\mathcal{M}_{4+}^{(3l)s/o} + \frac{1}{2}\mathcal{M}_{2-}^{(3l)s/o} + \frac{1}{2}\mathcal{M}_{4-}^{(3l)s/o} \\
m_q\mathcal{M}_3^{(2)s/o} &= \frac{1}{2}\mathcal{M}_{8+}^{(3l)s/o} - \frac{1}{2}\mathcal{M}_{9+}^{(3l)s/o} - \frac{1}{4}\mathcal{M}_{10+}^{(3h)s/o} + \frac{1}{2}\mathcal{M}_{5-}^{(3l)s/o} \qquad (22) \\
&\quad +\frac{1}{4}\mathcal{M}_{10-}^{(3h)s/o} - \frac{1}{2}\mathcal{M}_{8-}^{(3l)s/o} - \frac{1}{2}\mathcal{M}_{9-}^{(3l)s/o} \\
m_q\mathcal{M}_4^{(2)s/o} &= \mathcal{M}_{7+}^{(3h)s/o} - \mathcal{M}_{8+}^{(3h)s/o} - 2\mathcal{M}_{6-}^{(3l)s/o} + 2\mathcal{M}_{7-}^{(3l)s/o} - \mathcal{M}_{7-}^{(3h)s/o} \\
&\quad +\mathcal{M}_{8-}^{(3h)s/o} + \mathcal{M}_{3+}^{(3h)s/o} - \mathcal{M}_{3-}^{(3h)s/o}\,,
\end{aligned}
$$

in which momentum conservation has been already used.

Subtracting both equations (21) yields relations among the operators of dimension 7, written generically as:

$$
0 = [\bar{q}\Gamma^\mu i\not{D}^+ q][\bar{h}_v\Gamma'_\mu h_v] + [\bar{q}i\not{D}^-\Gamma^\mu q][\bar{h}_v\Gamma'_\mu h_v] \qquad (23)
$$

Choosing appropriate dirac matrices Γ_μ and Γ'_μ this identity allows us to remove the following operators from the basis (in favour of the operators on the r.h.s.):

$$
\begin{aligned}
\mathcal{M}_{4+}^{(3h)s/o} &= \mathcal{M}_{4-}^{(3h)s/o} \\
\mathcal{M}_{4+}^{(3l)s/o} &= -\mathcal{M}_{4-}^{(3l)s/o} + \mathcal{M}_{2+}^{(3l)s/o} - \mathcal{M}_{2-}^{(3l)s/o} \\
\mathcal{M}_{10+}^{(3h)s/o} &= 2\mathcal{M}_{8+}^{(3l)s/o} - 2\mathcal{M}_{9+}^{(3l)s/o} + 2\mathcal{M}_{8-}^{(3l)s/o} + 2\mathcal{M}_{9-}^{(3l)s/o} + \mathcal{M}_{10-}^{(3h)s/o}
\end{aligned}
$$

$$\begin{aligned}\mathcal{M}_{10+}^{(3l)s/o} &= \frac{1}{2}\mathcal{M}_{3+}^{(3h)s/o} - \frac{1}{2}\mathcal{M}_{7+}^{(3h)s/o} + \frac{1}{2}\mathcal{M}_{8+}^{(3h)s/o} - \frac{1}{2}\mathcal{M}_{3-}^{(3h)s/o} \\ &+ \frac{1}{2}\mathcal{M}_{7-}^{(3h)s/o} + \frac{1}{2}\mathcal{M}_{8-}^{(3h)s/o} \end{aligned} \tag{24}$$

3.4 Gluon EOM

Taking the functional derivative of the lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{q}(i\not{D} - m_q) + \bar{h}_v(ivD)h_v \tag{25}$$

with respect to the gluon field A_λ^b yields the gluon EOM

$$D_\mu^{ba} F^{a\mu\lambda} + g_s \bar{q}\gamma^\lambda T^b q + g_s v^\lambda \bar{h}_v T^b h_v = 0\,, \tag{26}$$

where $D^{ab} = \delta^{ab}\partial - g_s f^{abx} A^x$ is the covariant derivative in the adjoint representation. We multiply $T^b v_\lambda$ and sandwich the resulting expression between heavy quark spinors to get

$$\bar{h}_v[iD_\mu, [iD^\mu, (ivD)]]h_v = g_s^2[\bar{h}_v T^b h_v][\bar{q}\not{v} q] + g_s^2[\bar{h}_v T^b h_v][\bar{h}_v T^b h_v]\,. \tag{27}$$

The four–heavy–quark operator on the left hand side can be omitted, since at least to one loop order such operators only mix with themselves or appear as counterterms and therefore decouple from the renormalization group flow of the other operators.

