hep-ph/9801436
TTP 98–02
May 25, 1998

Renormalizing Heavy Quark Effective Theory at $\mathcal{O}(1/m_Q^3)$

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Abstract

We present a calculation of the renormalized HQET Lagrangian at order $\mathcal{O}(1/m_Q^3)$ in the one particle sector. The anomalous dimensions of local operators and time ordered products of dimension 7 contributing at this order are calculated in the one loop approximation. We show that a careful treatment of the time ordered products is necessary to arrive at a gauge independent renormalized lagrangian. Our result sets the stage for an investigation of reparametrization invariance at $\mathcal{O}(1/m_Q^3)$.

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1 Introduction

Heavy Quark Effective Field Theory (HQET) [1] has been established as the theoretical tool of choice for the description of mesons and baryons containing one heavy quark [2]. This derives from the fact that it is a systematic expansion in inverse powers of the heavy quark mass with well defined and calculable coefficients. Furthermore, its realization of the spin and flavor symmetry of the low energy theory is a phenomenologically powerful tool.

The $1/m_Q$ expansion has already been applied sucessfully to phenomenological problems such as the determination of V_{cb}. This involves a matrix element of the left handed current for the $b \to c$ transition between two heavy hadrons. In order to match the experimental precision corrections ($1/m_Q$ as well as radiative corrections) are indispensable.

The present paper considers partial aspects of the $1/m_Q^3$ corrections, which eventually are needed for phenomenological applications. Here the lagrangian of QCD is expanded up to order $1/m_Q^3$ and the one-loop renormalization of the corresponding operators is calculated. Results for the lower order terms [3, 4, 5, 6, 7, 8] as well as a matching calculation for terms of order $1/m_Q^3$ [9] are already known, but the full renormalization has not yet been studied.

The lagrangian itself is also of theoretical interest, since on one side it is relevant for the $1/m_Q$ expansion of the heavy hadron mass, and on the other side allows a study of reparametrization invariance.

To keep things as simple as possible we restrict ourselves to the one particle sector of HQET, disregarding four fermion operators as well as pure gluonic operators. The calculation of the anomalous dimensions of the corresponding operator basis sets the stage for an extensive study of reparametrization invariance at $\mathcal{O}(1/m_Q^3)$, which will be subject of a sequel to this paper.

This note is organized as follows: in section 2 we introduce our operator basis and discuss its reduction to a set of linearly independent operators. Our result for the anomalous dimensions and some checks of its consistency are presented in section 3. In section 4 we give insight into some aspects of the calculation. Finally, we present the logarithmic contributions to the short distance coefficients of the effective lagrangian in section 5 and conclusions in section 6.

2 Operator basis

The effective HQET lagrangian is defined by a systematic expansion of QCD in inverse powers of the heavy quark mass

$$\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v(ivD)h_v + \sum_{n=1}^{\infty} \frac{1}{(2m_Q)^n} \sum_i C_i^{(n)} \mathcal{O}_i^{(n)} . \qquad (1)$$

To lowest order only one operator, $\bar{h}_v(ivD)h_v$, contributes, which is independent of the flavor and the spin of the heavy quark, resulting in the well known spin-flavor symmetry of HQET. However, these symmetries are valid only in the limit $m_Q \to \infty$ and broken by local operators $\mathcal{O}_i^{(n)}$ of dimension 5 and higher. The coefficients $C_i^{(n)}$ are short distance corrections, which compensate for the modified UV behavior of the effective theory. They are determined perturbatively by matching HQET to QCD order by order in the strong coupling α_s and the inverse heavy quark mass supplemented by a renormalization group analysis of the respective operators.

We start with a review of the local operators appearing at $\mathcal{O}(1/m_Q)$ and $\mathcal{O}(1/m_Q^2)$, respectively. At $\mathcal{O}(1/m_Q)$ we choose the conventional basis

$$\begin{aligned} \mathcal{O}_1^{(1)} &= \bar{h}_v (iD)^2 h_v \\ \mathcal{O}_2^{(1)} &= \frac{g}{2} \bar{h}_v \sigma^{\mu\lambda} F_{\mu\lambda} h_v \\ \mathcal{O}_3^{(1)} &= \bar{h}_v (ivD)^2 h_v \end{aligned} \tag{2}$$

and at $\mathcal{O}(1/m_Q^2)$

$$\begin{aligned} \mathcal{O}_1^{(2)} &= \bar{h}_v iD_\mu (ivD) iD^\mu h_v & \mathcal{O}_2^{(2)} &= \bar{h}_v i\sigma^{\mu\lambda} iD_\mu (ivD) iD_\lambda h_v \\ \mathcal{O}_3^{(2)} &= \bar{h}_v (ivD)(iD)^2 h_v & \mathcal{O}_4^{(2)} &= \bar{h}_v (iD)^2 (ivD) h_v \\ \mathcal{O}_5^{(2)} &= \bar{h}_v (ivD)^3 h_v & \mathcal{O}_6^{(2)} &= \bar{h}_v (ivD) i\sigma^{\mu\lambda} iD_\mu iD_\lambda h_v \\ \mathcal{O}_7^{(2)} &= \bar{h}_v i\sigma^{\mu\lambda} iD_\mu iD_\lambda (ivD) h_v \,. \end{aligned} \tag{3}$$

The definition of the covariant derivative $iD = i\partial + g_s T^a A^a$ and field strength tensor $F^a_{\mu\nu} T^a = -i/g_s [iD_\mu, iD_\nu]$ follows usual conventions. Note, that we refrain from skipping operators which vanish by the heavy quark equation of motion (EOM) $(ivD)h_v = 0$. This will proove important, since once inserted into time ordered products at $\mathcal{O}(1/m_Q^3)$, such operators contribute to physical operators, as will be explained in detail below.

