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Penguin Topologies, Rescattering Effects and Penguin Hunting with $B_{u,d} \to K\overline{K}$ and $B^{\pm} \to \pi^{\pm}K$

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Abstract

In the recent literature, constraints on the CKM angle γ arising from the branching ratios for $B^{\pm} \to \pi^{\pm}K$ and $B_d \to \pi^{\mp}K^{\pm}$ decays received a lot of attention. An important theoretical limitation of the accuracy of these bounds is due to rescattering effects, such as $B^+ \to \{\pi^0K^+\} \to \pi^+K^0$. We point out that these processes are related to penguin topologies with internal up quark exchanges and derive $SU(2)$ isospin relations among the $B^+ \to \pi^+K^0$ and $B_d^0 \to \pi^-K^+$ decay amplitudes by defining "tree" and "penguin" amplitudes in a proper way, allowing the derivation of generalized bounds on the CKM angle γ. We propose strategies to obtain insights into the dynamics of penguin processes with the help of the decays $B_{u,d} \to K\overline{K}$ and $B^{\pm} \to \pi^{\pm}K$, derive a relation among the direct CP-violating asymmetries arising in these modes, and emphasize that rescattering effects can be included in the generalized bounds on γ completely this way. Moreover, we have a brief look at the impact of new physics.

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1 Introduction

As was pointed out in [1]–[3], the decays $B^+\to\pi^+K^0$, $B^0_d\to\pi^-K^+$ and their charge-conjugates may play an important role to determine the angle γ of the usual non-squashed unitarity triangle [4] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [5] at future B-factories (BaBar, BELLE, CLEO III; interesting feasibility studies can be found in [3, 6, 7]). The corresponding decay amplitudes can be expressed as

$$A(B^+\to\pi^+K^0) = P^{(s)} + c_d\,P_{\rm EW}^{(s){\rm C}} + A^{(s)} \tag{1}$$

$$A(B^0_d\to\pi^-K^+) = -\left[\left(\mathcal{P}^{(s)} + c_u\,\mathcal{P}_{\rm EW}^{(s){\rm C}}\right) + \mathcal{T}^{(s)}\right], \tag{2}$$

where $P^{(s)}$, $\mathcal{P}^{(s)}$ denote QCD penguin amplitudes, $P_{\rm EW}^{(s){\rm C}}$, $\mathcal{P}_{\rm EW}^{(s){\rm C}}$ correspond to "colour-suppressed" electroweak penguin contributions, $A^{(s)}$ is due to annihilation processes, and $\mathcal{T}^{(s)}$ is usually referred to as a "colour-allowed" $\bar b\to\bar u u\bar s$ "tree" amplitude. The label s reminds us that we are dealing with $\bar b\to\bar s$ modes, the minus sign in (2) is due to our definition of meson states, and $c_u=+2/3$ and $c_d=-1/3$ are the up- and down-type quark charges, respectively.

The CLEO collaboration has recently reported the first results for the combined, i.e. averaged over decay and charge-conjugate, branching ratios $\mathrm{BR}(B^\pm\to\pi^\pm K)$ and $\mathrm{BR}(B_d\to\pi^\mp K^\pm)$ [8]. These quantities may lead to interesting constraints on the CKM angle γ, if their ratio

$$R\equiv\frac{\mathrm{BR}(B_d\to\pi^\mp K^\pm)}{\mathrm{BR}(B^\pm\to\pi^\pm K)} \tag{3}$$

is found experimentally to be smaller than 1 [2]. Since the present CLEO data give $R=0.65\pm0.40$, this may indeed be the case. The bounds on γ obtained in this manner turn out to be complementary to the present range for this angle arising from the usual fits of the unitarity triangle (for a review, see for instance [9]), and are hence of particular phenomenological interest. A detailed analysis of the implications of these bounds for the determination of the unitarity triangle has been performed in [10].

An important limitation of the theoretical accuracy of the "naïve" bounds on γ derived in [2] – besides the "colour-suppressed" electroweak penguin contributions – is due to rescattering processes of the kind $B^+\to\{\pi^0K^+,\,\pi^0K^{*+},\,\rho^0K^{*+},\,\ldots\}\to\pi^+K^0$, which have received a lot of attention in the recent literature [11]–[16] (for earlier references, see [17]). In this paper, we focus on these rescattering effects. Following closely [18, 19], we show in Section 3 that they are related to penguin topologies with internal up quark exchanges. Analogously, rescattering processes such as $B^+\to\{\overline{D^0}D_s^+\}\to\pi^+K^0$ can be regarded as long-distance contributions to penguins with internal charm quarks, which also received considerable interest in the recent literature [18]–[21].

Our paper is organized as follows: in Section 2, we discuss the penguin topologies in general terms. In particular, we establish the relation between the penguin diagram pictures used by CP practitioners and the formal operator method. Here we also recall useful expressions for the $\bar b\to\bar s$ and $\bar b\to\bar d$ penguin amplitudes. In Section 3, we derive a simple isospin relation between the $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ decay amplitudes by defining the "tree" and "penguin" amplitudes in a proper way, and address the question of rescattering effects in penguin-induced B decays. These isospin relations play a key role to probe γ and allow the derivation of generalized bounds on this CKM angle. In Section 4, we point out that decays of the type $B_{u,d} \to K\overline{K}$ play an important role to obtain insights into rescattering processes and the dynamics of penguin topologies. Using their combined branching ratios and $\mathrm{BR}(B^\pm \to \pi^\pm K)$, interesting relations and bounds can be derived, including a relation between the direct CP-violating asymmetries arising in these modes. Moreover, it is even possible to take into account the rescattering effects *completely* in the generalized bounds on γ by following these lines. After a brief look at the impact of new-physics contributions to B_d^0–$\overline{B_d^0}$ mixing in Section 5, we collect our conclusions in Section 6. Technical details related to the isospin structure of the relevant amplitudes are relegated to an appendix.

