MPI/PhT/97-51
TTP97-44[†]
ZU-TH-17/97
TUM-HEP-285/97
IFT-12/97
hep-ph/9711280

$|\Delta F| = 1$ Nonleptonic Effective Hamiltonian in a Simpler Scheme

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Abstract

We consider $|\Delta F| = 1 \quad (F = S, C \text{ or } B)$ nonleptonic effective hamiltonian in a renormalization scheme which allows to consistently use fully anticommuting γ_5 at any number of loops, but at the leading order in the Fermi coupling G_F. We calculate two-loop anomalous dimensions and one-loop matching conditions for the effective operators in this scheme. Finally, we transform our results to one of the previously used renormalization schemes, and find agreement with the original calculations.

[†] The complete postscript file of this preprint, including figures, is available via anonymous ftp at www-ttp.physik.uni-karlsruhe.de (129.13.102.139) as /ttp97-44/ttp97-44.ps or via www at http://www-ttp.physik.uni-karlsruhe.de/cgi-bin/preprints.

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1. Introduction

Performing multiloop calculations is usually most convenient within the framework of dimensional regularization. However, when chiral fermions are considered, one often encounters technical difficulties with extending γ_5 to $D = 4 - 2\epsilon$ dimensions. These difficulties are related to Dirac traces like

$$T_{\mu\nu\rho\sigma} = Tr(\gamma_\alpha \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma^\alpha \gamma_5). \tag{1}$$

Such a trace is not uniquely defined in D dimensions, so long as one assumes that γ_5 anticommutes with all the other gamma matrices. One can easily check that the totally antisymmetric part of $T_{\mu\nu\rho\sigma}$ equals to

$$T_{[\mu\nu\rho\sigma]} = -D\, Tr(\gamma_{[\mu} \gamma_\nu \gamma_\rho \gamma_{\sigma]} \gamma_5) \tag{2}$$

when cyclicity of the trace and $\{\gamma_\alpha, \gamma_5\} = 0$ is used, or to

$$T_{[\mu\nu\rho\sigma]} = (D-8)\, Tr(\gamma_{[\mu} \gamma_\nu \gamma_\rho \gamma_{\sigma]} \gamma_5) \tag{3}$$

when the two contracted γ_α's are brought to each other through the whole string of other gamma matrices with use of $\{\gamma_\alpha, \gamma_\beta\} = 2g_{\alpha\beta}$.

These two results agree with each other only when $D = 4$ or when the whole trace vanishes. Neither ambiguous results for $D \neq 4$ nor the vanishing trace can be accepted in the Dirac algebra in a consistent dimensional regularization scheme. The Dirac algebra in an acceptable scheme must satisfy the following requirements:

- (i) It should give unique results independently of whether some diagram is considered as a subdiagram, and independently of the order in which subdiagrams are calculated. Otherwise subdivergences could not be properly subtracted in multiloop diagrams.

- (ii) When $D \to 4$, its rules must analytically tend to the rules of 4-dimensional Dirac algebra.

Several schemes satisfying these requirements have been proposed [1]–[7]. What they have in common is that their use requires complicated or tedious algebraic manipulations.

Calculations become much simpler if occurrence of traces with γ_5 can be avoided in a given problem. Then, one can define the D-dimensional γ_5 as

$$\gamma_5 = i^{\frac{(D-1)(D-2)}{2}} \gamma^0 ... \gamma^{D-1}. \tag{4}$$

With such an explicit definition at hand, the requirement (i) is automatically satisfied. The requirement (ii) would not be satisfied for diagrams containing traces with γ_5. (The trace (2) would then identically vanish for $D \neq 4$). However, in the absence of traces with γ_5, the requirement (ii) is satisfied, too. The matrix defined in eqn. (4) anticommutes with all the remaining gamma matrices, and its square is equal to unity.

Flavor changing $|\Delta F| = 1 \quad (F = S, C \text{ or } B)$ processes in the Standard Model (SM) are described by Feynman diagrams with no traces containing γ_5. This statement is true only at the leading order in weak interactions, but to all orders in QCD and QED. While higher orders in weak interactions are usually negligible, one is often interested in resumming large logarithms $\ln(M_W^2/m_{light}^2)$ from all orders of the QCD perturbation series. This is most conveniently done with use of an effective hamiltonian built out of light fields only. It is possible to define this hamiltonian in such a way that no traces with γ_5 are introduced.

In the standard approach to $|\Delta F| = 1$ processes [8], the applied form of the effective hamiltonian induced appearance of traces with γ_5. It was harmless at one loop, but caused many difficulties in the next-to-leading order (NLO) 2-loop calculations [9, 10, 11, 12, 13]. Applying the same scheme in even higher order computations would be very inefficient. Instead, it is better to start from the outset with a modified form of the effective hamiltonian which is free of the γ_5 problem.[2]

In the present paper, we present our calculation of the next-to-leading QCD corrections to the $|\Delta F| = 1$ nonleptonic effective hamiltonian. We use an operator basis in which traces with γ_5 do not occur. Consequently, we are allowed to use fully anticommuting γ_5 to all orders in QCD. Such an approach is much more natural and useful than using the "standard" operator basis [9, 10, 11]. In our case, the two-loop calculation can be made completely automatic and almost trivial. We do not face subtleties related to the use of Fierz symmetry arguments in D dimensions, as it was the case when fully anticommuting γ_5 was used in the "standard" operator basis. The calculation is much simpler than in the 't Hooft-Veltman scheme [1] for γ_5, too. In this latter scheme, D-dimensional Lorentz invariance is violated, which causes appearance of a large number of complicated evanescent operators.

Our main motivation for undertaking the present work was that its outcome was directly applicable in our 3-loop calculation of NLO corrections to $b \to X_s\gamma$ decay [15, 16]. However,

[2] A similar issue concerning renormalization of four-quark operators has been discussed in ref. [14], in the context of QCD sum rules.

we find it useful to discuss the nonleptonic hamiltonian separately from 3-loop $b \to X_s \gamma$ technicalities.

Our calculation proceeds along the standard lines [9, 10, 11]. We evaluate tree-level and one-loop matching conditions for the operators in the effective hamiltonian, as well as their one- and two-loop anomalous dimensions. We only use another basis of effective operators. Even with our results at hand, it is quite nontrivial to verify whether they agree with the previous calculations [9, 10, 11, 17]. Such a comparison is essential because of the phenomenological relevance of the considered quantities. Therefore, we devote a sizable part of the present article to show how this verification is performed.

Our paper is organized as follows. In the next section, the effective hamiltonian is introduced. In section 3, we calculate its Wilson coefficients and the necessary anomalous dimension matrix. Section 4 is devoted to showing how our results can be transformed to the previously used renormalization scheme, and to verifying that we can indeed confirm results of the previous calculations. The appendix contains a list of the nonphysical counterterms relevant in our calculation.

2. The effective hamiltonian

For definiteness, we shall consider here the $\Delta B = -\Delta S = 1$ nonleptonic effective hamiltonian,[3] neglecting the small CKM matrix element V_{ub}. Generalization to other $|\Delta F| = 1$ processes and/or to processes where V_{ub} cannot be neglected is straightforward.