At $\mathcal{O}(1/m_Q^3)$ we find 13 local operators contributing to physical matrixelements:

$$\begin{aligned} \mathcal{O}_1^{(3)} &= \bar{h}_v iD_\mu (ivD)^2 iD^\mu h_v & \mathcal{O}_2^{(3)} &= \bar{h}_v (iD)^2 (iD)^2 h_v \\ \mathcal{O}_3^{(3)} &= \bar{h}_v iD_\mu (iD)^2 iD^\mu h_v & \mathcal{O}_4^{(3)} &= \bar{h}_v iD_\mu iD_\nu iD^\mu iD^\nu h_v \\ \mathcal{O}_5^{(3)} &= \bar{h}_v i\sigma^{\mu\nu} iD_\mu (ivD)^2 iD_\nu h_v & \mathcal{O}_6^{(3)} &= \bar{h}_v i\sigma^{\mu\nu} iD_\mu iD_\nu (iD)^2 h_v \\ \mathcal{O}_7^{(3)} &= \bar{h}_v i\sigma^{\mu\nu} iD_\rho iD_\mu iD_\nu iD^\rho h_v & \mathcal{O}_8^{(3)} &= \bar{h}_v i\sigma^{\mu\nu} (iD)^2 iD_\mu iD_\nu h_v \\ \mathcal{O}_9^{(3)} &= \bar{h}_v i\sigma^{\mu\nu} iD_\mu (iD)^2 iD_\nu h_v & \mathcal{O}_{10}^{(3)} &= \bar{h}_v i\sigma^{\mu\nu} iD_\mu iD_\rho iD_\nu iD^\rho h_v \\ \mathcal{O}_{11}^{(3)} &= \bar{h}_v i\sigma^{\mu\nu} iD_\rho iD_\mu iD^\rho iD_\nu h_v & \mathcal{O}_{12}^{(3)} &= g_s^2 \bar{h}_v F^{a\mu\nu} F^a_{\mu\nu} h_v \\ \mathcal{O}_{13}^{(3)} &= g_s^2 \bar{h}_v v_\nu v^\rho F^{a\mu\nu} F^a_{\mu\rho} h_v \end{aligned} \tag{4}$$

Below these operators will be denoted collectively by $\vec{\mathcal{O}}^{(3)}$.

Another 11 operators contain the EOM piece $(ivD)h_v$ and are not considered any further, since at fixed order $1/m_Q^3$ their matrix elements vanish.

In addition to the local operators, there are the time-ordered products of the lower dimensional operators $\mathcal{O}_i^{(1)}$ and $\mathcal{O}_i^{(2)}$. Their contributions at $\mathcal{O}(1/m_Q^2)$ and $\mathcal{O}(1/m_Q^3)$ are written generically as

$$\mathcal{T}_{ij}^{(11)} = (1 - \frac{1}{2}\delta_{ij}) i \, \mathrm{T}\left[\mathcal{O}_i^{(1)}, \mathcal{O}_j^{(1)}\right], \quad i,j = 1 \ldots 3, \quad i \leq j \tag{5}$$

and

$$\mathcal{T}_{ij}^{(12)} = i \, \mathrm{T}\left[\mathcal{O}_i^{(1)}, \mathcal{O}_j^{(2)}\right], \quad i = 1 \ldots 3, \quad j = 1 \ldots 7 \tag{6}$$

$$\mathcal{T}_{ijk}^{(111)} = -\mathcal{S}_{ijk} \, \mathrm{T}\left[\mathcal{O}_i^{(1)}, \mathcal{O}_j^{(1)}, \mathcal{O}_k^{(1)}\right], \quad i,j,k = 1 \ldots 3, \quad i \leq j \leq k\,. \tag{7}$$

The symmetry factor $\mathcal{S}_{ijk}$ equals $1, 1/2$ or $1/6$, if no, two or all inserted operators are identical. There is a total of 31 time ordered products but not all of them are physical. The reason is that time ordered products, which contain at least one operator vanishing by the EOM, can contract to local operators if the $(ivD)h_v$ term acts on an internal heavy quark line. This is illustrated graphically as follows:

$$\mathrm{T}\left[[\ldots O_1(ivD)h_v][\bar{h}_v O_2 \ldots]\right] \qquad \propto \qquad \mathrm{T}\left[\ldots O_1 \cdot O_2 \ldots\right]$$