In our conventions (27) reads

$$\mathcal{M}_2^{(2)o} = -2\mathcal{O}_1^{(2)} + \mathcal{O}_3^{(2)} + \mathcal{O}_4^{(2)}\,. \tag{28}$$

To derive an analogous relation among operators of dimension 7 we multiply (26) with $T^b\Gamma^{\nu\lambda}$ and sandwich between $-\bar{h}_v iD_\nu^-$ and h_v

$$\begin{aligned} -g_s^2[\bar{q}\gamma_\lambda T^b q][\bar{h}_v iD_\nu^- T^b \Gamma^{\nu\lambda} h_v] &= -2\bar{h}_v\Gamma^{\nu\lambda} iD_\nu iD_\mu iD_\lambda iD^\mu h_v \\ &+ \bar{h}_v \Gamma^{\nu\lambda} iD_\nu iD_\lambda (iD)^2 h_v \\ &+ \bar{h}_v\Gamma^{\nu\lambda} iD_\nu (iD)^2 iD_\lambda h_v \end{aligned} \tag{29}$$

or between $\bar{h}_v$ and $iD_\nu^+ h_v$

$$\begin{aligned} g_s^2[\bar{q}\gamma_\lambda T^b q][\bar{h}_v T^b \Gamma^{\nu\lambda} iD_\nu^+ h_v] &= -2\bar{h}_v\Gamma^{\lambda\nu} iD_\mu iD_\nu iD^\mu iD_\lambda h_v \\ &+ \bar{h}_v\Gamma^{\lambda\nu}(iD)^2 iD_\nu iD_\lambda h_v \\ &+ \bar{h}_v\Gamma^{\lambda\nu} iD_\nu (iD)^2 iD_\lambda h_v\,. \end{aligned} \tag{30}$$

Choosing $\Gamma_{\nu\lambda} = v_\nu v_\lambda, g_{\nu\lambda}, i\sigma_{\nu\lambda}$ yields the following relations:

$$\begin{aligned} \mathcal{M}_{2-}^{(3h)o} &= -2\mathcal{O}_{18}^{(3)} + \mathcal{O}_{14}^{(3)} + \mathcal{O}_{15}^{(3)} \\ \mathcal{M}_{2+}^{(3h)o} &= -2\mathcal{O}_{17}^{(3)} + \mathcal{O}_{15}^{(3)} + \mathcal{O}_{16}^{(3)} \end{aligned}$$

$$
\begin{aligned}
\mathcal{M}_{3-}^{(3h)o} &= 2\mathcal{O}_{10}^{(3)} - \mathcal{O}_{6}^{(3)} - \mathcal{O}_{9}^{(3)} \\
\mathcal{M}_{3+}^{(3h)o} &= -2\mathcal{O}_{11}^{(3)} + \mathcal{O}_{8}^{(3)} + \mathcal{O}_{9}^{(3)} \\
\mathcal{M}_{4-}^{(3h)o} &= -2\mathcal{O}_{4}^{(3)} + \mathcal{O}_{2}^{(3)} + \mathcal{O}_{3}^{(3)} \\
\mathcal{M}_{4+}^{(3h)o} &= -2\mathcal{O}_{4}^{(3)} + \mathcal{O}_{2}^{(3)} + \mathcal{O}_{3}^{(3)}
\end{aligned} \tag{31}
$$

Note that the relations involving $\mathcal{M}_{4\pm}^{(3h)o}$ are consistent with the first relation in (24) and the relations involving $\mathcal{M}_{2\pm}^{(3h)o}$ with the heavy quark EOM.