The effective hamiltonian is built out of operators P_i multiplied by their Wilson coefficients C_i

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i \left[P_i \quad + \quad (\text{counterterms})_i \right], \tag{5}$$

where G_F is the Fermi coupling.

The specific structure of the operators P_i is determined from the requirement that the hamiltonian reproduces the Standard Model $\Delta B = -\Delta S = 1$ nonleptonic amplitudes at the leading order in (external momenta)$/M_W$ and in the electroweak gauge couplings, but to all orders in strong interactions. If this requirement was applied to off-shell amplitudes, then EOM-vanishing operators, i.e. operators which vanish by the QCD equations of motion would need to be present among P_i. Here, we do not include such operators. Consequently, our $\mathcal{H}_{eff}$ reproduces only on-shell SM amplitudes (both partonic and hadronic ones) [18].

[3]The hamiltonian is assumed to conserve flavors other than B and S.

However, we still require renormalizability of 1PI off-shell Green's functions generated by $\mathcal{H}_{eff}$. Therefore, some EOM-vanishing operators are present among counterterms.

The above requirements imply that the set of operators $\{P_i\}$ must be closed under QCD renormalization, up to counterterms proportional to nonphysical operators, i.e. operators whose renormalized matrix elements between physical states vanish. Such nonphysical operators can be either EOM-vanishing or evanescent (i.e. algebraically vanishing in four dimensions) [19, 20]. Potentially, one could also encounter BRS-exact operators (i.e. BRS-variations of some other operators) as nonphysical counterterms [19, 21]. However, they turn out to be unnecessary up to three loops for the operators P_i considered below.

A set of operators P_i which satisfies the imposed requirements consists of dimension-six operators $P_1, ..., P_6$

$$\begin{aligned} P_1 = & \ (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L), \\ P_2 = & \ (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L), \\ P_3 = & \ (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q} \gamma^\mu q), \\ P_4 = & \ (\bar{s}_L \gamma_\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma^\mu T^a q), \\ P_5 = & \ (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L) \textstyle\sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q), \\ P_6 = & \ (\bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L) \textstyle\sum_q (\bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q), \end{aligned} \tag{6}$$

and one dimension-five operator proportional to[4]

$$(\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}, \tag{7}$$

called the chromomagnetic moment operator. Here, T^a is the $SU(3)_{color}$ generator, and $G^a_{\mu\nu}$ stands for the gluon stress-energy tensor.

Certainly, the above choice of the operator basis is not unique. One could choose other linear combinations of the operators and/or redefine them by adding some nonphysical ones.

Let us briefly argue why all the operators given in eqns. (6) and (7) must be present in $\mathcal{H}_{eff}$. The operator P_2 must be present because it enters the matching condition shown in fig. 1. In this figure, the W-boson exchange is approximated by the effective Fermi weak interaction of quarks.

All the other operators in eqns. (6) and (7) are generated from P_2 in the process of QCD renormalization of various amplitudes: The diagrams with P_2-vertices shown in fig. 2 require counterterms proportional to P_1, P_2 and to the nonphysical evanescent operator $E_1^{(1)}$. (The relevant nonphysical operators are collected together in the appendix.) All the remaining

[4]The s-quark mass is neglected here. Therefore, s_R decouples from flavor-changing processes.

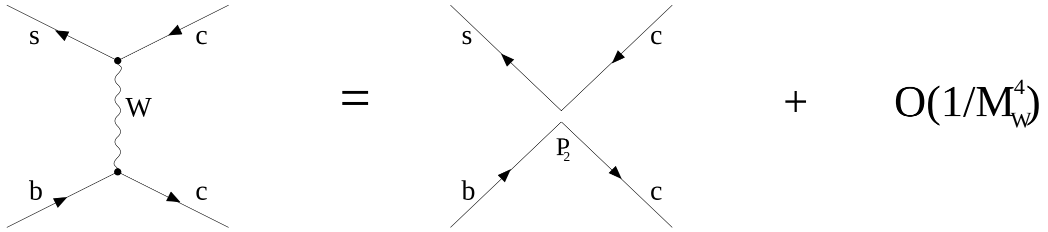

Figure 1: Tree-level matching condition.

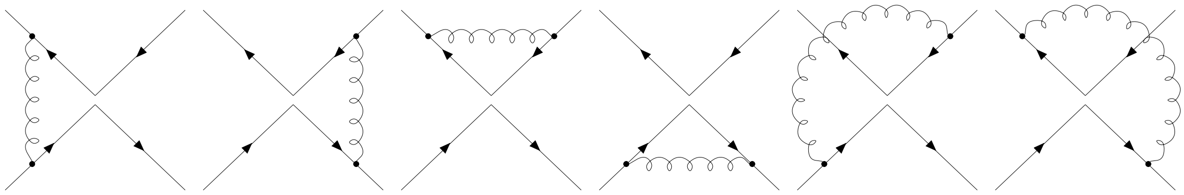

Figure 2: One-loop current-current diagrams.

divergent 1-loop 1PI diagrams with P_2-insertion are shown in fig. 3. They are renormalized by a single counterterm proportional to

$$(\bar{s}_L T^a \gamma_\mu b_L) D_\nu G^{a\;\mu\nu} \tag{8}$$

where D_ν is the QCD covariant derivative. For convenience in the future two-loop calculation, we write the above operator as a linear combination of P_4 and the EOM-vanishing nonphysical operator N_1 (see the appendix)

$$N_1 = \frac{1}{g}(\bar{s}_L T^a \gamma_\mu b_L) D_\nu G^{a\;\mu\nu} \quad + \quad P_4. \tag{9}$$

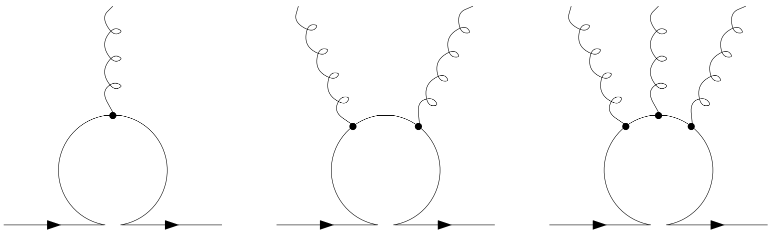

Figure 3: One-loop penguin diagrams with no Dirac trace.

Inserting P_4 into the diagrams of fig. 2, one finds that counterterms proportional to P_5 and P_6 are necessary. Finally, the same diagrams with insertions of P_6 require a counterterm proportional to P_3.

The set of operators $\{P_1, ..., P_6\}$ is closed under one-loop QCD renormalization, up to nonphysical counterterms. A set which closes to all orders in QCD is obtained by including only one more operator, i.e. the chromomagnetic moment operator (7). The fact that no more physical operators of dimension 5 and 6 need to be present in $\mathcal{H}_{eff}$ can be shown analogously to ref. [22] by writing all the possible $\Delta B = -\Delta S = 1$ operators allowed by gauge symmetry and reducing them by the equations of motion.