Below such operator relations will be denoted *contraction identities* (CI). They can be derived by manipulating the generating functional of Greensfunctions in the presence of the relevant operators in much the same way, as one usually establishes the validity of the EOM for local operators. As a concrete example consider the triple insertion

$$\mathcal{T}_{113}^{(111)} = -\frac{1}{2} \, \mathrm{T}\left[\bar{h}_v (iD)^2 h_v, \bar{h}_v (ivD)^2 h_v, \bar{h}_v (iD)^2 h_v\right]. \tag{8}$$

Naively acting with the EOM operator on the heavy quark propagators to the left and the right, one would expect that

$$\mathcal{T}_{113}^{(111)} = \bar{h}_v (iD)^2 (iD)^2 h_v + \ldots\,. \tag{9}$$

On the other hand an exact derivation of the contraction identity leads to

$$\begin{aligned} \mathcal{T}_{113}^{(111)} = & -i \, \mathrm{T}\left[\bar{h}_v (ivD)(iD)^2 h_v, \bar{h}_v (iD)^2 h_v\right] - i \, \mathrm{T}\left[\bar{h}_v (iD)^2 (ivD) h_v, \bar{h}_v (iD)^2 h_v\right] \\ & - \bar{h}_v (iD)^2 (iD)^2 h_v. \end{aligned} \tag{10}$$

The apparent contradiction in the sign of the local operator is resolved by the fact that the double insertions on the right hand side themselves contain a local contribution proportional to $\bar{h}_v (iD)^2 (iD)^2 h_v$:

$$i \, \mathrm{T}\left[\bar{h}_v (ivD)(iD)^2 h_v, \bar{h}_v (iD)^2 h_v\right] = -\bar{h}_v (iD)^2 (iD)^2 h_v + [\mathrm{EOM}] \tag{11}$$

Here the symbol [EOM] collects time ordered products that vanish by the EOM and can be neclected. The CI allow us to remove all time ordered products from the operator basis that contain EOM operator components. That way we are left with 4 triple insertions and 4 double insertions collectively denoted as $\vec{\mathcal{T}}^{(111)}$ and $\vec{\mathcal{T}}^{(12)}$ and composed only of the lower order physical operators $\mathcal{O}^{(1)}_{1,2}$ and $\mathcal{O}^{(2)}_{1,2}$. In the next section we will show explicitly that a consistent implementation of CI into the renormalization procedure is necessary to maintain gauge independence of the renormalized lagrangian.

3 Anomalous dimensions

To treat CI correctly during renormalization we start with the calculation of the anomalous dimensions corresponding to the *full operator basis*. This basis contains all local operators of dimension 7 allowed by Lorentz invariance and the symmetries of HQET as well as all time ordered products which can be constructed from lower order operators disregarding any operator identities (such as CI). The anomalous dimension matrix corresponding to the full basis is then projected onto the *physical basis* with all CI applied and EOM operators removed. The physical basis is minimal in the sense that it contains only linearly independent operators.

To clarify this procedure let us for a moment forget about time ordered products and consider renormalization of an overcomplete operator basis consisting of physical operators $\mathcal{O}_i$ and operators $\hat{\mathcal{O}}_j$ which linearely depend on the $\mathcal{O}_i$:

$$\hat{\mathcal{O}}_i = \sum_k c_{ik}\mathcal{O}_k \tag{12}$$

The full basis renormalizes as

$$\mathcal{O}_i = \sum_j Z^{(1)}_{ij}\mathcal{O}^{bare}_j + \sum_j \hat{Z}^{(1)}_{ij}\hat{\mathcal{O}}^{bare}_j \tag{13}$$

$$\hat{\mathcal{O}}_i = \sum_j Z^{(2)}_{ij}\mathcal{O}^{bare}_j + \sum_j \hat{Z}^{(2)}_{ij}\hat{\mathcal{O}}^{bare}_j\,, \tag{14}$$

wherein we use (12) to eleminate the bare operators $\hat{\mathcal{O}}^{bare}_j$ in favor of the physical ones:

$$\mathcal{O}_i = \sum_j Z^{(1)}_{ij}\mathcal{O}^{bare}_j + \sum_{jk} \hat{Z}^{(1)}_{ij}c_{jk}\mathcal{O}^{bare}_k = \sum_j \tilde{Z}^{(1)}_{ij}\mathcal{O}^{bare}_j \tag{15}$$

$$\hat{\mathcal{O}}_i = \sum_j Z^{(2)}_{ij}\mathcal{O}^{bare}_j + \sum_{jk} \hat{Z}^{(2)}_{ij}c_{jk}\mathcal{O}^{bare}_k = \sum_j \tilde{Z}^{(2)}_{ij}\mathcal{O}^{bare}_j \tag{16}$$