2 Penguin Topologies

In many phenomenological analyses of B decays in the literature, it is customary to represent penguin contributions by penguin diagrams with explicit $W^\pm$, t, c and u exchanges, as shown in Fig. 1 (a). On the other hand, the proper treatment of B decays at scales $\mathcal{O}(m_b)$ is an effective five-quark field theory, which deals with local operators. Here $W^\pm$ and t do not appear explicitly as dynamical fields and the local operators are built out of lighter flavours only. The effects of $W^\pm$ and t are present only in the short-distance Wilson coefficients of these operators.

The purpose of this section is to clarify the relation between these two approaches to describe B decays and in particular to state explicitly what is meant by the $\bar{b} \to \bar{s}$ penguin amplitude in (1), and similarly by the $\bar{b} \to \bar{d}$ penguin amplitude contributing to the decay $B^+ \to K^+\overline{K^0}$, which will play an important role in Section 4. To this end, let us recall the effective Hamiltonian for $\Delta B = +1$ decays by concentrating on the $\bar{b} \to \bar{s}$ penguin transitions. The $\bar{b} \to \bar{d}$ case can be analysed in the same manner with the obvious replacement $s \to d$ in the formulae given below. We have [22]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \left[\lambda_u^{(s)} \left(C_1(\mu) Q_1^u + C_2(\mu) Q_2^u \right) + \lambda_c^{(s)} \left(C_1(\mu) Q_1^c + C_2(\mu) Q_2^c \right) - \lambda_t^{(s)} \sum_{i=3}^{6} C_i(\mu) Q_i \right], \tag{4}$$

where μ is the renormalization scale $\mathcal{O}(m_b)$, $\lambda_i^{(s)} \equiv V_{is} V_{ib}^*$, and

$$Q_1^c = (\bar{c}_\alpha s_\beta)_{\mathrm{V-A}} \, (\bar{b}_\beta c_\alpha)_{\mathrm{V-A}} \,, \qquad Q_2^c = (\bar{c}s)_{\mathrm{V-A}} \, (\bar{b}c)_{\mathrm{V-A}} \tag{5}$$

$$Q_1^u = (\bar{u}_\alpha s_\beta)_{\mathrm{V-A}} \, (\bar{b}_\beta u_\alpha)_{\mathrm{V-A}} \,, \qquad Q_2^u = (\bar{u}s)_{\mathrm{V-A}} \, (\bar{b}u)_{\mathrm{V-A}} \tag{6}$$

$$Q_3 = (\bar{b}s)_{\mathrm{V-A}} \sum_{q=u,d,s,c,b} (\bar{q}q)_{\mathrm{V-A}} \,, \qquad Q_4 = (\bar{b}_\alpha s_\beta)_{\mathrm{V-A}} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{\mathrm{V-A}} \tag{7}$$

$$Q_5 = (\bar{b}s)_{\mathrm{V-A}} \sum_{q=u,d,s,c,b} (\bar{q}q)_{\mathrm{V+A}} \,, \qquad Q_6 = (\bar{b}_\alpha s_\beta)_{\mathrm{V-A}} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{\mathrm{V+A}} , \tag{8}$$

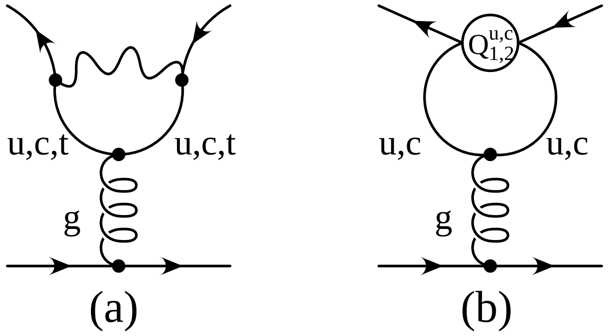

Figure 1: Penguin diagrams in the full (a) and effective (b) theory. $Q^{u,c}_{1,2}$ denotes operator insertions.

with α and β denoting colour indices. Here $Q^{u,c}_{1,2}$ are the current–current operators and the Q_i (with $i = 3, \ldots, 6$) the QCD penguin operators. We do not show the electroweak penguin operators, which can be found for instance in [9]. A discussion similar to the one presented below can also be made for the latter operators [15]. Since quark charges enter in the electroweak penguin operators, they exhibit, however, a different isospin structure as the QCD penguin operators.

The explicit derivation of (4) can be found in an appendix of [22]. It involves in particular the matching of the full six-quark theory containing $W^{\pm}$ with the effective theory, in which $W^{\pm}$ and the top quark do not appear as dynamical fields: the operators Q_i do not involve the top quark and are built out of u, c, d, s and b quarks only.

Let us now see how the penguin diagrams with internal $W^{\pm}$, t, c and u exchanges, in Fig. 1 (a), are represented in the effective Hamiltonian (4). The effect of the penguin diagrams with internal $W^{\pm}$ and top quark exchanges is obviously represented by the last term: the coefficients C_{3-6} depend on $x_t \equiv m_t^2/M_W^2$, and this dependence is represented by the well-known Inami–Lim function $E(x_t)$ [23]. On the other hand, there is no trace in (4) of the penguin diagrams with internal $W^{\pm}$, u and c exchanges. In particular, the coefficients $C_{1,2}$ do not carry any information on these penguin topologies. Indeed, as explicitly demonstrated in [22], the penguin diagrams with internal u and c quarks and with the W propagator have been cancelled in the process of matching by similar diagrams, where the W propagators have been replaced by local operators, as shown in Fig. 1 (b). Strictly speaking, there remains a renormalization scheme dependent constant, which is added with the help of the unitarity of the CKM matrix to the function $E(x_t)$ in C_{3-6}. This constant is equal to $-2/3$ in the NDR scheme, and vanishes in the HV scheme [22]. A discussion of these features and applications to non-leptonic B decays can also be found in [24].