In the standard approach [8, 9, 10, 11], triple products of gamma matrices occuring in P_5 and P_6 have been reduced with help of the four-dimensional identity

$$\gamma_\mu \gamma_\nu \gamma_\rho \underset{D=4}{=\!=\!=} g_{\mu\nu}\gamma_\rho + g_{\nu\rho}\gamma_\mu - g_{\mu\rho}\gamma_\nu + i\epsilon_{\mu\nu\rho\sigma}\gamma^\sigma \gamma_5. \tag{10}$$

Doing this step in a two-loop calculation requires introducing several more evanescent operators and leads to problematic traces with γ_5. Therefore, we do not perform this step, and just leave the operators P_5 and P_6 as they stand.[5] We also find it convenient to leave their color structure as it is, rather than to remove the generators T^a with help of the identity $T^a_{\alpha\beta} T^a_{\gamma\delta} = \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta} - \frac{1}{6}\delta_{\alpha\beta}\delta_{\gamma\delta}$.

In the remaining part of this paper, we shall concentrate on finding the Wilson coefficients and the anomalous dimension matrix of the operators $P_1, ..., P_6$, leaving aside the chromomagnetic moment operator (7). The latter dimension-five operator does not require counterterms proportional to dimension-six operators. Thus, it does not affect renormalization group evolution of their Wilson coefficients. Consequently, the operators $P_1, ..., P_6$ can be considered separately, which makes the necessary formulae more compact. We discuss the chromomagnetic moment operator together with its photonic counterpart up to three loops in other publications [15, 16].

3. Matching conditions for the Wilson coefficients and the anomalous dimension matrix

We now turn to the values of the coefficients $C_1, ..., C_6$. They are found by matching the SM and effective theory amplitudes perturbatively in α_s. When performing the matching, one sets the renormalization scale μ close to M_W, in order to reduce size of $\ln(M_W/\mu)$ which could otherwise worsen the perturbative expansion. In the following, we use the $\overline{MS}$ scheme with fully anticommuting γ_5, and take $\mu = M_W$ as the matching scale.

The tree-level matching condition is shown in fig. 1. The one loop matching conditions for the coefficients C_1 and C_2 are found by requiring that the one-loop SM diagrams in fig. 4

[5] An essentially identical procedure was used in the calculation of one-loop corrections to the coefficient functions of four-quark operators appearing in the operator product expansion of two quark currents [23].

be reproduced by the effective theory diagrams in fig. 2.[6] The one-loop contribution to the coefficient C_4 can be found by on-shell matching of the 1PR amplitudes presented in fig. 5. The coefficients C_3, C_5 and C_6 acquire no one-loop contributions.

The obtained coefficients $C_i(\mu = M_W)$ read

$$\begin{aligned} C_1(M_W) &= \frac{15\alpha_s}{4\pi} + \mathcal{O}(\alpha_s^2), &(11)\\ C_2(M_W) &= 1 + \mathcal{O}(\alpha_s^2), &(12)\\ C_4(M_W) &= \frac{\alpha_s}{4\pi}\left[E(x) - \frac{2}{3}\right] + \mathcal{O}(\alpha_s^2), &(13)\\ C_3(M_W) &= C_5(M_W) = C_6(M_W) = \mathcal{O}(\alpha_s^2), &(14) \end{aligned}$$

where

$$E(x) = \frac{x(18 - 11x - x^2)}{12(1-x)^3} + \frac{x^2(15 - 16x + 4x^2)}{6(1-x)^4}\ln x - \frac{2}{3}\ln x. \qquad (15)$$

and $x = m_t^2/M_W^2$.

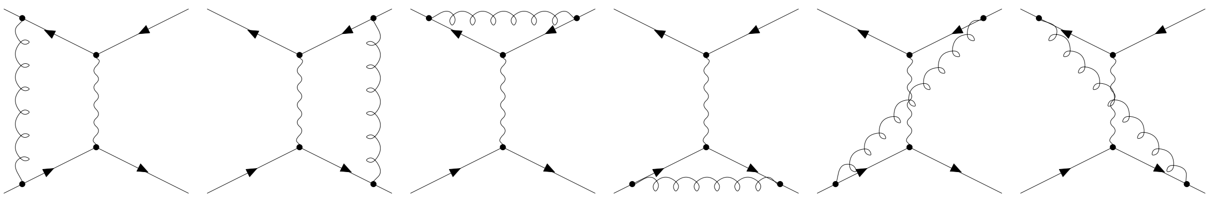

Figure 4: One-loop SM current-current diagrams.

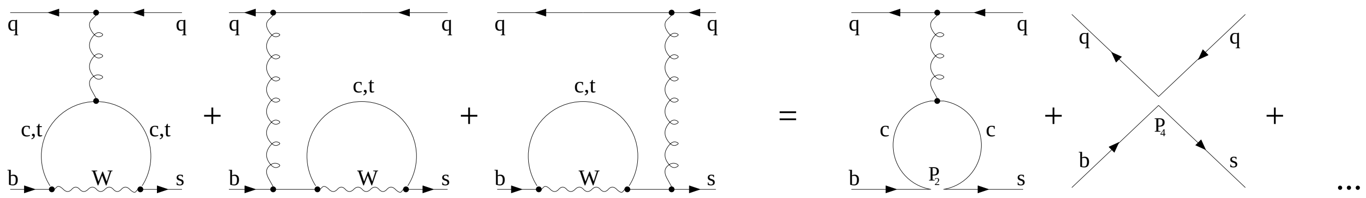

Figure 5: One-loop matching for C_4. Dots stand for $\mathcal{O}(1/M_W^4)$, a counterterm, and for a diagram with insertion of the chromomagnetic moment operator given in eqn. (7).

Many flavor changing processes occur at energy scales $m_{light} << M_W$. Perturbative expansion of their amplitudes is efficient when the renormalization scale μ is close to m_{light} rather than to M_W. Wilson coefficients C_i at $\mu \sim m_{light}$ are found from $C_i(M_W)$ with help of the Renormalization Group Equations (RGE)

$$\mu\frac{d}{d\mu}C_i(\mu) = \sum_{j=1}^{6} C_j(\mu)\gamma_{ji}(\mu). \qquad (16)$$

[6]Tree diagrams with counterterms need to be added on both the SM and the effective theory sides before the matching is performed.