$\tilde{Z}^{(1)}_{ij}$ are effective renormalization constants which describe the renormalization of the physical operator basis $\mathcal{O}_i$ among themselves and defines their anomalous dimensions. The $\tilde{Z}^{(2)}_{ij}$ express the renormalization counterterms of the redundant operators in terms of the physical ones. They are not linearly independent since the renormalized operators on the left hand side of (15,16) should also fulfill (12), which in turn leads to the consistency condition

$$\sum_k c_{ik}\tilde{Z}^{(1)}_{kj} = \tilde{Z}^{(2)}_{ij}. \tag{17}$$

To one loop order and in the MS scheme, the anomalous dimensions are the negative of the pole parts of the renormalization constants:

$$\tilde{Z}^{(1)}_{ij} = \delta_{ij} - (\frac{\alpha}{\pi})\frac{1}{\epsilon}\gamma_{ij} + \mathcal{O}((\frac{\alpha}{\pi})^2) \tag{18}$$

Subsequently (17) leads to relations among the anomalous dimensions of the physical and unphysical operators, which provides a powerful consistency check of the calculation.

The discussion above applies to our operator basis, if we identify the $\hat{\mathcal{O}}_i$ with time ordered products which are redundant after application of the CI and the $\mathcal{O}_i$ with the residing physical operators. In our case the removal of the redundant operators prooves important to ensure gauge independence of the physical operator basis. In the following we demonstrate this explicitly in the case of the renormalized triple insertion $\mathcal{T}^{(111)}_{111}$ of the kinetic energy operator $\mathcal{O}^{(1)}_1$. Before applying the CI, the renormalized triple insertion expressed in terms of bare operators reads

$$\begin{aligned}\mathcal{T}^{(111)}_{111} = -(\frac{\alpha}{\pi})\frac{1}{\epsilon}\Big[\quad & C_F(2+\bar{\xi})(-\frac{1}{2})\,\mathrm{T}\left[\bar{h}_v(iD)^2h_v, \bar{h}_v(ivD)^2h_v, \bar{h}_v(iD)^2h_v\right]^{bare} \\ & + (\frac{1}{12}C_A + C_F(\frac{10}{3}+\bar{\xi}))\,i\,\mathrm{T}\left[\bar{h}_v(iD)^2(ivD)h_v, \bar{h}_v(iD)^2h_v\right]^{bare} \\ & + (\frac{1}{12}C_A + C_F(\frac{10}{3}+\bar{\xi}))\,i\,\mathrm{T}\left[\bar{h}_v(ivD)(iD)^2h_v, \bar{h}_v(iD)^2h_v\right]^{bare} \\ & + (\frac{1}{6}C_A + C_F(\frac{14}{3}+\bar{\xi}))\,\bar{h}_v(iD)^2(iD)^2h_v^{bare} \qquad \Big] + \ldots .\end{aligned} \tag{19}$$

$\bar{\xi} = 1-\xi$ is the usual gauge parameter and the ellipses denote terms not relevant for our discussion. The first term on the right hand side results from local renormalization of the kinetic energy operator

$$\bar{h}_v(iD)^2h_v = \bar{h}_v(iD)^2h_v^{bare} - (\frac{\alpha}{\pi})\frac{1}{\epsilon}C_F(2+\bar{\xi})\bar{h}_v(ivD)^2h_v^{bare}. \tag{20}$$

The fact that the kinetic energy operator is not renormalized multiplicatively is a consequence of the celebrated reparametrization invariance of HQET [10]. However, the kinetic energy operator requires renormalization by the EOM operator $\mathcal{O}^{(1)}_3$, which in turn finds it way into renormalization of the triple insertion $\mathcal{T}^{(111)}_{111}$.

The appearance of the operators $\bar{h}_v(iD)^2(ivD)h_v$ and $\bar{h}_v(ivD)(iD)^2h_v$ in the double time ordered products on the right hand side of (19) derives from the fact that the double insertion of the kinetic energy operator needs renormalization by a local operator of dimension 6:

$$\frac{i}{2}\,\mathrm{T}\left[\bar{h}_v(iD)^2h_v, \bar{h}_v(iD)^2h_v\right] = -(\frac{\alpha}{\pi})\frac{1}{\epsilon}\left(\frac{1}{12}C_A + C_F(\frac{10}{3}+\bar{\xi})\right) \cdot \left[\bar{h}_v(ivD)(iD)^2h_v^{bare} + \bar{h}_v(iD)^2(ivD)h_v^{bare}\right] + \ldots . \tag{21}$$

If we now replace the bare triple insertion $\mathcal{T}_{113}^{(111)}$ on the right hand side of (19) with the help of CI (10), the gauge dependencies cancel and we are left with a gauge independent result. We stress that the latter property is not spoiled by additional application of CI (11) for the double insertions since their coefficients are gauge independent already. The result is that apart from the above example similar gauge cancellations occur in the case of all time ordered products which are related to physical operators by CI. To summarize, as long as one works with an overcomplete operator basis in which some of the operators are not linearely independent, physical operators may have gauge dependent coefficients. However, if one finally projects onto to the physical operator basis, these gauge artefacts must drop out.