The fact that there is no trace of penguin diagrams with internal u and c quarks in (4) is consistent with the general structure of the operator product expansion combined with renormalization group methods. At scales $\mu_b = \mathcal{O}(m_b)$, the effect of quarks with $m_i < \mu_b$ is absent in the Wilson coefficients and can only be found in the matrix elements of the operators Q_i.

The $\bar{b} \to \bar{s}$ QCD penguin amplitude $P^{(s)}$ contributing to $B^+ \to \pi^+ K^0$ can be decomposed as follows:

$$P^{(s)} = \lambda_u^{(s)} P_u^{(s)} + \lambda_c^{(s)} P_c^{(s)} + \lambda_t^{(s)} P_t^{(s)}. \tag{9}$$

Analogously, the $\bar{b} \to \bar{d}$ QCD penguin amplitude $P^{(d)}$ contributing to $B^+ \to K^+ \overline{K^0}$ can be written as

$$P^{(d)} = \lambda_u^{(d)} P_u^{(d)} + \lambda_c^{(d)} P_c^{(d)} + \lambda_t^{(d)} P_t^{(d)}. \tag{10}$$

Note that similar expressions hold also for the electroweak penguin amplitudes. The strong amplitudes $P_q^{(s)}$ and $P_q^{(d)}$ ($q \in \{u, c, t\}$) in (9) and (10) are related to each other by interchanging all d and s quarks, i.e. through the so-called U spin of the $SU(3)$ flavour symmetry of strong interactions. Let us focus in the following discussion on the decay $B^+ \to \pi^+ K^0$. In the formal operator language presented above, the meaning of $P_u^{(s)}$, $P_c^{(s)}$ and $P_t^{(s)}$ is simply as follows:

$$P_u^{(s)} = \frac{G_{\rm F}}{\sqrt{2}} \left[C_1(\mu)\, \langle K^0 \pi^+ | Q_1^u(\mu) | B^+ \rangle_{\rm P} + C_2(\mu)\, \langle K^0 \pi^+ | Q_2^u(\mu) | B^+ \rangle_{\rm P} \right] \tag{11}$$

$$P_c^{(s)} = \frac{G_{\rm F}}{\sqrt{2}} \left[C_1(\mu)\, \langle K^0 \pi^+ | Q_1^c(\mu) | B^+ \rangle_{\rm P} + C_2(\mu)\, \langle K^0 \pi^+ | Q_2^c(\mu) | B^+ \rangle_{\rm P} \right] \tag{12}$$

$$P_t^{(s)} = -\frac{G_{\rm F}}{\sqrt{2}} \sum_{i=3}^{6} C_i(\mu)\, \langle K^0 \pi^+ | Q_i(\mu) | B^+ \rangle. \tag{13}$$

Here $\langle K^0 \pi^+ | Q_{1,2}^u | B^+ \rangle_{\rm P}$ and $\langle K^0 \pi^+ | Q_{1,2}^c | B^+ \rangle_{\rm P}$ denote hadronic matrix elements with insertions of the current–current operators $Q_{1,2}^u$ and $Q_{1,2}^c$ into penguin diagrams with internal u and c quark exchanges. The importance of such diagrams in connection with certain strategies for CKM determinations has been pointed out for the first time in [18], although such contributions have been considered already a long time ago in a different context [25]. Recently the importance of $\langle Q_{1,2}^c \rangle_{\rm P}$, in particular in connection with the CLEO data and the determination of the angle α through the CP asymmetry in $B_d \to \pi^+ \pi^-$, has also been emphasized by the authors of [20], who have named them "charming penguins".

Let us next observe that the matrix elements of the penguin operators Q_{3-6} do not carry the subscript P. Indeed, what is meant by $P_t^{(s)}$ in the literature are hadronic matrix elements with insertions of the penguin operators not only into the penguin diagrams, but also into other topologies, in particular tree diagrams.

At this point, it should be stressed that the current–current operators $Q_{1,2}^c$ and $Q_{1,2}^u$ contribute to the mode $B^+ \to \pi^+ K^0$ only through the penguin topologies discussed above, and through annihilation topologies. The latter, which will be discussed in more detail in the following section, are described by the $A^{(s)}$ amplitude in (1) and are only due to the $Q_{1,2}^u$ operators [12]. Such annihilation processes are absent in the case of $B_d^0 \to \pi^- K^+$. However, in contrast to $B^+ \to \pi^+ K^0$, this decay receives also contributions from hadronic matrix elements of the $Q_{1,2}^u$ operators with insertions into tree-diagram-like topologies, which are represented in (2) by the amplitude $\mathcal{T}^{(s)}$.

We hope that this discussion shows the relation between the formal operator method and the more phenomenological picture used by CP asymmetry practitioners. Simultaneously, this discussion demonstrates that attributing the usual Feynman diagrams with full W propagators and internal t, c and u quarks to $P_t^{(s)}$, $P_c^{(s)}$ and $P_u^{(s)}$, respectively – although roughly correct – does not fully describe what happens at scales $\mathcal{O}(m_b)$ and may sometimes be confusing.