Here, $\hat{\gamma}(\mu)$ is the anomalous dimension matrix which has the following perturbative expansion

$$\hat{\gamma}(\mu) = \frac{\alpha_s(\mu)}{4\pi}\hat{\gamma}^{(0)} \; + \; \frac{\alpha_s^2(\mu)}{(4\pi)^2}\hat{\gamma}^{(1)} \; + \; \mathcal{O}(\alpha_s^3). \tag{17}$$

The anomalous dimension matrix in the MS or $\overline{MS}$ schemes is found from one- and two-loop counterterms in the effective theory, according to the following relations:

$$\hat{\gamma}^{(0)} \;\; = \;\; 2\hat{a}^{11}, \tag{18}$$

$$\hat{\gamma}^{(1)} \;\; = \;\; 4\hat{a}^{12} - 2\hat{b}\hat{c}. \tag{19}$$

The matrices $\hat{a}^{11}$, $\hat{a}^{12}$ and $\hat{b}$ in the above equations parameterize the MS-scheme counterterms proportional to $C_1, ..., C_6$ in eqn. (5)

$$\begin{aligned}
(\text{counterterms})_i \;\; &= \;\; \frac{\alpha_s}{4\pi\epsilon}\left[\sum_{k=1}^{6} a_{ik}^{11} P_k \; + \; \sum_{k=1}^{4} b_{ik} E_k^{(1)} \; + \; n_i N_1\right] \; + \; \frac{\alpha_s^2}{(4\pi)^2}\sum_{k=1}^{6}\left(\frac{1}{\epsilon^2}a_{ik}^{22} + \frac{1}{\epsilon}a_{ik}^{12}\right) P_k \\
&+ \;\; (\text{two-loop chromomagnetic counterterm}) \\
&+ \;\; (\text{two-loop nonphysical counterterms}) + \mathcal{O}(\alpha_s^3).
\end{aligned} \tag{20}$$

The matrix $\hat{c}$ occurs in the one-loop matrix elements of evanescent operators. Let us denote by $\langle E_k^{(1)}\rangle_{1loop}$ any one-loop amplitude with an insertion of some evanescent operator $E_k^{(1)}$. Dependently of what external lines are chosen, such an amplitude can be given by the diagrams in fig. 2 or in figs. 3 and 6. Pole parts of such amplitudes are proportional to evanescent operators. The remaining parts in the limit $D \to 4$ equal to linear combinations of tree-level matrix elements of the physical operators and N_1.

$$\begin{aligned}
\langle E_k^{(1)}\rangle_{1loop} \;\; &= \;\; -\sum_{j=0}^{4}\frac{1}{\epsilon}\left[d_{kj}\langle E_j^{(1)}\rangle_{tree} + e_{kj}\langle E_j^{(2)}\rangle_{tree}\right] \; - \; \sum_{i=0}^{6} c_{ki}\langle P_i\rangle_{tree} \\
&\quad + x_k\langle(\bar{s}_L\sigma^{\mu\nu}T^a b_R)G_{\mu\nu}^a\rangle_{tree} \; - \; \tilde{n}_k\langle N_1\rangle_{tree} \; + \; \mathcal{O}(\epsilon).
\end{aligned} \tag{21}$$

The coefficients in these linear combinations (which define the matrix $\hat{c}$) are independent of what particular external lines are chosen. However, recovering the matrix $\hat{c}$ usually requires considering several choices of the external lines, so that all the potentially relevant operators can enter into eqn. (21) with nonvanishing tree-level matrix elements.

In ref. [20], one can find a more detailed explanation why evanescent operators affect the anomalous dimension matrix precisely in the manner we have just described.

The matrices $\hat{a}^{11}$, $\hat{a}^{12}$, $\hat{b}$ and $\hat{c}$ which determine $\hat{\gamma}^{(0)}$ and $\hat{\gamma}^{(1)}$ are found by evaluating various one- and two-loop Feynman diagrams with insertions of the operators P_i and $E_k^{(1)}$. The one-loop diagrams are shown in figs. 2, 3 and 6. All the two-loop diagrams we calculate

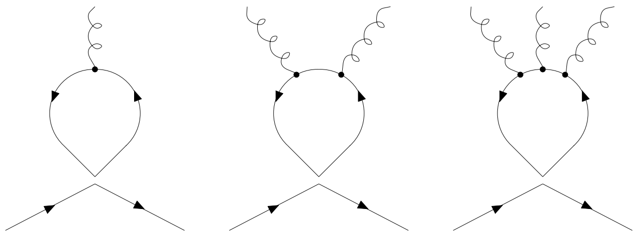

Figure 6: One-loop penguin diagrams with Dirac traces. The traces contain no γ_5.

(where only P_i need to be inserted) are identical to 1PI parts of the two-loop diagrams shown in the figures of ref. [10]. Most of them can be obtained from the one-loop ones by adding an additional internal gluon.

We evaluate pole parts of these diagrams using the method we have described in refs. [24, 25] where a common mass parameter was used as an infrared regulator. Next, we decompose the obtained ultraviolet divergences into terms proportional to the physical and nonphysical operators. This gives us the necessary renormalization constants and, in the end, the anomalous dimension matrix for the operators $P_1, ..., P_6$.

We find (both in the MS and $\overline{MS}$ schemes)

$$\hat{\gamma}^{(0)} = \begin{bmatrix} -4 & \frac{8}{3} & 0 & -\frac{2}{9} & 0 & 0 \\ 12 & 0 & 0 & \frac{4}{3} & 0 & 0 \\ 0 & 0 & 0 & -\frac{52}{3} & 0 & 2 \\ 0 & 0 & -\frac{40}{9} & -\frac{160}{9} + \frac{4}{3}f & \frac{4}{9} & \frac{5}{6} \\ 0 & 0 & 0 & -\frac{256}{3} & 0 & 20 \\ 0 & 0 & -\frac{256}{9} & -\frac{544}{9} + \frac{40}{3}f & \frac{40}{9} & -\frac{2}{3} \end{bmatrix} \tag{22}$$

and

$$\hat{\gamma}^{(1)} = \begin{bmatrix} -\frac{145}{3} + \frac{16}{9}f & -26 + \frac{40}{27}f & -\frac{1412}{243} & -\frac{1369}{243} & \frac{134}{243} & -\frac{35}{162} \\ -45 + \frac{20}{3}f & -\frac{28}{3} & -\frac{416}{81} & \frac{1280}{81} & \frac{56}{81} & \frac{35}{27} \\ 0 & 0 & -\frac{4468}{81} & -\frac{29129}{81} - \frac{52}{9}f & \frac{400}{81} & \frac{3493}{108} - \frac{2}{9}f \\ 0 & 0 & -\frac{13678}{243} + \frac{368}{81}f & -\frac{79409}{243} + \frac{1334}{81}f & \frac{509}{486} - \frac{8}{81}f & \frac{13499}{648} - \frac{5}{27}f \\ 0 & 0 & -\frac{244480}{81} - \frac{160}{9}f & -\frac{29648}{81} - \frac{2200}{9}f & \frac{23116}{81} + \frac{16}{9}f & \frac{3886}{27} + \frac{148}{9}f \\ 0 & 0 & \frac{77600}{243} - \frac{1264}{81}f & -\frac{28808}{243} + \frac{164}{81}f & -\frac{20324}{243} + \frac{400}{81}f & -\frac{21211}{162} + \frac{622}{27}f \end{bmatrix}. \tag{23}$$

where f is the number of active flavors (equal to 5 for $\mu \in [m_b, M_W]$).