We now come to the presentation of our results. The anomalous dimensions corresponding to the physical operator basis can be cast into block diagonal form

$$\hat{\gamma}^{(3)} = \begin{array}{c c} & \begin{array}{ccc} \vec{\mathcal{O}}^{(3)} & \vec{\mathcal{T}}^{(12)} & \vec{\mathcal{T}}^{(111)} \end{array} \\ \begin{array}{c} \vec{\mathcal{O}}^{(3)} \\ \vec{\mathcal{T}}^{(12)} \\ \vec{\mathcal{T}}^{(111)} \end{array} & \begin{pmatrix} \hat{\gamma}_l^{(3)} & 0 & 0 \\ \hat{\gamma}_l^{(12)} & \hat{\gamma}_{nl}^{(12)} & 0 \\ \hat{\gamma}_l^{(111)} & \hat{\gamma}_{nl}^{(111)} & \hat{\gamma}_{nl}^{(111)} \end{pmatrix} \end{array} \tag{22}$$

The first column of this matrix is the result of our calculation and is presented in appendix A. Weinbergs theorem [11] guarantees, that only local operators are needed as renormalization counterterms. However, apart from mixing with local operators the triple insertions $\mathcal{T}_{ijk}^{(111)}$ also require nonlocal renormalization by the double insertions $\mathcal{T}_{ij}^{(12)}$. This reflects the fact that two operator components of the triple insertions are themselves renormalized by a local counterterm of dimension 6 (see equation (21)).

4 Aspects of the calculation

To calculate the anomalous dimensions of an operator basis of definite dimensionality one generally has to insert every operator into potentially UV divergent 1PI Greenfunctions and extract the pole piece. In the background field method

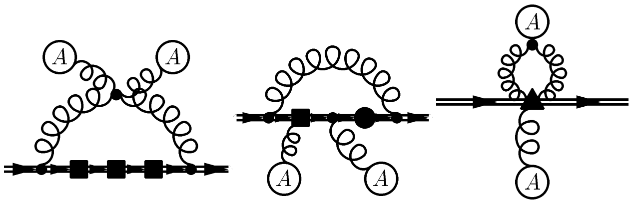

Figure 1: Examples of abelian and nonabelian diagrammatic contributions to the renormalization of the triple, double and local insertion. The square, blob and triangle represent operators of dimensions 5, 6 and 7, respectively.

[12, 13], gauge invariance is maintained explicitly, which allows one to get the full result from the calculation of a subset of Feyman diagrams with a specified number of external background fields. This number is fixed by the highest power of gluon field strength tensors appearing in the basis, because such operators do not contribute at tree level to 1PI Greensfunctions with a smaller number of external background fields. In our case the local operators $\mathcal{O}^{(3)}_{12/13}$ are bilinear in the field strength. This in turn requires the calculation of all one loop diagrams with one incoming and outgoing heavy quark and two external background fields to get the counterterm contributions of all operators. Figure 1 shows some diagrammatic examples. The calculation has been performed with the help of algebraic manipulations provided by the package FORM [14].

In the first step every integrand corresponding to a specific diagram is expanded in a Taylor series with respect to its external momenta, i.e. the heavy quark momentum and two background field momenta. This procedure is justified since we are only interested in UV divergences. The latter are easily extracted at this order of the expansion where the power of external momenta agrees with the UV degree of divergence of the diagrams. They appear in the pole parts of simple one loop tensor integrals of generic structure,

$$I_{n,m} = \int \frac{d^d k}{(2\pi)^d} \frac{k^{\mu_1} \cdots k^{\mu_n}}{(vk)^m (k^2)^{(n+4-m)/2}} \,, \tag{23}$$

which can be related recursively to two basis integrals $I_{0,0}$ and $I_{0,2}$ via Pasarino–Veltman reduction. In the calculation integrals with up to 10 indices are needed.

All diagrammatic contributions to a specific operator insertion are then summed up and expressed in terms of tree level contributions of the local operators $\mathcal{O}^{(3)}_i$:

$$\langle \mathcal{A}_i \rangle^{(1)}_{1PI} = (\frac{\alpha}{\pi}) \frac{1}{\epsilon} \sum_{i=0}^{13} \gamma_{ij} \langle \mathcal{O}^{(3)}_i \rangle^{(0)}_{1PI} \tag{24}$$

Here $\mathcal{A}_i$ denotes collectively local operators or time ordered products of the full operator basis and $\langle \ldots \rangle_{1PI}$ indicates insertion of the given operator in a 1PI Greensfunction to the loop order specified by the superscript.

In this step explicit gauge invariance in the background field ensures that no gauge variant operators are needed in the series on the right hand side of (24). This property provides a powerful consistency check of the calculation, since the tree level contributions contain up to 30 terms which must match in such a way that all gauge variant terms cancel.

In (24) the coefficients γ_{ij} are easily identified as the entries of the anomalous dimension matrix $\hat{\gamma}_l^{(3)}$, $\hat{\gamma}_l^{(12)}$ and $\hat{\gamma}_l^{(111)}$ corresponding to mixing with the local operators $\mathcal{O}_i^{(3)}$. In the case of time ordered products the anomalous dimensions of their operator components contribute just additively and can therefore be derived from the anomalous dimensions of the operator bases appearing at lower $\mathcal{O}(1/m_Q)$.