Let us next have a closer look at the general structure of the $\bar{b} \to \bar{q}$ ($q \in \{s, d\}$) QCD penguin amplitudes $P^{(s)}$ and $P^{(d)}$. Employing the unitarity of the CKM matrix and making furthermore use of the Wolfenstein parametrization [26], we arrive at

$$P^{(s)} = -\left(1-\frac{1}{2}\lambda^2\right)\,\lambda^2\,A\left[1-\Delta P^{(s)}\,+\,\left(\frac{\lambda^2\,R_b}{1-\lambda^2/2}\right)\,e^{i\gamma}\right]\left|P_{tu}^{(s)}\right|\,e^{i\delta_{tu}^{(s)}} \tag{14}$$

$$P^{(d)} = A\,\lambda^3\,R_t\,\left[e^{-i\beta}-\frac{1}{R_t}\Delta P^{(d)}+\mathcal{O}(\lambda^4)\right]\left|P_{tu}^{(d)}\right|\,e^{i\delta_{tu}^{(d)}}\,, \tag{15}$$

where the notation

$$P_{q_1q_2}^{(q)} \equiv \left|P_{q_1q_2}^{(q)}\right|\,e^{i\delta_{q_1q_2}^{(q)}} \equiv P_{q_1}^{(q)}-P_{q_2}^{(q)} \tag{16}$$

has been introduced, and

$$\Delta P^{(q)} \equiv \left|\Delta P^{(q)}\right|\,e^{i\delta_{\Delta P}^{(q)}} \equiv \frac{P_{cu}^{(q)}}{P_{tu}^{(q)}} = \frac{P_c^{(q)}-P_u^{(q)}}{P_t^{(q)}-P_u^{(q)}} \tag{17}$$

describes the contributions of penguins with up and charm quarks running as virtual particles in the loops. The present status of the relevant CKM factors in (14) and (15) is given by

$$A \equiv \frac{1}{\lambda^2}\,|V_{cb}| = 0.81 \pm 0.06\,,\quad R_b \equiv \frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right| = 0.36 \pm 0.08\,,\quad R_t \equiv \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right| = \mathcal{O}(1)\,, \tag{18}$$

where $\lambda \equiv |V_{us}| = 0.22$. Strategies to fix these parameters have recently been reviewed in [9].

At this point, it should be emphasized that whereas the $P^{(q)}$ amplitudes are μ and renormalization scheme independent, this is not the case for the different contributions in (9) and (10). This is evident, if one inspects equations (11)–(13). To this end, one can simply evaluate the penguin-like matrix elements of $Q_{1,2}^{u,c}$ in a perturbative framework to find that their μ dependences, related to the mixing between $Q_{1,2}^{u,c}$ and the penguin operators, cannot be cancelled by the μ dependence of $C_{1,2}(\mu)$. Similarly, the non-logarithmic terms in these matrix elements are renormalization scheme dependent, and this scheme dependence cannot be cancelled by the scheme dependence of $C_{1,2}(\mu)$. On the other hand, $P_{tu}^{(q)}$ and $\Delta P^{(q)}$ are μ independent and renormalization scheme independent, because these dependences cancel in (16). Consequently, $\Delta P^{(q)}$ is a physical quantity and can therefore be determined experimentally, as we will see in Section 4.

Formulae (14) and (15) can already be found in the literature [18, 19]. In the case of the $P^{(s)}$ amplitude (14), we have kept terms of $\mathcal{O}(\lambda^4)$, which have been neglected in these papers, and also in the bound on the CKM angle γ derived in [2]. The highly CKM-suppressed $\lambda^2 R_b e^{i\gamma} = \mathcal{O}(0.02)$ phase factor may lead to direct CP violation in the decay $B^+ \to \pi^+ K^0$. Model calculations performed at the perturbative quark level indicate that $\Delta P^{(s)}$ is not close to 1, i.e. that the large CKM suppression of the $e^{i\gamma}$ term in (14) is not compensated, and give CP asymmetries of at most a few percent [24, 25, 27]. However, as was pointed out recently [11]–[14], rescattering effects of the kind $B^+ \to \{\pi^0 K^+\} \to \pi^+ K^0$ may lead to CP asymmetries as large as $\mathcal{O}(10\%)$, and represent an important limitation of the theoretical accuracy of the "original" bounds on γ arising from the $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ decays that were proposed in [2]. Let us have a closer look at these final-state interaction effects, which are closely related to penguin topologies, in the following section.

3 Isospin Relations and the Connection Between Rescattering Processes and Penguin Topologies

As we have already noted, the decays $B^+ \to \pi^+ K^0$, $B^0_d \to \pi^- K^+$ and their charge conjugates provide a fertile ground to probe the CKM angle γ [1]–[3]. In this context, one makes use of the $SU(2)$ isospin symmetry of strong interactions, which appears – in contrast to the $SU(3)$ flavour symmetry – to be a very good and safe working assumption. It allows us to relate the QCD penguin amplitudes contributing to $B^+ \to \pi^+ K^0$ and $B^0_d \to \pi^- K^+$ to each other, yielding the following relations:

$$A(B^+ \to \pi^+ K^0) \equiv P \tag{19}$$

$$A(B^0_d \to \pi^- K^+) = -\left[T + P + P_{\rm ew}\right], \tag{20}$$

where the $B^+ \to \pi^+ K^0$ decay amplitude *defines* the $\bar{b} \to \bar{s}$ "penguin" amplitude P, the quantity $P_{\rm ew} \equiv c_u\, \mathcal{P}^{(s)\rm C}_{\rm EW} - c_d\, P^{(s)\rm C}_{\rm EW}$ is due to "colour-suppressed" electroweak penguins, and the generalized "colour-allowed" $\bar{b} \to \bar{u} u \bar{s}$ "tree" amplitude takes the form

$$T = e^{i\gamma} e^{i\delta_T} |T|, \tag{21}$$

where δ_T is a CP-conserving strong phase. In order to express (1) and (2) as in (19) and (20) with the help of the $SU(2)$ isospin symmetry, special care has to be taken. In particular, the "tree" amplitude T has to be defined in a proper way. The point is that an application of the isospin symmetry requires that we replace all u and d quarks in the $B^+ \to \pi^+ K^0$ decay processes by d and u quarks, respectively, in order to relate them to those of the transition $B^0_d \to \pi^- K^+$. While such a replacement is straightforward in the case of