In the end of this section, we shall use the obtained anomalous dimension matrix to find explicit expressions for the Wilson coefficients

$$C_i(\mu) = C_i^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_i^{(1)}(\mu) + \mathcal{O}(\alpha_s^2) \tag{24}$$

at the renormalization scale $\mu_b \sim m_b$ which is appropriate for studying b-quark decays. Using the general solution of the next-to-leading RGE (given eg. in ref. [9]), we find

$$C_i^{(0)}(\mu_b) \;=\; \sum_{j=1}^{6} A_{ij}\eta^{a_j}, \tag{25}$$

$$C_i^{(1)}(\mu_b) \;=\; \sum_{j=1}^{6}\left[B_{ij} + B'_{ij}\eta + B^E_{ij}E(x)\eta\right]\eta^{a_j}. \tag{26}$$

where $\eta = \alpha_s(M_W)/\alpha_s(\mu_b)$ and

$$\vec{a} \simeq \begin{bmatrix} \frac{6}{23} & -\frac{12}{23} & 0.4086 & -0.4230 & -0.8994 & 0.1456 \end{bmatrix}, \tag{27}$$

$$\hat{A} \simeq \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{63} & -\frac{1}{27} & -0.0659 & 0.0595 & -0.0218 & 0.0335 \\ \frac{1}{21} & \frac{1}{9} & 0.0237 & -0.0173 & -0.1336 & -0.0316 \\ -\frac{1}{126} & \frac{1}{108} & 0.0094 & -0.0100 & 0.0010 & -0.0017 \\ -\frac{1}{84} & -\frac{1}{36} & 0.0108 & 0.0163 & 0.0103 & 0.0023 \end{bmatrix}, \tag{28}$$

$$\hat{B} \simeq \begin{bmatrix} 5.9606 & 1.0951 & 0 & 0 & 0 & 0 \\ 1.9737 & -1.3650 & 0 & 0 & 0 & 0 \\ -0.5409 & 1.6332 & 1.6406 & -1.6702 & -0.2576 & -0.2250 \\ 2.2203 & 2.0265 & -4.1830 & -0.7135 & -1.8215 & 0.7996 \\ 0.0400 & -0.1860 & -0.1669 & 0.1887 & 0.0201 & 0.0304 \\ -0.2614 & -0.1918 & 0.4197 & 0.0295 & 0.1474 & -0.0640 \end{bmatrix}, \tag{29}$$

$$\hat{B}' \simeq \begin{bmatrix} 2.0394 & 5.9049 & 0 & 0 & 0 & 0 \\ 1.3596 & -1.9683 & 0 & 0 & 0 & 0 \\ 0.0647 & 0.2187 & -0.2979 & -0.6218 & 0.1880 & -0.1318 \\ 0.0971 & -0.6561 & 0.1071 & 0.1806 & 1.1520 & 0.1242 \\ -0.0162 & -0.0547 & 0.0423 & 0.1041 & -0.0085 & 0.0067 \\ -0.0243 & 0.1640 & 0.0489 & -0.1700 & -0.0889 & -0.0091 \end{bmatrix}, \tag{30}$$

$$\hat{B}^E \simeq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1933 & 0.1579 & 0.1428 & -0.1074 \\ 0 & 0 & 0.0695 & -0.0459 & 0.8752 & 0.1012 \\ 0 & 0 & 0.0274 & -0.0264 & -0.0064 & 0.0055 \\ 0 & 0 & 0.0317 & 0.0432 & -0.0675 & -0.0074 \end{bmatrix}. \tag{31}$$

For $\alpha_s(M_Z) = 0.118$ and $\mu = 5$ GeV, the ratio η at NLO is $\eta = \alpha_s(M_W)/\alpha_s(\mu_b) \simeq 0.1203/0.2117 \simeq 0.568$, and

$$\begin{aligned} \vec{C}(\mu) &= \begin{pmatrix} -0.480 & 1.023 & -0.0045 & -0.0640 & 0.00043 & 0.00091 \end{pmatrix} \\ &\quad + \tfrac{\alpha_s(\mu)}{4\pi} \begin{pmatrix} 12.12 & -0.965 & 0.0230 & -0.779 & -0.0093 & -0.0083 \end{pmatrix} + \mathcal{O}(\alpha_s^2) \\ &= \begin{pmatrix} -0.276 & 1.007 & -0.0041 & -0.0771 & 0.00028 & 0.00077 \end{pmatrix} + \mathcal{O}(\alpha_s^2), \end{aligned} \tag{32}$$

where $m_t = 175$ GeV has been used.

When processes taking place at energy scales much lower than m_b are considered, the b-quark must be decoupled at $\mu \sim m_b$. The same refers to the c-quark at $\mu \sim m_c$ in the case of kaon mixing or decays. The number of active flavors f in the anomalous dimension matrix (22)–(23) changes at the heavy quark thresholds. One-loop matching between effective theories needs to be performed then (see eg. ref. [26]). Explicit formulae for the Wilson coefficients become more complex in such cases.

Having evolved the effective hamiltonian to a low-energy scale, one calculates its matrix element between some physical states of interest. The actual way of calculating the matrix element depends on the process under consideration. One must always make sure that the same renormalization scheme is used in the renormalization group evolution and in evaluating the matrix element.

We shall not consider any examples of calculating matrix elements here. Instead, in the next section, we shall demonstrate that our anomalous dimension matrix is in agreement with the one found previously in the standard operator basis. Consequently, phenomenological results obtained in our basis cannot be different from what is already known. The advantage of introducing our new basis is not improving the existing two-loop phenomenology, but rather forming a much more convenient starting point for even higher loop computations like the one in refs. [15, 16].

4. Transformation of the anomalous dimension matrix to the "standard" basis

The present section is devoted to demonstrating that the anomalous dimension matrix (22)–(23) found by us in the new operator basis (6) agrees with the one found previously in the following "standard" basis of operators [8, 9, 10, 11]:

$$
\begin{aligned}
O_1 =&\ (\bar{s}^\alpha_L \gamma_\mu c^\beta_L)(\bar{c}^\beta_L \gamma^\mu b^\alpha_L),\\
O_2 =&\ (\bar{s}^\alpha_L \gamma_\mu c^\alpha_L)(\bar{c}^\beta_L \gamma^\mu b^\beta_L),\\
O_3 =&\ (\bar{s}^\alpha_L \gamma_\mu b^\alpha_L) \textstyle\sum_q (\bar{q}^\beta_L \gamma^\mu q^\beta_L),\\
O_4 =&\ (\bar{s}^\alpha_L \gamma_\mu b^\beta_L) \textstyle\sum_q (\bar{q}^\beta_L \gamma^\mu q^\alpha_L),\\
O_5 =&\ (\bar{s}^\alpha_L \gamma_\mu b^\alpha_L) \textstyle\sum_q (\bar{q}^\beta_R \gamma^\mu q^\beta_R),\\
O_6 =&\ (\bar{s}^\alpha_L \gamma_\mu b^\beta_L) \textstyle\sum_q (\bar{q}^\beta_R \gamma^\mu q^\alpha_R),
\end{aligned}
\tag{33}
$$