However, up to now the anomalous dimensions correspond to the full operator basis and some artificial gauge dependencies of physical operators show up. Therefore in the last step of the calculation the projection onto the physical operator basis has to be performed. This is where unphysical gauge artefacts drop out and one is left with the gauge independent anomalous dimension matrix of the physical operator basis. These nontrivial cancellations provide another powerful cross–check of our results.

5 Renormalization group logarithms

With the one loop anomalous dimensions and the tree level matching coefficients we are now in the position to solve the renormalization group (RG) equation for the Wilson coefficients of the effective lagrangian at $\mathcal{O}(1/m_Q^3)$:

$$\frac{d}{d\ln\mu}\vec{C}^{(3)}(\mu) + \hat{\gamma}^{(3)\top}\vec{C}^{(3)}(\mu) = 0 \tag{25}$$

Here $\vec{C}^{(3)}(\mu)$ denotes the coefficents of the physical operators $(\vec{\mathcal{O}}^{(3)}, \vec{\mathcal{T}}^{(12)}, \vec{\mathcal{T}}^{(111)})$. However, an analytical diagonalization of $\hat{\gamma}^{(3)}$ seems to be difficult. Instead we restrict ourselves to the calculation of the first logarithmic correction $\propto \alpha_s \ln(\mu/m_Q)$ in the coefficients $\vec{C}^{(3)}(\mu)$. The exact solution of (25) reads

$$\vec{C}^{(3)}(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{\frac{\hat{\gamma}^{(3)\top}}{2\beta^{(0)}}} \vec{C}^{(3)}(m_Q) \tag{26}$$

with the one loop running coupling

$$\frac{\alpha_s(\mu)}{\alpha_s(m_Q)} = 1 - 2\beta^{(0)}(\frac{\alpha_s(\mu)}{\pi})\ln(\frac{\mu}{m_Q}) \tag{27}$$

where $\beta^{(0)} = (33 - 2n_f)/12$ in the presence of n_f light flavours. Expanding (26) to first order in the strong coupling we get

$$\vec{C}^{(3)}(\mu) = \vec{C}^{(3)}(m_Q) - (\frac{\alpha_s(\mu)}{\pi})\ln(\frac{\mu}{m_Q})\hat{\gamma}^{(3)\top}\vec{C}^{(3)}(m_Q) + \mathcal{O}((\frac{\alpha_s(\mu)}{\pi})^2). \tag{28}$$

With our result $\hat{\gamma}^{(3)\top}$ and the tree level matching coefficients $\vec{C}^{(3)}(m_Q)$ the Wilson coefficients are easily calculated. The result is shown in the table below.

$C_i^{(3)}(\mu)$	tree level	coefficient of $(\alpha_s(\mu)/\pi)\ln(\mu/m_Q)$
1	2	$25/3\,C_A - 23/3\,C_F$
2	-1	$-1/2\,C_A$
3	-1	$4\,C_A + 8/3\,C_F$
4	1	$-17/6\,C_A$
5	-2	$5/3\,C_A + 8/3\,C_F$
6	1	$-\,C_A$
7	1	$-13/6\,C_A - 8/3\,C_F$
8	1	$-\,C_A$
9	1	$-4\,C_A$
10	-1	$9/2\,C_A$
11	-1	$9/2\,C_A$
12	0	$1/12\,C_A$
13	0	$-1/3\,C_A$

We only present the coefficients of local operators, since the coefficients of the time ordered products are the products of the coefficients of their operator components. We note that there is a subtlety in the determination of the tree level coefficients $\vec{C}^{(3)}(m_Q)$. Again the CI forbid their naive extraction from the tree level lagrangian at $\mathcal{O}(1/m_Q^3)$

$$\mathcal{L}^{(3)} = \bar{h}_v iD_\mu (ivD)^2 iD^\mu h_v - \bar{h}_v (ivD)^4 h_v - \bar{h}_v i\sigma^{\mu\nu} iD_\mu (ivD)^2 iD_\nu h_v. \qquad (29)$$

The reason is that some of the time ordered products with nonvanishing tree level coefficients are related to physical operators by CI. Since we are studying the RG flow of the physical basis, we first have to reduce the tree level matrix elements of the full operator basis with the CI to the ones of the physical basis and then apply the RG controlled by the anomalous dimensions of the physical basis.

For example, the triple insertion $\mathcal{T}_{113}^{(111)}$ contributes at tree level with coefficient -1. However, after application of CI (10,11) this coefficient is attributed to the local operator $\mathcal{O}_2^{(3)} = \bar{h}_v (iD)^2 (iD)^2 h_v$ which does not appear in the tree level lagrangian. The same occurs for all time ordered products which contribute at tree level and in addition are affected by a CI. This explains the nonvanishing tree level contributions of coefficients of operators not appearing in the tree level lagrangian. Note that the operators $\mathcal{O}_{12/13}^{(3)}$ are protected from CI contributions because of their color structure.