$$\mathcal{P}^{(s)}_c = \frac{G_{\rm F}}{\sqrt{2}}\left[C_1(\mu)\,\langle K^+\pi^-|Q^c_1(\mu)|B^0_d\rangle_{\rm P} + C_2(\mu)\,\langle K^+\pi^-|Q^c_2(\mu)|B^0_d\rangle_{\rm P}\right] = P^{(s)}_c \tag{22}$$

$$\mathcal{P}^{(s)}_t = -\frac{G_{\rm F}}{\sqrt{2}}\sum_{i=3}^{6} C_i(\mu)\,\langle K^+\pi^-|Q_i(\mu)|B^0_d\rangle = P^{(s)}_t, \tag{23}$$

at first sight a problem shows up in the case of

$$\mathcal{P}_u^{(s)}=\frac{G_{\rm F}}{\sqrt{2}}\left[C_1(\mu)\,\langle K^+\pi^-|Q_1^u(\mu)|B^0_d\rangle_{\rm P}+\,C_2(\mu)\,\langle K^+\pi^-|Q_2^u(\mu)|B^0_d\rangle_{\rm P}\right], \tag{24}$$

since isospin symmetry implies the relation

$$\begin{aligned} P_u^{(s)} &= \frac{G_{\rm F}}{\sqrt{2}}\left[C_1(\mu)\,\langle K^0\pi^+|Q_1^u(\mu)|B^+\rangle_{\rm P}+\,C_2(\mu)\,\langle K^0\pi^+|Q_2^u(\mu)|B^+\rangle_{\rm P}\right]\\ &= -\frac{G_{\rm F}}{\sqrt{2}}\left[C_1(\mu)\,\langle K^+\pi^-|Q_1^d(\mu)|B^0_d\rangle_{\rm P}+\,C_2(\mu)\,\langle K^+\pi^-|Q_2^d(\mu)|B^0_d\rangle_{\rm P}\right]. \end{aligned} \tag{25}$$

Here $Q^d_{1,2}$ can be obtained easily from (6) by substituting u through d quarks. Consequently, isospin does not imply that $P_u^{(s)}$ is equal to $\mathcal{P}_u^{(s)}$. The difference between these amplitudes can, however, be absorbed in the proper definition of the T amplitude in (20). It has to be defined as

$$\begin{aligned} T\equiv -\frac{G_{\rm F}}{\sqrt{2}}\,\lambda_u^{(s)}\Big[&C_1(\mu)\langle K^+\pi^-|Q_1^u(\mu)|B^0_d\rangle_{\rm T}+C_2(\mu)\langle K^+\pi^-|Q_2^u(\mu)|B^0_d\rangle_{\rm T}\\ &+\Big\{C_1(\mu)\langle K^+\pi^-|Q_1^u(\mu)|B^0_d\rangle_{\rm P}+C_2(\mu)\langle K^+\pi^-|Q_2^u(\mu)|B^0_d\rangle_{\rm P}\\ &-C_1(\mu)\langle K^+\pi^-|Q_1^d(\mu)|B^0_d\rangle-C_2(\mu)\langle K^+\pi^-|Q_2^d(\mu)|B^0_d\rangle\Big\}\Big] \end{aligned} \tag{26}$$

to arrive at (19) and (20).

Equations (22), (23) and (25) are completely general and rely only on the isospin decomposition of the $B^+\to\pi^+K^0$ and $B^0_d\to\pi^-K^+$ decay amplitudes, which is performed explicitly by using the Wigner–Eckart theorem in the appendix. The second term in (26), which is required by this isospin decomposition, shows that T is not only given by hadronic matrix elements with insertions of the $Q^u_{1,2}$ current–current operators into tree-diagram-like topologies, as naïvely expected. The $Q^d_{1,2}$ operators contribute both through insertions into penguin topologies and through annihilation processes. The latter correspond to the annihilation amplitude $A^{(s)}$ appearing in (1). Consequently, the term in curly brackets in (26) consists both of the difference of contributions from penguin topologies with internal up and down quarks, which is shown in Fig. 2, and of contributions from annihilation topologies.

In order to address the question of rescattering effects in $B\to\pi K$ decays, let us first return to the expressions given in (14) and (15). These formulae are completely general. In several previous analyses of penguin-induced B decays, it was, however, assumed that penguin processes are dominated by internal top quark exchanges, corresponding to $\Delta P^{(q)}=0$. An important implication of this special case would be that the $P^{(d)}$ penguin amplitude is proportional to the phase factor $e^{-i\beta}$, i.e. exhibits a very simple phase structure. As was pointed out in [18], this feature is spoiled by penguin contributions with internal up and charm quarks, leading to sizeable values of $\Delta P^{(d)}$. Since the $e^{i\gamma}$ factor

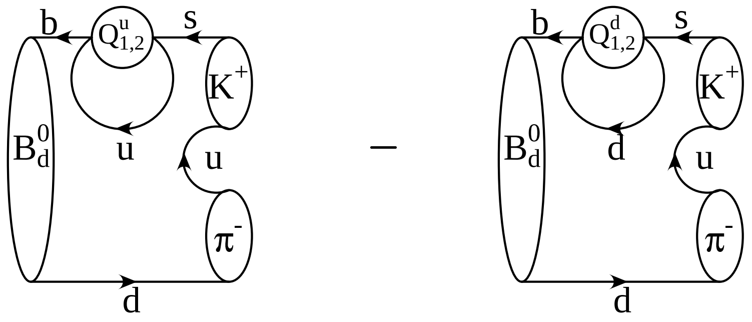

Figure 2: Penguin topologies contributing to the amplitude T.

in (14) is highly CKM-suppressed by $\lambda^2 R_b = \mathcal{O}(0.02)$, there is to a good approximation no non-trivial CP-violating phase present in the $\bar{b} \to \bar{s}$ penguin amplitude, provided the relation $|1-\Delta P^{(s)}| \gg \lambda^2 R_b$ is fulfilled. Model calculations performed at the perturbative quark-level to estimate the penguin amplitudes $P_q^{(s)}$ ($q \in \{u, c, t\}$) indicate that this requirement is indeed satisfied, i.e. that the very large CKM suppression of the CP-violating phase factor in (14) is not compensated, and that there is direct CP violation in $B^\pm \to \pi^\pm K$ of at most a few percent [25, 27].