supplemented with the following evanescent operators:

$$
\begin{aligned}
O_1^E = &\ (\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} c_L^\beta)(\bar{c}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} b_L^\alpha) + (-16+4\epsilon)\ O_1, \\
O_2^E = &\ (\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} c_L^\alpha)(\bar{c}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} b_L^\beta) + (-16+4\epsilon)\ O_2, \\
O_3^E = &\ (\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L^\alpha) \textstyle\sum_q (\bar{q}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q_L^\beta) + (-16+4\epsilon)\ O_3, \\
O_4^E = &\ (\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L^\beta) \textstyle\sum_q (\bar{q}_L^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q_L^\alpha) + (-16+4\epsilon)\ O_4, \\
O_5^E = &\ (\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L^\alpha) \textstyle\sum_q (\bar{q}_R^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q_R^\beta) + (-4-4\epsilon)\ O_5, \\
O_6^E = &\ (\bar{s}_L^\alpha \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} b_L^\beta) \textstyle\sum_q (\bar{q}_R^\beta \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q_R^\alpha) + (-4-4\epsilon)\ O_6.
\end{aligned}
\tag{34}
$$

Such evanescent operators are necessary to renormalize one-loop current-current diagrams (fig. 2) with insertions of the operators $O_1, ..., O_6$. We have found their explicit form by imposing the requirements given in eqns. (4.3) and (4.8) of ref. [10] up to $\mathcal{O}(\epsilon^2)$.

We shall pass from the new operator basis (6,62) to the old one (33,34) in a series of subsequent redefinitions of operators. First, we shall redefine the physical operators by adding some evanescent ones to them. Next, some of the evanescent operators will be redefined by adding "$\epsilon \times$ (physical operators)" to them. The final (and easiest) step will be an ϵ-independent linear transformation of operators.

Before going into details, let us mention how the renormalization constants and anomalous dimensions transform when passing from one operator basis to another. Let us start with a set of physical operators $\{P_i\}$ and a set of evanescent ones $\{E_k^{(1)}\}$. The quantities we would like to consider are the one- and two-loop anomalous dimensions $\hat{\gamma}^{(0)}$ and $\hat{\gamma}^{(1)}$, the one-loop renormalization constants $\hat{b}$ and $\hat{d}$, as well as the matrix $\hat{c}$ (see eqns. (20) and (21)).

Suppose we want to pass to a new (primed) operator basis which differs from the initial one by adding some linear combinations of evanescent operators to the physical ones

$$
P'_i = P_i + \sum_k W_{ik} E_k^{(1)}, \qquad\qquad E'^{(1)}_k = E_k^{(1)}. \tag{35}
$$

The quantities under consideration transform as follows:

$$
\hat{\gamma}'^{(0)} = \hat{\gamma}^{(0)}, \qquad\qquad \hat{\gamma}'^{(1)} = \hat{\gamma}^{(1)} - \left[\hat{W}\hat{c},\ \hat{\gamma}^{(0)}\right] - 2\beta_0 \hat{W}\hat{c}, \tag{36}
$$

$$
\hat{b}' = \hat{b} + \hat{W}\hat{d} - \frac{1}{2}\hat{\gamma}^{(0)}\hat{W}, \qquad\qquad \hat{c}' = \hat{c}, \qquad\qquad \hat{d}' = \hat{d}. \tag{37}
$$

where $\beta_0 = 11 - \frac{2}{3} f$ is the well-known one-loop β-function coefficient in QCD.

Next, we redefine the evanescent operators by adding "$\epsilon \times$ (physical operators)" to them

$$
P''_i = P'_i, \qquad\qquad E''^{(1)}_k = E'^{(1)}_k + \epsilon \sum_i U_{ki} P'_i. \tag{38}
$$

The corresponding transformations read

$$\hat{\gamma}''^{(0)} = \hat{\gamma}'^{(0)}, \qquad \hat{\gamma}''^{(1)} = \hat{\gamma}'^{(1)} + \left[\hat{b}'\hat{U},\ \hat{\gamma}'^{(0)}\right] + 2\beta_0\hat{b}'\hat{U}, \tag{39}$$

$$\hat{b}'' = \hat{b}', \qquad \hat{c}'' = \hat{c}' + \frac{1}{2}\hat{U}\hat{\gamma}'^{(0)} - \hat{d}'\hat{U}, \qquad \hat{d}'' = \hat{d}'. \tag{40}$$

Finally, we perform ϵ-independent linear transformations of the physical and evanescent operators (separately), passing to the triply primed basis

$$P_i''' = \sum_j R_{ij} P_j'', \qquad E_k'''^{(1)} = \sum_l M_{kl} E_l''^{(1)}. \tag{41}$$

The transformations are now rather trivial

$$\hat{\gamma}'''^{(0)} = \hat{R}\hat{\gamma}''^{(0)}\hat{R}^{-1}, \qquad \hat{\gamma}'''^{(1)} = \hat{R}\hat{\gamma}''^{(1)}\hat{R}^{-1}, \tag{42}$$

$$\hat{b}''' = \hat{R}\hat{b}''\hat{M}^{-1}, \qquad \hat{c}''' = \hat{M}\hat{c}''\hat{R}^{-1}, \qquad \hat{d}''' = \hat{M}\hat{d}''\hat{M}^{-1}, \tag{43}$$

Before we start to transform our particular basis of the physical operators P_i (eqn. (6)), and the one-loop evanescent ones (eqn. (62)), we extend it by introducing four extra evanescent operators

$$\begin{aligned} E_5^{(1)} &= P_5 - 10P_3 + 6\tilde{P}_3, \\ E_6^{(1)} &= P_6 - 10P_4 + 6\tilde{P}_4, \\ E_7^{(1)} &= \tilde{P}_5 - 10\tilde{P}_3 + 6P_3, \\ E_8^{(1)} &= \tilde{P}_6 - 10\tilde{P}_4 + 6P_4, \end{aligned} \tag{44}$$

where tilde over an operator denotes inserting γ_5 in front of the last quark field. If some operator P_i is a linear combination of terms $(\bar{s}_L\Gamma b_L)(q\Gamma' q)$, then $\tilde{P}_i$ is the same linear combination of $(\bar{s}_L\Gamma b_L)(q\Gamma'\gamma_5 q)$, where Γ and Γ' stand for arbitrary Dirac matrices.

The operators $E_5^{(1)}, ..., E_8^{(1)}$ are not needed as counterterms in the initial basis. However, some linear combinations of them will become parts of either the physical or the evanescent operators in the triply primed basis which will be equivalent to the "standard" one (eqns. (33) and (34)).