Taking the time ordered product of (28) with a dimension 5 operator $\mathcal{O}_i^{(1)}$ yields

$$
\mathcal{T}_{i2}^{(12hl)o} = -2\mathcal{T}_{i1}^{(12)} + \mathcal{T}_{i3}^{(12)} + \mathcal{T}_{i4}^{(12)} \, , \tag{32}
$$

where the last two T–products on the right hand side obey a contraction identity and are related to local operators $\mathcal{O}_i^{(3)}$. Note that in the case $i = 3$ (32) is consistent with the contraction identity (19), if the heavy quark EOM is applied.

3.5 Reduced Operator basis

Applying the identities of the previous subsections to the full operator basis leaves us with the set of physical operators listed in figure 1. In terms of this set of lineareiy independent operators the effective lagrangian at $\mathcal{O}(1/m_Q^3)$ reads

$$
\mathcal{L}^{(3)} = \vec{C}_{\mathcal{B}} \cdot \vec{\mathcal{B}} + \vec{C}_{\mathcal{O}_m} \cdot \vec{\mathcal{O}}_m + \vec{C}_{\mathcal{M}} \cdot \vec{\mathcal{M}} + \vec{C}_{\mathcal{T}} \cdot \vec{\mathcal{T}} \, . \tag{33}
$$

The different types of operators and their coefficients are collected in vectors as defined in figure 1.

4 Anomalous dimensions

In order to compute the anomalous dimensions of the operators $(\vec{\mathcal{B}}, \vec{\mathcal{O}}_m, \vec{\mathcal{M}}, \vec{\mathcal{T}})$, in general one has to calculate the pole parts of all divergent 1PI Greensfunctions with a certain operator inserted and express the result in terms of tree–level operators

$$
\langle \mathcal{A}_i \rangle_{1PI}^{(1)} = (\frac{\alpha}{\pi}) \frac{1}{\epsilon} \sum_{j=0} \gamma_{ij} \langle \mathcal{A}_j \rangle_{1PI}^{(0)} \tag{34}
$$

where $\mathcal{A}_i$ runs over all operators of the basis. From this one can directly read off the anomalous dimensions γ_{ij}.

Since we are working in the background field gauge [11, 12], it suffices to consider 1PI Greensfunctions with either two external heavy quark legs and, depending on the dimension, one or two background fields, or two heavy and two light fermion fields. Figure 2 shows the mixing properties of the operator basis by diagrammatic examples. The procedure how to take care of the var-

operator	index	shorthand (number)
$\mathcal{O}_i^{(3)}$	$i = 1, \dots, 13$	$\vec{\mathcal{B}}$ (21)
$\mathcal{T}_{ijk}^{(111)}$	$i,j,k = 1,2 \quad i \le j \le k$	
$\mathcal{T}_{ij}^{(12)}$	$i,j = 1,2$	
$m_q^2 \mathcal{O}_i^{(1)}$	$i = 1,2$	$\vec{\mathcal{O}}_m$ (7)
$m_q \mathcal{O}_i^{(2)}$	$i = 1,2$	
$m_q \mathcal{T}_{ij}^{(11)}$	$i,j = 1,2 \quad i \le j$	
$\mathcal{M}_{i+}^{(3h)s}$	$i = 3,5,6,7,9$	$\vec{\mathcal{M}}$ (43)
$\mathcal{M}_{i-}^{(3h)s}$	$i = 3,4,5,6,7,9$	
$\mathcal{M}_{i+}^{(3h)o}$	$i = 5,6,7,9$	
$\mathcal{M}_{i-}^{(3h)o}$	$i = 5,6,7,9$	
$\mathcal{M}_{i+}^{(3l)s}$	$i = 2,8,9$	
$\mathcal{M}_{i-}^{(3l)s}$	$i = 1, \dots, 9$	
$\mathcal{M}_{i+}^{(3l)o}$	$i = 2,8,9$	
$\mathcal{M}_{i-}^{(3l)o}$	$i = 1, \dots, 9$	
$\mathcal{T}_{ij}^{(12hl)s}$	$i = 1,2 \quad j = 1, \dots, 4$	$\vec{\mathcal{T}}$ (14)
$\mathcal{T}_{ij}^{(12hl)o}$	$i = 1,2 \quad j = 1,3,4$	