The QCD penguin amplitudes $P_q^{(d,s)}$ receive, however, also long-distance contributions from rescattering processes, which can be divided into two categories and are not included in these simple model calculations. Let us discuss these effects by focusing on the channel $B^+ \to \pi^+ K^0$. In the first class of rescattering processes, we have to deal with decays such as $B^+ \to \overline{D^0} D_s^+$, which are caused by the $Q_{1,2}^c$ current–current operators through insertions into tree-diagram-like topologies, and may rescatter into the final state $\pi^+ K^0$, i.e. we have

$$B^+ \to \{F_c^{(s)}\} \to \pi^+ K^0, \tag{27}$$

where $F_c^{(s)} \in \{\overline{D^0} D_s^+,\, \overline{D^0} D_s^{*+},\, \overline{D^{*0}} D_s^{*+},\, \ldots\}$. Here the dots include also intermediate multibody states. These final-state interaction effects are related to penguin topologies with internal charm quark exchanges, as can be seen in Fig. 3, and are included in (14) as long-distance contributions to the $P_c^{(s)}$ amplitude. They are expected to contribute significanlty to the magnitude of the $\bar{b} \to \bar{s}$ QCD penguin amplitude [18, 20].

In the second class of rescattering processes, channels of the kind $B^+ \to \pi^0 K^+$, which receive $Q_{1,2}^u$ current–current operator contributions with insertions into tree-diagram-like topologies, rescatter into $\pi^+ K^0$, i.e.

$$B^+ \to \{F_u^{(s)}\} \to \pi^+ K^0, \tag{28}$$

where $F_u^{(s)} \in \{\pi^0 K^+,\, \pi^0 K^{*+},\, \rho^0 K^{*+},\, \ldots\}$. As can be seen in Fig. 4, these final-state interaction effects are related to penguin topologies with internal up quarks and appear in (14) as long-distance contributions to the $P_u^{(s)}$ amplitude. Moreover, we also

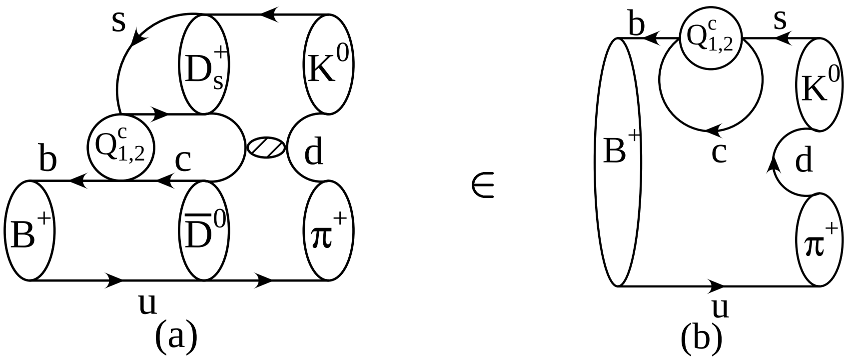

Figure 3: Illustration of a rescattering processes of the kind $B^+ \to \{\overline{D^0} D_s^+\} \to \pi^+ K^0$ (a), which is contained in penguin topologies with internal charm quarks (b). $Q_{1,2}^c$ denotes an operator insertion.

get contributions from the rescattering processes (28) to the amplitude $A^{(s)}$ through annihilation topologies. Usually it is assumed that annihilation processes are suppressed relative to tree-diagram-like processes by a factor of f_B/m_B. However, this feature may no longer hold in the presence of rescattering effects [12, 28], so that annihilation topologies may play a more important role than naïvely expected. Model calculations [28] based on Regge phenomenology typically give an enhancement of the ratio $|A^{(s)}|/|\mathcal{T}^{(s)}|$ from $f_B/m_B \approx 0.04$ to $\mathcal{O}(0.2)$. Rescattering processes of this kind can be probed, e.g. by the $\Delta S=0$ decay $B_d \to K^+K^-$. A future stringent bound on $\mathrm{BR}(B_d \to K^+K^-)$ at the level of 10^{-7} or lower may provide a useful limit on these rescattering effects [3]. The present upper bound obtained by the CLEO collaboration is 4.3×10^{-6} [8]. In order to take into account annihilation processes in the $B^+ \to \pi^+ K^0$ decay amplitude, we simply have to perform the replacement $P_u^{(s)} \to P_u^{(s)} + \widetilde{A}^{(s)}$ in (17), yielding

$$\Delta\widetilde{P}^{(s)} \equiv \frac{P_c^{(s)} - P_u^{(s)} - \widetilde{A}^{(s)}}{P_t^{(s)} - P_u^{(s)} - \widetilde{A}^{(s)}}, \tag{29}$$

where $\widetilde{A}^{(s)}$ is defined by $A^{(s)} \equiv V_{us}V_{ub}^*\widetilde{A}^{(s)}$.