As an phenomenologic application we consider the expansion of the physical mass M_H of a heavy meson in the HQET

$$M_H = m_Q + \bar{\Lambda} - \langle \mathcal{L}_{int}(0) e^{i \int d^4x \mathcal{L}_{int}(x)} \rangle \tag{30}$$

with $\mathcal{L}_{int}$ given by the powercorrections on the right hand side of (1) and $\langle \cdots \rangle$ denoting the expectation value between heavy meson states in the heavy mass limit. Expanding up to $\mathcal{O}(1/m_Q^3)$ we get

$$M_H = m_Q + \bar{\Lambda} - \frac{1}{2m_Q} M_H^{(1)} - \frac{1}{(2m_Q)^2} M_H^{(2)} - \frac{1}{(2m_Q)^3} M_H^{(3)} \tag{31}$$

where

$$M_H^{(1)} = \sum_{i=1}^{2} C_i^{(1)}(\mu) \langle \mathcal{O}_i^{(1)} \rangle \tag{32}$$

$$M_H^{(2)} = \sum_{i=1}^{2} C_i^{(2)}(\mu) \langle \mathcal{O}_i^{(2)} \rangle + \sum_{i \le j=1}^{2} C_i^{(1)}(\mu) C_j^{(1)}(\mu) \langle \mathcal{T}_{ij}^{(11)} \rangle \tag{33}$$

$$\begin{aligned} M_H^{(3)} = & \sum_{i=1}^{13} C_i^{(3)}(\mu) \langle \mathcal{O}_i^{(3)} \rangle + \sum_{i,j=1}^{2} C_i^{(1)}(\mu) C_j^{(2)}(\mu) \langle \mathcal{T}_{ij}^{(12)} \rangle \\ & + \sum_{i \le j \le k=1}^{2} C_i^{(1)}(\mu) C_j^{(1)}(\mu) C_k^{(1)}(\mu) \langle \mathcal{T}_{ijk}^{(111)} \rangle \, . \end{aligned} \tag{34}$$

Since the coefficients $C_i^{(1/2)}(\mu)$ are allready known from lower order calculations, with our result for the coefficients $C_i^{(3)}(\mu)$ all short distance contributions up to $\mathcal{O}(1/(2m_Q)^3)$ are known. However the problem is the nonperturbative input, i.e. the hadronic matrix elements on the right hand side of (32,33,34) which cannot be calculated from first principles. The hadronic matrixelements at $\mathcal{O}(1/m_Q)$ are parametrized by the well known parameters λ_1 and λ_2. At $\mathcal{O}(1/m_Q^2)$ 5 parameters are needed and at $\mathcal{O}(1/m_Q^3)$ a further proliferation of parameters takes place: here 21 parameters are needed.

6 Conclusions

We have presented a new calculation of the anomalous dimensions of the operator basis appearing at $\mathcal{O}(1/m_Q^3)$ of the HQET lagrangian. Local operators as well as all time ordered products – composed of lower dimension operators – of dimension 7 contribute to the operator basis at this order. We have shown that there exist non trivial relations among some time ordered products and local operators, so called contraction identities, which have to be properly taken into account during renormalization. Otherwise gauge independence of the physical operators is violated.

The calculation of the one loop anomalous dimensions of the physical operators was the goal of this paper. Supplemented by the tree level matching coefficients, it was possible to extract the first logarithmic corrections $\propto \alpha_s(\mu)\ln(\mu/m_Q)$ to the Wilson coefficents. Of course, to get the non–logarithmic contributions at $\mathcal{O}(\alpha_s)$, a one loop matching calculation has to be performed, which is beyond the scope of this work.

In [9] the short distance coefficients of the $\mathcal{O}(1/m_Q^3)$ effective lagrangian have been derived by matching the full QCD vertex function with one external gluon to HQET in an on–shell renormalization scheme. In this case all loop diagrams in the effective theory vanish and the coefficients can be read of directly from an expansion of the QCD vertex in powers of the heavy quark momentum and the gluon momenta to the appropriate order. It is clear that the coeffients of operators bilinear in the gluon fields cannot be determined by this method. Using the equation of motion we have transformed the operator basis of [9] into ours. This way only the coefficients of $\mathcal{O}_2^{(3)}$, $\mathcal{O}_6^{(3)}$ and $\mathcal{O}_8^{(3)}$ are determined. The other coefficients are related to coefficients which have not been calculated in [9]. Unfortunately there is a mismatch in the coefficient of $\mathcal{O}_2^{(3)}$. A possible reason for this mismatch may be found in the fact that the tensor structures appearing in the expansion of the one loop QCD vertex function at $\mathcal{O}(1/m_Q^3)$ cannot be uniquely attributed to tree level insertions of the local operators $\mathcal{O}_i^{(3)}$ with one external background field. For example, in our conventions the operators $\mathcal{O}_3^{(3)}$ and $\mathcal{O}_4^{(3)}$ contribute at tree level identically to the vertex function with only one external gluon. Therefore one should expand the one loop QCD vertex function with two external background fields in inverse powers of the large mass to perform the matching at $\mathcal{O}(1/m_Q^3)$ correctly.

