Standard Model Calculations for Precision Observables Beyond One Loop

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This talk gives a review of recent two- and three-loop calculations to electroweak precision observables. Emphasis is put on the calculation of low energy observables, in particular the muon anomalous magnetic moment and the electric dipole moment of the neutron.

1. Introduction

In order to match with the increasing precision of electroweak data from experiments at LEP, hadron colliders or low energy experiments theorists are forced to perform two- and three-loop calculations, thus testing theory on the quantum level. Important goals from the comparison between theory and experiment include

- precision tests of the Standard Model,
- possible deviations as a signal for new physics,
- improvement of indirect constraints on the Higgs mass.

Due to the high experimental precision, the following list of electroweak input parameters is currently used: $\alpha(0)$, G_μ, M_Z, all observables being known at least to $\mathcal{O}(10^{-5})$. The relation between the basic parameters $\alpha(0)$, G_μ, M_Z, M_W is

$$G_\mu = \frac{\pi\alpha}{\sqrt{2}}\frac{1}{M_W^2(1-M_W^2/M_Z^2)}\left[1+\Delta r\right] \qquad (1)$$

where Δr parametrizes the radiative corrections. The theoretical prediction for Δr is

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{remainder} \qquad (2)$$

Several improvements have been achieved on the SM radiative corrections:

- *Electroweak Precision Observables:* Numerically the most important contribution to Δr is from $\Delta\alpha$ (see [1] for details). As for the ρ-parameter, its contribution increases with the mass splitting in each isospin doublet. For this reason the contribution from the top-bottom-doublet is dominant. QCD-corrections to $\Delta\rho$ have been computed including three loops [2,3]. These corrections are well under control and have been implemented in the analysis of LEP data.

- *Vector Boson Masses:* In eq.(1) the relation between M_W, M_Z and the Fermi constant G_μ is established. To Δr one-loop and leading parts $\mathcal{O}(G_\mu^2 m_t^4)$ of the two-loop contributions have been available since some time already (see [1]). An estimate of the theoretical uncertainty by investigation of the renormalization scheme dependence in ref. [5] revealed, however, that there was still a significant uncertainty ($\Delta M_W \approx 16$ MeV, $\Delta s_W^2 \approx 1.4\cdot 10^{-4}$ and a shift in the upper bound of M_H by up to 100 GeV) coming from the unknown non-leading terms $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$. Two new calculations are now available which significantly reduce this theoretical uncertainty: the complete $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ calculation of [6] shows that the subleading terms in the m_t-expansion can be equally important as the m_t^4 terms (see also [8]). In the analysis of [7] the M_H dependence of Δr has been investigated. This is a subset of the diagrams calculated in [6], however in [7] the full m_t, M_H dependence is retained. The comparison between [6] and [7] shows fair agreement, giving some confidence to the assumption that it is enough to include the $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ terms in order to control two-loop electroweak corrections to Δr.

- *Z-Boson Observables:* There is a new evaluation of non-factorizable QCD and electroweak corrections to the hadronic Z boson decay rates [12]. A negative correction to the Z hadronic width was found reducing the SM prediction for Γ_h by 0.59 MeV after summing over the four light quark flavors. The corresponding effect for the partial width $\Gamma(Z \to$ '$b\bar{b}$') has not been evaluated. This shift leads to an enhancement of the α_s value extracted from the electroweak data by 0.001.

2. Low-Energy Observables

Although there is now clear evidence for electroweak loop effects in high energy physics, it still remains a challenge to prove the validity of the Standard Model on the quantum level also at very low energies, ten orders of magnitude below the physics of the Z resonance. Two observables are of particular interest for electroweak physics: the anomalous magnetic moment of the muon and the electric dipole moment (EDM) of the neutron, d_n.

2.1. Anomalous Magnetic Moment of the Muon

One of the most precisely measured electroweak quantities is the anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)/2$. The error of the current experimental average [9]

$$a_\mu^{\rm exp} = 116592300(840) \times 10^{-11} \tag{3}$$

is going to be reduced by at least a factor of 20 in the new Brookhaven National Laboratory experiment E821 to $\Delta a_\mu^{exp} = \pm 40 \cdot 10^{-11}$. At that level of accuracy, the experiment will be sensitive to loop effects of the W and Z bosons, thus testing the full Standard Model at very low energies. The electroweak one-loop contribution to a_μ

$$a_\mu^{ew}(1-loop) = 195 \cdot 10^{-11} \tag{4}$$

has been supplemented with the corresponding two-loop corrections [10,11]

$$a_\mu^{ew}(2-loop) = -44 \cdot 10^{-11}\,. \tag{5}$$

In fig. 2 the complete subset of bosonic diagrams which contributes to the large logarithms $\ln \frac{M_W^2}{m_\mu^2}$ is depicted. Although only a small fraction of bosonic diagrams, they account for the numerically most important contribution to $a_\mu^{ew}(bos.)$. The diagrams which contain a closed fermion loop have been discussed in detail in [10,12]. The electroweak two-loop contribution turned out to be of the same size as the anticipated uncertainty of the BNL experiment, reducing the total electroweak contribution to

$$a_\mu^{ew} = 151(4) \cdot 10^{-11} \tag{6}$$

with a very small theoretical uncertainty.