The quantity $\Delta\widetilde{P}^{(s)}$ would not be close to 1 if rescattering processes of the type (27) played the dominant role in $B^+ \to \pi^+ K^0$, or if both (27) and (28) were similarly important. In the former case, $\Delta\widetilde{P}^{(s)}$ would carry a sizeable CP-conserving strong phase, whereas there would be a cancellation in (29) in the latter case, leading to $|\Delta\widetilde{P}^{(s)}| \ll 1$. On the other hand, $\Delta\widetilde{P}^{(s)}$ may be close to 1, if the final-state interactions arising from (28) would dominate $B^+ \to \pi^+ K^0$. In a recent attempt to calculate final-state interaction effects of the kind $B^+ \to \{\pi^0 K^+\} \to \pi^+ K^0$ with the help of Regge phenomenology [13], it is found that such rescattering processes may in fact play a dominant role, i.e. $|\mathcal{P}_{uc}|/|\mathcal{P}_{tc}| = \mathcal{O}(5)$, thereby leading to values of $|1-\Delta\widetilde{P}^{(s)}|$ as small as $\mathcal{O}(0.2)$. An important phenomenological implication of these rescattering effects would be CP violation in

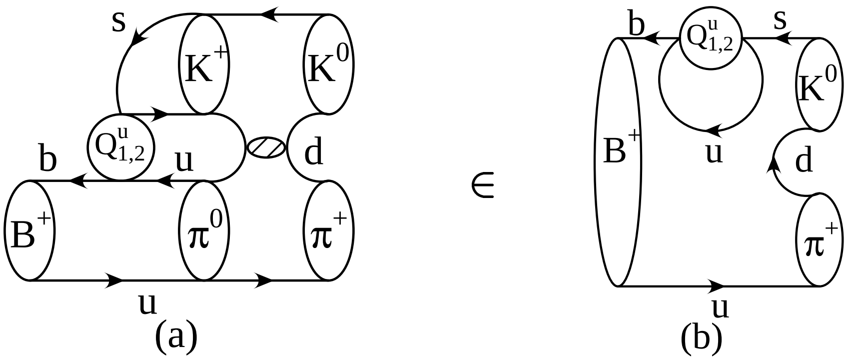

Figure 4: Illustration of a rescattering processes of the kind $B^+ \to \{\pi^0 K^+\} \to \pi^+ K^0$ (a), which is contained in penguin topologies with internal up quarks (b). $Q_{1,2}^u$ denotes an operator insertion.

$B^+ \to \pi^+ K^0$ as large as $\mathcal{O}(10\%)$. Similar features are also found in a different approach to deal with final-state interactions in $B \to \pi K$ decays [11, 12].

Although the "factorization" hypothesis [29] is in general questionable, it may work reasonably well for the colour-allowed amplitude $\mathcal{T}^{(s)}$ [30]. Since the intrinsic "strength" of decays such as $B^+ \to \pi^0 K^+$, representing the "first step" of the rescattering processes (28), is given by $\mathcal{T}^{(s)}$, we may derive a "plausible" upper bound $\lambda^2 R_b/|1-\Delta\widetilde{P}^{(s)}| \lesssim 0.15$, where the recent CLEO data on the combined $B^\pm \to \pi^\pm K$ branching ratio [8], and the BSW form factors [31] have been used to evaluate $\mathcal{T}^{(s)}$ within "factorization" (see [15]). Note that the ratio $\lambda^2 R_b/|1-\Delta\widetilde{P}^{(s)}|$ is typically one order of magnitude smaller, if rescattering processes do not play the dominant role in $B^+ \to \pi^+ K^0$.

In the following discussion we will not comment further on quantitative estimates of rescattering effects. A reliable theoretical treatment is very difficult and requires knowledge of the dynamics of strong interactions that is unfortunately not available at present. We rather advocate to use additional experimental information to obtain insights into final-state interactions. Before turning to such strategies in Section 4, let us first point out an interesting feature of the rescattering processes. The ratio R of the combined $B \to \pi K$ branching ratios introduced in (3) does not only imply constraints on the CKM angle γ, but also on the magnitude of the "tree" amplitude T. Since the present central value $R = 0.65$ obtained by the CLEO collaboration [8] is significanlty smaller than 1, these constraints are at the edge of compatibility with "factorization" [2, 15]. In particular, larger values of $|T|$ are favoured by the CLEO data. However, as we have seen in (26), the properly defined amplitude T is not just a "colour-allowed tree amplitude" as $\mathcal{T}^{(s)}$, but actually receives also contributions from penguin and annihilation topologies. Consequently, if the rescattering effects characterized by (28) and related to such topologies play in fact an important role, the value of $|T|$ could in principle be shifted significantly from its "factorized" value. In particular, the small central value of $R = 0.65$ may indicate already that $|T|$ is enhanced considerably by final-state interactions, and it may well be possible that future measurements will stabilize around this naïvely small value [15].

The bounds on γ derived in [2] are constructed in such a way that they do *not* depend on $|T|$. Generalized bounds, making not only use of the ratio R of the combined $B \to \pi K$ branching ratios, but also of the "pseudo-asymmetry" A_0, which is defined by

$$A_0 \equiv \frac{\mathrm{BR}(B^0_d \to \pi^- K^+) - \mathrm{BR}(\overline{B^0_d} \to \pi^+ K^-)}{\mathrm{BR}(B^+ \to \pi^+ K^0) + \mathrm{BR}(B^- \to \pi^- \overline{K^0})}, \tag{30}$$

were derived in [15]. They are due to the fact that the amplitude relations (19) and (20) imply the following minimal value of R:

$$R_{\min} = \kappa \, \sin^2\gamma \, + \, \frac{1}{\kappa}\left(\frac{A_0}{2\,\sin\gamma}\right)^2, \tag{31}$$

where rescattering and electroweak penguin effects are included through the parameter κ, which is given by

$$\kappa = \frac{1}{w^2}\left[\,1 + 2\,(\epsilon\, w)\cos\Delta + (\epsilon\, w)^2\,\right] \tag{32}$$

with

$$w = \sqrt{1 + 2\,\rho\,\cos\theta\cos\gamma + \rho^2}. \tag{33}$$

Note that no approximations were made in order to derive (31). The quantities ρ and ϵ measure the "strength" of the rescattering and electroweak penguin effects, respectively, and can be expressed as

$$\rho\, e^{i\theta} = \frac{\lambda^2 R_b}{1-\lambda^2/2}\left(\frac{1}{1-\Delta\widetilde{P}^{(s)}}\right) = \frac{\lambda^2 R_b}{1-\lambda^2/2}\left[1 - \left(\frac{P^{(s)}_{uc} + \widetilde{A}^{(s)}}{P^{(s)}_{tc}}\right)\right] \tag{34}$$

and

$$\epsilon \equiv \frac{|P_{\rm ew}|}{\sqrt{\langle |P|^2\rangle}}\,, \tag{35}$$

where

$$\langle |P|^2\rangle \equiv \frac{1}{2}\left(|P|^2 + |\overline{P}|^2\right). \tag{36}$$