Figure 1: Physical operator basis at $\mathcal{O}(1/m_Q^3)$.

ious operator identities consistently follows the lines of [13]. We present our result for the anomalous dimension matrix corresponding to the operator basis $(\vec{\mathcal{B}}, \vec{\mathcal{O}}_m, \vec{\mathcal{M}}, \vec{\mathcal{T}})$ as a block matrix:

$$\hat{\gamma}^{(3)} = \begin{pmatrix} \hat{\gamma}_{\mathcal{B}\mathcal{B}} & 0 & \hat{\gamma}_{\mathcal{B}\mathcal{M}} & 0 \\ 0 & \hat{\gamma}_{\mathcal{O}_m\mathcal{O}_m} & \hat{\gamma}_{\mathcal{O}_m\mathcal{M}} & 0 \\ \hat{\gamma}_{\mathcal{M}\mathcal{B}} & \hat{\gamma}_{\mathcal{M}\mathcal{O}_m} & \hat{\gamma}_{\mathcal{M}\mathcal{M}} & 0 \\ \hat{\gamma}_{\mathcal{T}\mathcal{B}} & 0 & \hat{\gamma}_{\mathcal{T}\mathcal{M}} & \hat{\gamma}_{\mathcal{T}\mathcal{T}} \end{pmatrix} \tag{35}$$

The mixing of $\vec{\mathcal{T}}$ with $\vec{\mathcal{B}}$ is a consequence of the gluon EOM removing HL–operators in favour of $\mathcal{O}_i^{(3)}$ (see (31)), since to one loop order there are no penguin diagrams which would require $\mathcal{O}_i^{(3)}$ as counterterms.

Furthermore the anomalous dimensions of the H–operators (i.e. the entry $\hat{\gamma}_{\mathcal{B}\mathcal{B}}$) are affected by the gluon EOM since some of the $\vec{\mathcal{M}}$–counterterms may be

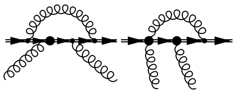
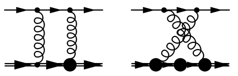
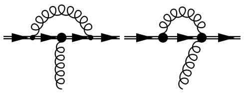
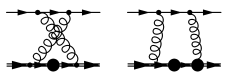
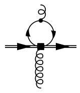
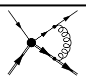
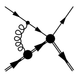
	diagram (example)	local counterterm	$\hat{\gamma}^{(3)}$
$\vec{\mathcal{B}}$		$\mathcal{O}_i^{(3)}$	$\hat{\gamma}_{\mathcal{B}\mathcal{B}}$
		$\mathcal{M}_{i\pm}^{(3h/l)s/o}$	$\hat{\gamma}_{\mathcal{B}\mathcal{M}}$
		$\mathcal{M}_{i\pm}^{(3h/l)s/o} \to \mathcal{O}_i^{(3)}$(gluon EOM)	$\hat{\gamma}_{\mathcal{B}\mathcal{B}}$
$\vec{\mathcal{O}}_m$		$m_q\mathcal{O}_i^{(2)}$, $m_q^2\mathcal{O}_i^{(1)}$	$\hat{\gamma}_{\mathcal{O}_m\mathcal{O}_m}$
		$m_q\mathcal{M}_i^{(2)s/o} \to \mathcal{M}_{i\pm}^{(3h/l)s/o}$ (light EOM)	$\hat{\gamma}_{\mathcal{O}_m\mathcal{M}}$
$\vec{\mathcal{M}}$		$\mathcal{O}_i^{(3)}$, $m_q\mathcal{O}_i^{(2)}$, $m_q^2\mathcal{O}_i^{(1)}$	$\hat{\gamma}_{\mathcal{M}\mathcal{B}}, \hat{\gamma}_{\mathcal{M}\mathcal{O}_m}$
		$\mathcal{M}_{i\pm}^{(3h/l)s/o}$	$\hat{\gamma}_{\mathcal{M}\mathcal{M}}$
$\vec{\mathcal{T}}$		$\mathcal{M}_{i\pm}^{(3h/l)s/o}$	$\hat{\gamma}_{\mathcal{T}\mathcal{M}}$
		$\mathcal{M}_{i\pm}^{(3h/l)s/o} \to \mathcal{O}_i^{(3)}$(gluon EOM)	$\hat{\gamma}_{\mathcal{T}\mathcal{B}}$

Figure 2: Mixing properties and operator identities. The black blob represents an operator of appopriate mass dimension.

removed using (31). This in turn implies corrections to the Wilson coefficients of the local operators in $\vec{\mathcal{B}}$.