The anomalous dimension matrices $\hat{\gamma}^{(0)}$ and $\hat{\gamma}^{(1)}$ in the extended basis are just the same as in eqns. (22) and (23). The matrices $\hat{b}$, $\hat{c}$ and $\hat{d}$ in the extended basis are found from one-loop diagrams (figs. 2, 3 and 6) with insertions of $P_1, ..., P_6$ and $E_1^{(1)}, ..., E_8^{(1)}$. We find

$$\hat{b} = \begin{bmatrix} \frac{5}{12} & \frac{2}{9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2}{9} & \frac{5}{12} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{45}$$

$$\hat{c} = \begin{bmatrix} 64 & \frac{32}{3} & 0 & \frac{4}{9} & 0 & 0 \\ 48 & -64 & 0 & -\frac{8}{3} & 0 & 0 \\ 0 & 0 & \frac{8960}{3} & -2432 & -\frac{1280}{3} & 320 \\ 0 & 0 & -\frac{4480}{9} & -\frac{8864}{3} - 40f & \frac{640}{9} & \frac{1280}{3} \\ 0 & 0 & \frac{320}{3} & -\frac{256}{3} & -\frac{32}{3} & 8 \\ 0 & 0 & -\frac{160}{9} & -\frac{952}{9} - 4f & \frac{16}{9} & \frac{32}{3} \\ ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? \end{bmatrix}, \tag{46}$$

$$\hat{d} = \begin{bmatrix} -7 & -\frac{4}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{64}{3} & -14 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{28}{9} & \frac{13}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 6 \\ 0 & 0 & \frac{2}{9} & \frac{5}{12} & 0 & -\frac{9}{2} & \frac{4}{3} & \frac{5}{2} \\ ? & ? & ? & ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? & ? & ? & ? \end{bmatrix}. \tag{47}$$

Question marks have been put into the entries which we do not know, but which do not affect the quantities calculated below.

Transforming two-loop anomalous dimensions to the "standard" basis requires knowledge of one-loop diagrams with insertions of the extra evanescent operators $E_5^{(1)}, ..., E_8^{(1)}$, which introduces traces with γ_5 to the calculation. Fortunately, from among all the relevant quantities listed above, only c_{64} and c_{84} depend on such traces. (In the cases of c_{54} and c_{74}, the relevant color factors vanish.) The trace which occurs in the case of c_{64} is $Tr(\gamma_\mu \gamma_\nu \gamma_5)$ which can be safely set to zero. On the other hand, c_{84} is irrelevant in the particular transformation we perform below.

At this point, we are ready to start the explicit transformation. In the first step, we subtract some evanescent operators from the operators P_5 and P_6

$$P'_5 = P_5 - E_5^{(1)}, \qquad\qquad P'_6 = P_6 - E_6^{(1)}, \tag{48}$$

which is equivalent to taking the 6×8 matrix $\hat{W}$ in eqn. (35) equal to

$$W_{ik} = \begin{cases} -1 & \text{when } i = k = 5 \text{ or } i = k = 6, \\ 0 & \text{otherwise.} \end{cases} \tag{49}$$

In the second step, we redefine the evanescent operators according to eqn. (38), with

$$\hat{U} = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{20}{3} & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & -\frac{20}{3} & 0 & \frac{2}{3} \\ 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 \end{bmatrix}. \tag{50}$$

Finally, we perform ϵ-independent linear transformations according to eqn. (41), with

$$\hat{R} = \begin{bmatrix} 2 & \frac{1}{3} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & 0 & \frac{1}{12} & 0 \\ 0 & 0 & -\frac{1}{9} & -\frac{2}{3} & \frac{1}{36} & \frac{1}{6} \\ 0 & 0 & \frac{4}{3} & 0 & -\frac{1}{12} & 0 \\ 0 & 0 & \frac{4}{9} & \frac{8}{3} & -\frac{1}{36} & -\frac{1}{6} \end{bmatrix} \tag{51}$$

and

$$\hat{M} = \begin{bmatrix} 2 & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & 1 & -\frac{1}{6} & -1 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{6} & 1 & \frac{1}{6} & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{52}$$

It is only a matter of tedious but simple algebra to verify that the triply primed basis is identical to the "standard" one, i.e. $P'''_i = O_i$ and $E'''^{(1)}_i = O^E_i$ for $i = 1, ..., 6$. The operators

$E_7'''^{(1)}$ and $E_8'''^{(1)}$ play role of extra (unnecessary) evanescent operators in the triply primed basis, i.e. the physical operators do not mix into them at one loop.

Finding the anomalous dimension matrix in the "standard" basis is now only a matter of simple matrix multiplication. Combining eqns. (36), (37), (39) and (42), one finds

$$\hat{\gamma}'''^{(0)} = \hat{R}\hat{\gamma}^{(0)}\hat{R}^{-1}, \tag{53}$$

$$\hat{\gamma}'''^{(1)} = \hat{R}\left\{\hat{\gamma}^{(1)} + \left[\Delta\hat{r},\ \hat{\gamma}^{(0)}\right] + 2\beta_0\,\Delta\hat{r}\right\}\hat{R}^{-1}, \tag{54}$$

where

$$\Delta\hat{r} = \left(\hat{b} + \hat{W}\hat{d} - \frac{1}{2}\hat{\gamma}^{(0)}\hat{W}\right)\hat{U} - \hat{W}\hat{c}. \tag{55}$$

The formula (54) resembles eqn. (3.4) of ref. [9] where renormalization-scheme dependence of $\gamma^{(1)}$ was considered in general.

Substituting the matrices from eqns. (22), (23), (45), (46), (47), (49), (50) and (51) to the above equations, one obtains

$$\hat{\gamma}'''^{(0)} = \begin{bmatrix} -2 & 6 & 0 & 0 & 0 & 0 \\ 6 & -2 & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} \\ 0 & 0 & -\frac{22}{9} & \frac{22}{3} & -\frac{4}{9} & \frac{4}{3} \\ 0 & 0 & 6-\frac{2}{9}f & -2+\frac{2}{3}f & -\frac{2}{9}f & \frac{2}{3}f \\ 0 & 0 & 0 & 0 & 2 & -6 \\ 0 & 0 & -\frac{2}{9}f & \frac{2}{3}f & -\frac{2}{9}f & -16+\frac{2}{3}f \end{bmatrix} \tag{56}$$

and

$$\hat{\gamma}'''^{(1)} = \begin{bmatrix} -\frac{21}{2}-\frac{2}{9}f & \frac{7}{2}+\frac{2}{3}f & \frac{79}{9} & -\frac{7}{3} & -\frac{65}{9} & -\frac{7}{3} \\ \frac{7}{2}+\frac{2}{3}f & -\frac{21}{2}-\frac{2}{9}f & -\frac{202}{243} & \frac{1354}{81} & -\frac{1192}{243} & \frac{904}{81} \\ 0 & 0 & -\frac{5911}{486}+\frac{71}{9}f & \frac{5983}{162}+\frac{1}{3}f & -\frac{2384}{243}-\frac{71}{9}f & \frac{1808}{81}-\frac{1}{3}f \\ 0 & 0 & \frac{379}{18}+\frac{56}{243}f & -\frac{91}{6}+\frac{808}{81}f & -\frac{130}{9}-\frac{502}{243}f & -\frac{14}{3}+\frac{646}{81}f \\ 0 & 0 & -\frac{61}{9}f & -\frac{11}{3}f & \frac{71}{3}+\frac{61}{9}f & -99+\frac{11}{3}f \\ 0 & 0 & -\frac{682}{243}f & \frac{106}{81}f & -\frac{225}{2}+\frac{1676}{243}f & -\frac{1343}{6}+\frac{1348}{81}f \end{bmatrix} \tag{57}$$

The above anomalous dimension matrices are in agreement with those found in refs. [9, 10, 11] in the so-called NDR scheme with fully anticommuting γ_5 (see eg. eqns. (4.5) and (4.8) in ref. [9]).