It is mandatory to also include leading and higher order hadronic contributions to a_μ. The $\mathcal{O}(\alpha^3)$ hadronic contributions fall into two classes: vacuum polarization insertions into the photon propagator and the so-called light-by-light-contribution. The former can be calculated by using experimental data for the ratio $R(s) = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ and has recently be re-evaluated [13]

$$a_\mu^{had}(\text{higher order}) = -101(6) \cdot 10^{-11}\,. \tag{7}$$

For the light-by-light-contribution, however, there is no experimental data available. Instead, the Extended Nambu-Jona-Lasinio model has been used [14]. The latest evaluation yields [15]

$$a_\mu^{had}(\text{light}-\text{by}-\text{light}) = -79.2(15.4) \cdot 10^{-11}\,. \tag{8}$$

It should be kept in mind, however, that due to the model dependence it is difficult to give an error estimate.

2.2. Electric Dipole Moment of the Neutron

The EDM of the neutron is a quantity of great interest since, if non-zero, would be the first manifestation of CP-violation outside of the $K^0 - \bar{K}^0$ system. Current experimental bounds are already very stringent: $|d_n| < 1.1 \times 10^{-25} e \cdot cm$ [16]. In the Standard Model, EDM of elementary particles are highly suppressed, the first non-vanishing contribution for quarks arising only at the three-loop level. The corresponding final result for the

Standard Model calculation has been published in [19]. A particularly important issue which has been clarified in the explicit calculation of [19] was the rôle of the top quark. Here we report on details of the top contribution. Previous estimates [17] and the calculation in the Fermi theory [18] date back to the early 80s and were based on the assumption $M_W > m_t$. Since we know that the top quark is very heavy and that some electroweak observables are enhanced with m_t^2/M_W^2 a reinvestigation of the EDM of the neutron seems to be in order. The relevant diagrams with contributions to d_n from the top-quark are depicted in fig.1.

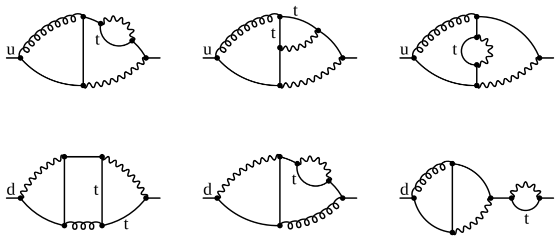

Figure 1. *Contribution of the top-quark to the EDM of up- and down-quarks, respectively. The coupling of the external photon to all charged lines has been omitted for brevity. Wiggled lines denote W bosons, spiral lines the Gluon. A sum over all light quarks in the loop is understood.*

The sum of the diagrams containing the top quark is separately finite and yields for the EDM of the down-and up-quark respectively:

$$\frac{d_d(top)}{e} = -\frac{m_d m_c^2 \alpha_s G_F^2 \tilde{\delta}}{48\pi^5} \times \times \left\{ -\frac{13}{3} + \frac{37}{27}\cdot\frac{\pi^2}{6} + \frac{59}{27} L_{bc} + \frac{37}{54} L_{bc}^2 \right\} \qquad (9)$$

The corresponding result for the up-quark is

$$\frac{d_u(top)}{e} = -\frac{m_u m_s^2 \alpha_s G_F^2 \tilde{\delta}}{48\pi^5} \times \times \left\{ \frac{95}{27}\left[\frac{\pi^2}{6} - \frac{1}{2} - \frac{1}{2} L_{bs}\right] - \frac{37}{108} L_{bs}^2 \right\} \qquad (10)$$

where $\tilde{\delta}$ denotes the CP-violating invariant

$$\tilde{\delta} = s_1^2 s_2 s_3 c_1 c_2 c_3 \delta \ . \qquad (11)$$

and $L_{ab} \equiv \ln(m_a^2/m_b^2)$. In eq.(10) and (11) terms suppressed with additional powers of light quark masses have been neglected. The main observation is that terms enhanced with m_t^2 are actually absent. This is true also in next order in the expansion in $1/M_W$, i.e. there are no terms of the type $m_t^2 m_c^2 m_d/M_W^6$ (for the down-quark) or $m_t^2 m_s^2 m_u/M_W^6$ (for the up-quark). In leading order, the top quark contribution does not depend on the top mass at all. Its numerical impact on d_n is not very important, and as a consequence the Standard Model contribution to the electric dipole moment of the neutron will be definitively unobservable for the experiments in the near future.

3. Conclusions

Higher order Standard Model loop calculations of the past few years have been reviewed. Theoretical progress generally matches the present and anticipated experimental accuracy of precision observables like e.g. M_W or a_μ. It is interesting to note that in most of the recent higher order loop calculations the method of asymptotic expansions (for a recent review see e.g. [20]) has been applied and proven to be a convenient tool for practical calculations. The theoretical uncertainties in electroweak physics are now dominated by the experimental error in $\sigma(e^+e^- \to Hadrons)$. Reduction of this error is crucial for improved sensitivity to new physics and more stringent bounds on M_H.