The operators $\vec{\mathcal{O}}_m$ require local counterterms of the form $m_q\mathcal{M}_i^{(2)s/o}$ which are removed by means of (22) in favour of operators $\mathcal{M}_{i\pm}^{(3h/l)s/o}$, thereby explaining $\hat{\gamma}_{\mathcal{O}_m\mathcal{M}}$. The even more complicated situation that some of the latter obey the gluon EOM fortunately does not occure.

Since the submatrices of the block matrix (35) are too large we refrain from their presentation. They are available from the author on request.

5 Renormalization group logarithms

With the one loop anomalous dimensions and the tree level matching coefficients we are now in the position to solve the renormalization group equation for the

Wilson coefficients of the effective lagrangian at $\mathcal{O}(1/m_Q^3)$:

$$\frac{d}{d\ln\mu}\vec{\mathcal{C}}^{(3)}(\mu)+\hat{\gamma}^{(3)\top}\vec{\mathcal{C}}^{(3)}(\mu)=0 \tag{36}$$

Here $\vec{\mathcal{C}}^{(3)}(\mu)$ denotes collectively the coefficents $(\vec{C}_{\mathcal{B}},\vec{C}_{\mathcal{O}_m},\vec{C}_{\mathcal{M}},\vec{C}_{\mathcal{T}})$ of the physical operators $(\vec{\mathcal{B}},\vec{\mathcal{O}}_m,\vec{\mathcal{M}},\vec{\mathcal{T}})$ in the effective lagrangian (33). Since an analytical diagonalization of $\hat{\gamma}^{(3)}$ seems to be difficult we restrict ourselves to the calculation of the first logarithmic correction $\propto \alpha_s\ln(\mu/m_Q)$ in the coefficients $\vec{\mathcal{C}}^{(3)}(\mu)$. The exact solution of (36) reads

$$\vec{\mathcal{C}}^{(3)}(\mu)=\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{\frac{\hat{\gamma}^{(3)\top}}{2\beta^{(0)}}}\vec{\mathcal{C}}^{(3)}(m_Q) \tag{37}$$

with the one loop running coupling

$$\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}=1-2\beta^{(0)}(\frac{\alpha_s(\mu)}{\pi})\ln(\frac{\mu}{m_Q}) \tag{38}$$

where $\beta^{(0)}=(33-2n_f)/12$ in the presence of n_f light flavors. Expanding (37) to first order in the strong coupling we get

$$\vec{\mathcal{C}}^{(3)}(\mu)=\vec{\mathcal{C}}^{(3)}(m_Q)-(\frac{\alpha_s(\mu)}{\pi})\ln(\frac{\mu}{m_Q})\hat{\gamma}^{(3)\top}\vec{\mathcal{C}}^{(3)}(m_Q)+\mathcal{O}((\frac{\alpha_s(\mu)}{\pi})^2). \tag{39}$$

With our result $\hat{\gamma}^{(3)\top}$ and the tree level matching coefficients $\vec{\mathcal{C}}^{(3)}(m_Q)$ the Wilson coefficients are easily calculated. Note that the operators in $\vec{\mathcal{O}}_m$, $\vec{\mathcal{M}}$ and $\vec{\mathcal{T}}$ are not present at tree level, i.e. the corresponding entries in $\vec{\mathcal{C}}^{(3)}(m_Q)$ are zero. In figure 3 the non vanishing coefficients are shown. The remainig coefficients are $\mathcal{O}(\alpha_s^2)$ and do not contribute in our approximation.

The coefficients of the time ordered products are given by the product of the coefficients of their operator components.