Matching conditions in the "standard" basis can be obtained from eqns. (11)–(14), according to

$$\vec{C}'''(M_W) = \left(\hat{R}^{-1}\right)^T\left[1 - \frac{\alpha_s}{4\pi}\Delta\hat{r}^T\right]\vec{C}(M_W) + \mathcal{O}(\alpha_s^2), \tag{58}$$

with $\Delta\hat{r}$ given in eqn. (55). The above equation yields

$$C_1''''(M_W) = \frac{11\alpha_s}{8\pi} + \mathcal{O}(\alpha_s^2), \qquad C_2''''(M_W) = 1 - \frac{11\alpha_s}{24\pi} + \mathcal{O}(\alpha_s^2), \tag{59}$$

$$C_3''''(M_W) = -\frac{1}{3}C_4''''(M_W) = C_5''''(M_W) = -\frac{1}{3}C_6''''(M_W) = -\frac{\alpha_s}{24\pi}\left(E(x) - \frac{2}{3}\right) + \mathcal{O}(\alpha_s^2), \tag{60}$$

which agrees with refs. [9, 10, 11] as well (see eg. eqns. (5.5) and (5.6) in ref [9]).

5. Summary

The $|\Delta F| = 1$ nonleptonic effective hamiltonian has been considered in a renormalization scheme which allows to consistently use fully anticommuting γ_5 at any number of loops, but at the leading order in the Fermi coupling G_F. It is possible in the specific operator basis we have introduced. Such a basis is particularly suited for performing multiloop calculations.

We have evaluated two-loop anomalous dimensions of the relevant operators and one-loop matching conditions for their Wilson coefficients in the new basis. Renormalization group evolution of the coefficients has been studied numerically in the case $\Delta B = -\Delta S = 1$. In the end, the two-loop anomalous dimension matrix found by us was transformed to the previously used operator basis, and the existing NLO results have been confirmed.

Acknowledgments

M. Misiak thanks for hospitality at the University of Zurich where most of this research has been performed. K. Chetyrkin appreciates the warm hospitality of the Theoretical group of the Max Planck Institute in Munich where part of this work has been made.

This work has been partially supported by the German Bundesministerium für Bildung and Forschung under the contract 06 TM 874 and DFG Project Li 519/2-2. K. Chetyrkin has been partially supported by INTAS under Contract INTAS-93-0744-ext. M. Misiak has been supported in part by Schweizerischer Nationalfonds, by the Polish Committee for Scientific Research (under grant 2 P03B 180 09, 1995–97) and by the EC contract HCMP CT92004.

Appendix

Here, we give the nonphysical operators relevant in our discussion. In the process of renormalizing off-shell two-loop amplitudes with insertions of operators $P_1, ...P_6$, there arise many EOM-vanishing operators, i.e. operators which vanish by the QCD equations of motion.

However, the specific structure of only one of them

$$N_1 = \frac{1}{g}(\bar{s}_L T^a \gamma_\mu b_L) D_\nu G^{a\,\mu\nu} \quad + \quad P_4 \tag{61}$$

is relevant in finding one- and two-loop mixing of the operators given in eqn. (6).

Introduction of N_1 in section 2 was the reason why P_4 (and, consequently, P_3, P_5 and P_6) entered the considerations. However, if P_4 was not introduced this way, $P_3, ..., P_6$ would be generated anyway as counterterms to box diagrams (like the one in fig. 7) with insertions of the first term in N_1. This term would be then treated as a physical operator. Introducing the nonphysical operator N_1 makes considering such box diagrams unnecessary, because the whole N_1 cannot have physical counterterms [18, 19, 21].

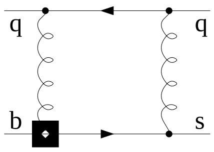

Figure 7: Sample box diagram with insertion of the first term in N_1 (denoted by the black square).

Another kind of nonphysical operators which we need to list here are the so-called evanescent operators i.e. operators which algebraically vanish in 4 dimensions [19, 20].

The one-loop diagrams shown in fig. 2 with insertions of $P_1, ..., P_6$ require counterterms proportional to $P_1, ..., P_6$ themselves and to the following evanescent operators:

$$\begin{aligned}
E_1^{(1)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a c_L)(\bar{c}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a b_L) - 16 P_1, \\
E_2^{(1)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} c_L)(\bar{c}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} b_L) - 16 P_2, \\
E_3^{(1)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} q) - 20 P_5 + 64 P_3, \\
E_4^{(1)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T^a b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} T^a q) - 20 P_6 + 64 P_4.
\end{aligned} \tag{62}$$

At the two-loop level, we encounter four more evanescent operators

$$\begin{aligned}
E_1^{(2)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T^a c_L)(\bar{c}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} T^a b_L) - 20 E_1^{(1)} - 256 P_1, \\
E_2^{(2)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5} T c_L)(\bar{c}_L \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} b_L) - 20 E_2^{(1)} - 256 P_2, \\
E_3^{(2)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\gamma_{\mu_6}\gamma_{\mu_7} b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}\gamma^{\mu_6}\gamma^{\mu_7} q) - 336 P_5 + 1280 P_3, \\
E_4^{(2)} &= (\bar{s}_L \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3}\gamma_{\mu_4}\gamma_{\mu_5}\gamma_{\mu_6}\gamma_{\mu_7} T^a b_L) \textstyle\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5}\gamma^{\mu_6}\gamma^{\mu_7} T^a q) - 336 P_6 + 1280 P_4.
\end{aligned} \tag{63}$$

The latter evanescent operators $E_1^{(2)}, ..., E_4^{(2)}$ are necessary in renormalizing the two-loop amplitudes from which one determines next-to-leading anomalous dimensions. They are also present in eqn. (21) which determines the matrix $\hat{c}$. However, a lot can be changed in their particular structure without affecting the two-loop anomalous dimension matrix of the physical operators. For instance, adding "$\epsilon \times$ (physical operator)" to $E_k^{(2)}$ affects $\hat{a}^{12}$ and $\hat{c}$ in eqn. (19), but leaves $\hat{\gamma}^{(1)}$ unchanged. This is in contrary to what happens when such a redefinition is applied to the one-loop evanescent operators $E_k^{(1)}$ (see eqns. (38) and (39)).

In section 4, the transformation of the two-loop anomalous dimension matrix of the physical operators has been successfully performed without specifying what two-loop evanescent operators were used in the "standard" basis. This is another illustration of insensitivity of two-loop results to the specific structure of $E_k^{(2)}$. These operators become more important at the three-loop level.

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