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REFERENCES

1. W. Hollik, KA-TP-14-1997, these proceedings, hep-ph/9708339.
2. L. Avdeev, J. Fleischer, S. M. Mikhailov, O. Tarasov, Phys. Lett. B 336 (1994) 560; E: Phys. Lett. B 349 (1995) 597.
3. K.G. Chetyrkin, J.H. Kühn, M. Steinhauser, Phys. Lett. B 351 (1995) 331.
4. A. Czarnecki, J.H. Kühn, Phys. Rev. Lett. 77 (1996) 3955
5. Precision Calculations for the Z Resonance, CERN 95-03 (1995), Eds. D. Bardin, W. Hollik, G. Passarino
6. G. Degrassi, P. Gambino, A. Vicini, Phys. Lett. B 383 (1996) 219; G. Degrassi, P. Gambino, A. Sirlin, Phys. Lett. B 394 (1997) 188
7. S. Bauberger, G. Weiglein, Nucl. Instrum. Meth. A 389 (1997) 318.
8. G. Degrassi, P. Gambino, M. Passera, A. Sirlin, CERN-TH-97-197 (1997), hep-ph/9708311.
9. Particle Data Group, Phys. Rev. **D54**, 1 (1996).
10. A. Czarnecki, B. Krause, and W. Marciano, Phys. Rev. **D52**, R2619 (1995);
11. A. Czarnecki, B. Krause, and W. Marciano, Phys. Rev. Lett. **76**, 3267 (1996).
12. A. Czarnecki and B. Krause, Nucl. Phys. B (Proc. Suppl.) **51C**, 148 (1996).
13. B. Krause, Phys. Lett. **B390**, 392 (1997).
14. J. Bijnens, E. Pallante, and J. Prades, Phys. Rev. Lett. **75**, 1447 (1995), Erratum: ibid. **75**, 3781 (1995); J. Bijnens, E. Pallante, and J. Prades, Nucl. Phys. **B474**, 379 (1996); M. Hayakawa, T. Kinoshita, and A. I. Sanda, Phys. Rev. Lett. **75**, 790 (1995); M. Hayakawa, T. Kinoshita, and A. I. Sanda, Phys. Rev. **D54**, 3137 (1996).
15. M. Hayakawa, T. Kinoshita, KEK-TH-530, hep-ph/9708227. (1996).
16. I. S. Altarev *et al.*, Phys. Lett. **B276**, 242 (1992); K. F. Smith *et al.*, Phys. Lett. **B234**, 191 (1990).
17. E. P. Shabalin, Sov. J. Nucl. Phys. **31**, 864 (1980).
18. I. B. Khriplovich, Sov. J. Nucl. Phys. **44**, 659 (1986).
19. A. Czarnecki and B. Krause, Phys. Rev. Lett. **78**, 4339 (1997).
20. V. A. Smirnov, Mod. Phys. Lett. **A10**, (1995) 1485.

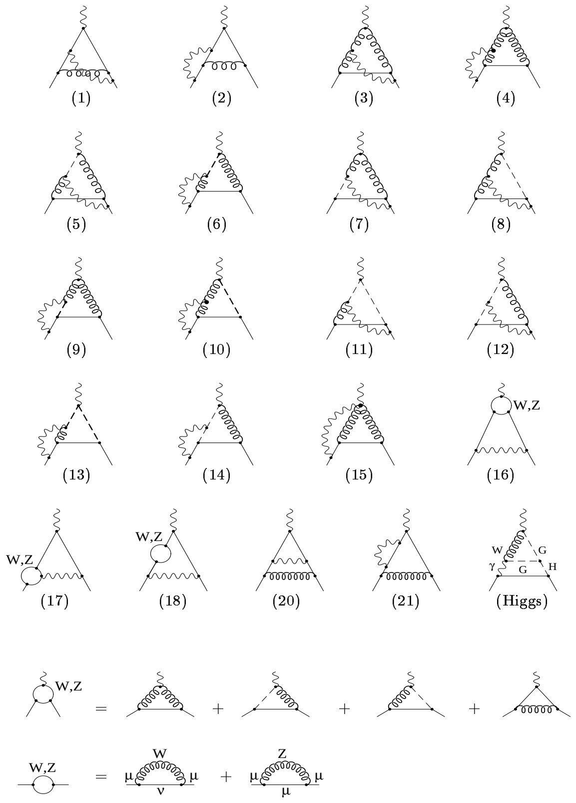

Figure 2. *Electroweak two-loop corrections to the anomalous magnetic moment of the muon: the complete set of bosonic diagrams with contributions to the leading logarithms* $\ln \frac{M_W^2}{m_\mu^2}$. *Wiggled lines denote photons, spiral lines the massive vector bosons* W, Z *and dashed lines the Goldstone bosons.*