Note, that the coefficients $C_i^{(3)}$ for $i=6,8,9,10,11$ are modified with respect to the value they would get if one disregards the HL–operators (in brackets). This correction is a consequence of the gluon EOM which relates certain HL–operators to H–operators (see (31)). In general the same is true for the coefficients $C_i^{(3)}$, $i=2,3,4$ but to one loop order the two operators $\mathcal{M}_{4\pm}^{(3h)o}$ which would cause the corrections always appear in the combination $\mathcal{M}_{4+}^{(3h)o}-\mathcal{M}_{4-}^{(3h)o}$ which vanishes identically according to (31) or the first relation in (24).

6 Conclusions

In this paper we have completed the one loop renormalization of the HQET lagrangian at $\mathcal{O}(1/m_Q^3)$. In addition to H–operators which are bilinear in the heavy quark field HL–operators consisting of two heavy and two light quark

Wilson coefficient	tree level	coefficient of $(\alpha_s(\mu)/\pi)\ln(\mu/m_Q)$
$C_1^{(3)}$	2	$-25/3\,C_A - 32/3\,C_F$
$C_2^{(3)}$	-1	$-1/2\,C_A$
$C_3^{(3)}$	-1	$4\,C_A + 8/3\,C_F$
$C_4^{(3)}$	1	$-17/6\,C_A$
$C_5^{(3)}$	-2	$5/3\,C_A + 8/3\,C_F$
$C_6^{(3)}$	1	$\mathbf{1/2\,C_A}\quad(-\,C_A)$
$C_7^{(3)}$	1	$-13/6\,C_A - 8/3\,C_F$
$C_8^{(3)}$	1	$\mathbf{1/2\,C_A}\quad(-\,C_A)$
$C_9^{(3)}$	1	$\mathbf{-\,C_A}\quad(-4\,C_A)$
$C_{10}^{(3)}$	-1	$\mathbf{3/2\,C_A}\quad(9/2\,C_A)$
$C_{11}^{(3)}$	-1	$\mathbf{3/2\,C_A}\quad(9/2\,C_A)$
$C_{12}^{(3)}$	0	$1/12\,C_A$
$C_{13}^{(3)}$	0	$-1/3\,C_A$
$C_{7+}^{(3h)s}$	0	$-2\,C_A\,C_F + 1/2\,C_A^2 + 2\,C_F^2 - 1/2$
$C_{7-}^{(3h)s}$	0	$-2\,C_A\,C_F + 1/2\,C_A^2 + 2\,C_F^2 - 1/2$
$C_{7+}^{(3h)o}$	0	$2\,C_A - 4\,C_F$
$C_{7-}^{(3h)o}$	0	$C_A - 4\,C_F$
$C_{1-}^{(3l)s}$	0	$-20/3\,C_A\,C_F + 5/3\,C_A^2 + 20/3\,C_F^2 - 5/3$
$C_{3-}^{(3l)s}$	0	$32/3\,C_A\,C_F - 8/3\,C_A^2 - 32/3\,C_F^2 + 8/3$
$C_{1-}^{(3l)o}$	0	$5\,C_A - 40/3\,C_F$
$C_{3-}^{(3l)o}$	0	$-8\,C_A + 64/3\,C_F$
$C_{6-}^{(3l)o}$	0	$-\,C_A$
$C_{7-}^{(3l)o}$	0	C_A

Figure 3: Wilson coefficients of the physical operator basis.

fields are included in the operator basis. We have shown that there exist several interdependencies between the operators of the complete operator basis which makes its reduction to a basis of linearely independent operators a non-trivial task. The anomalous dimensions of the H–operators have already been calculated in [13]. However in the presence of HL–operators the short distance coefficents of the H–operators are modified. This is a consequence of the gluon EOM which relates certain HL–operators to H–operators thereby modifying their anomalous dimensions and coefficients. This effect is well known at $\mathcal{O}(1/m_Q^2)$ where it causes corrections to the coefficient of the Darwin operator [7]. However, this correction is weak of $\mathcal{O}(\alpha_s^2)$ whereas at $\mathcal{O}(1/m_Q^3)$ the coefficients receive corrections already at $\mathcal{O}(\alpha_s)$.

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