In [7] the effect of four fermion operators at $\mathcal{O}(1/m_Q^2)$ has been studied. The upshot of this work was that the famous Darwin operator is renormalized by a four fermion operator involving two static and two light quarks, even if all four fermion operators are removed at the end of the renormalization procedure. Similiar effects cannot be excluded at $\mathcal{O}(1/m_Q^3)$, but in this case the higher dimension of the operators make life more complicated. Nevertheless it is not only a calculational challenge to include all possible four fermion operators into the operatorbasis at $\mathcal{O}(1/m_Q^3)$ and to investigate their renormalization properties [15].

Another topic which will be covered in a sequel to this publication [16], is the reparametrization invariance of HQET. This new symmetry of the HQET lagrangian predicts relations among Wilson coefficients of operators appearing at different $\mathcal{O}(1/m_Q)$. With our results it should be possible to check the validity of these relations. This in turn may help to draw a decision between several concepts of this symmetry appearing in the literature, all of them unfortunately agreeing in their predictions up to $\mathcal{O}(1/m_Q^2)$.

Acknowledgements

This work is supported by DFG contract Ma 1187/7-1,2 and Graduiertenkolleg “Elementarteilchenphysik an Beschleunigern”. The author wish to thank T. Mannel for useful and enlightening discussions.

A Anomalous dimension matrices

In the decomposition $\hat{\gamma}_l^{(\cdots)} = C_A \hat{\gamma}_l^{(\cdots)A} + C_F \hat{\gamma}_l^{(\cdots)F}$ the entries in the first column of (22) are:

$$\hat{\gamma}_l^{(3)A} =$$

$$\begin{pmatrix}
-\frac{11}{12} & 0 & \frac{11}{24} & -\frac{11}{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{288} & -\frac{11}{72} \\
-4 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{9} \\
-3 & \frac{5}{6} & \frac{5}{6} & -\frac{5}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{12} & -\frac{7}{18} \\
-\frac{7}{2} & \frac{5}{6} & \frac{4}{3} & -\frac{13}{6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{24} & -\frac{11}{36} \\
0 & 0 & 0 & 0 & -\frac{7}{6} & \frac{1}{8} & \frac{1}{12} & \frac{1}{8} & \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{7}{8} & \frac{1}{2} & \frac{5}{24} & \frac{1}{3} & -\frac{1}{4} & -\frac{5}{12} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{5}{24} & -1 & \frac{5}{24} & -\frac{1}{12} & \frac{1}{12} & \frac{1}{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{5}{24} & \frac{1}{2} & -\frac{7}{8} & \frac{1}{3} & -\frac{5}{12} & -\frac{1}{4} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & \frac{1}{4} & 0 & \frac{1}{8} & 0 & -\frac{1}{2} & -\frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{24} & \frac{1}{4} & \frac{1}{6} & \frac{11}{24} & -\frac{13}{12} & -\frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{6} & \frac{1}{4} & \frac{1}{24} & \frac{11}{24} & -\frac{1}{3} & -\frac{13}{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{24} & -\frac{1}{6}
\end{pmatrix}$$

$$\hat{\gamma}_l^{(111)A} = \begin{pmatrix}
\frac{16}{3} & \frac{1}{6} & -\frac{5}{2} & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{4}{3} & \frac{5}{3} & \frac{8}{3} & \frac{5}{3} & 3 & -4 & -4 & 0 & 0 \\
5 & \frac{1}{6} & -\frac{23}{6} & \frac{11}{3} & -1 & 0 & 1 & 0 & \frac{5}{2} & -\frac{5}{2} & -\frac{5}{2} & -\frac{1}{4} & \frac{1}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{8} & \frac{1}{4}
\end{pmatrix}$$

$$\hat{\gamma}_l^{(111)F} = \begin{pmatrix}
\frac{32}{3} & \frac{8}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{8}{3} & -\frac{4}{3} & \frac{8}{3} & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$
\hat{\gamma}_l^{(12)A} =
$$

$$
\begin{pmatrix}
4 & -\frac{1}{2} & -\frac{3}{2} & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{2}{9} \\
0 & 0 & 0 & 0 & -\frac{4}{3} & -\frac{7}{12} & -1 & -\frac{7}{12} & -\frac{1}{6} & \frac{2}{3} & \frac{2}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{24} & \frac{1}{2} & \frac{1}{24} & \frac{7}{12} & -\frac{7}{12} & -\frac{7}{12} & 0 & 0 \\
-\frac{2}{3} & \frac{1}{3} & \frac{17}{12} & -\frac{7}{4} & \frac{1}{6} & -\frac{5}{24} & -\frac{1}{3} & -\frac{5}{24} & -\frac{3}{4} & \frac{3}{4} & \frac{3}{4} & \frac{1}{144} & \frac{11}{36}
\end{pmatrix}
$$

$$
\hat{\gamma}_l^{(12)F} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{16}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$\hat{\gamma}_l^{(3)F}$ has only zero entries, a fact for which we have no simple explanation by now.

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