The phase Δ in (32) is given by the difference of CP-conserving strong phases of the QCD and electroweak penguins. Since the values of the CKM angle γ implying $R_{\min} > R_{\rm exp}$, where $R_{\rm exp}$ denotes the experimentally determined value of R, are excluded, we obtain an allowed range for γ. For values of R as small as 0.65, which is the central value of present CLEO data [8], a large region around $\gamma = 90^\circ$ is excluded. As soon as a non-vanishing experimental result for A_0 has been established, also an interval around $\gamma = 0^\circ$ and 180° can be ruled out, while the impact on the excluded region around 90° is rather small [15]. The "original" bounds derived in [2] correspond to $\rho = \epsilon = 0$ and were obtained without using information provided by the "pseudo-asymmetry" A_0, i.e. $R_{\rm min}^{(0)} = \sin^2\gamma$. The impact of rescattering and electroweak penguin effects on the constraints on γ implied by (31) was analysed in great detail in [15, 16]. As was pointed out in these papers, additional experimental data on the decay $B^+ \to K^+\overline{K^0}$ and its charge conjugate allows us to take into account rescattering effects in these bounds *completely.* Let us have a closer look at another strategy to accomplish this task in the following section.

4 Penguin Hunting with $B_{u,d} \to K\overline{K}$ and $B^\pm \to \pi^\pm K$ Decays

The penguin-induced $\bar b \to \bar d$ mode $B_d^0 \to K^0\overline{K^0}$, which is governed by a decay amplitude as the one given in (15), was proposed frequently in the literature as a probe for new physics. This mode would indeed provide a striking signal of physics beyond the Standard Model, if QCD penguin processes were dominated by internal top quarks. In that case, the weak decay and B_d^0–$\overline{B_d^0}$ mixing phases would cancel, implying vanishing CP violation in $B_d \to K^0\overline{K^0}$ (for a detailed discussion, see [19]). As we have seen in the previous section, this feature is, however, spoiled by penguin topologies with internal up and charm quark exchanges, i.e. by the $\Delta P^{(d)}$ term [32]. Interestingly, the CP-violating asymmetries induced by these contributions allow a determination of the CKM angle α that does not suffer from penguin uncertainties, if one relates $B_d \to K^0\overline{K^0}$ and $B_d \to \pi^+\pi^-$ to each other with the help of the $SU(3)$ flavour symmetry of strong interactions [33]. We shall briefly come back to the issue of new-physics effects in $B_d \to K^0\overline{K^0}$ in Section 5.

In the present section, we point out that $B_d \to K^0\overline{K^0}$ and its spectator-quark isospin partner $B^\pm \to K^\pm K$ play an important role to obtain insights into the dynamics of penguin and rescattering processes. As in the case of the $B \to \pi K$ decays discussed above, the observables that will be available first are probably the combined branching ratios

$$\mathrm{BR}(B_d \to K^0\overline{K^0}) \equiv \frac{1}{2}\left[\mathrm{BR}(B_d^0 \to K^0\overline{K^0}) + \mathrm{BR}(\overline{B_d^0} \to K^0\overline{K^0})\right] \tag{37}$$

$$\mathrm{BR}(B^\pm \to K^\pm K) \equiv \frac{1}{2}\left[\mathrm{BR}(B^+ \to K^+\overline{K^0}) + \mathrm{BR}(B^- \to K^-K^0)\right]. \tag{38}$$

At present, only the bounds $\mathrm{BR}(B_d \to K^0\overline{K^0}) < 1.7\times10^{-5}$ and $\mathrm{BR}(B^\pm \to K^\pm K) < 2.1\times10^{-5}$ are available from the CLEO collaboration, while the combined $B^\pm \to \pi^\pm K$ branching ratio

$$\begin{aligned}\mathrm{BR}(B^\pm \to \pi^\pm K) &\equiv \frac{1}{2}\left[\mathrm{BR}(B^+ \to \pi^+K^0) + \mathrm{BR}(B^- \to \pi^-\overline{K^0})\right]\\ &= \left(2.3^{+1.1}_{-1.0} \pm 0.3 \pm 0.2\right)\times10^{-5}\end{aligned} \tag{39}$$

has already been measured [8].

In order to obtain insights into the dynamics of penguin and final-state interaction processes, the ratio

$$
\begin{aligned}
H &\equiv R_{SU(3)}^2 \left(\frac{1-\lambda^2}{\lambda^2}\right) \frac{\mathrm{BR}(B^\pm \to K^\pm K)}{\mathrm{BR}(B^\pm \to \pi^\pm K)} \\
&= \frac{R_t^2 - 2\,R_t\,|\Delta P|\,\cos\delta_{\Delta P}\,\cos\beta + |\Delta P|^2}{1 + 2\,\lambda^2\,\widetilde{R}_b\,\cos\gamma + \lambda^4 \widetilde{R}_b^2 - 2\,|\Delta P|\,\cos\delta_{\Delta P}\left(1 + \lambda^2 \widetilde{R}_b \cos\gamma\right) + |\Delta P|^2}
\end{aligned} \tag{40}
$$

plays a key role. Here we have used the $SU(3)$ flavour symmetry of strong interactions to relate $\widetilde{P}^{(d)}$ to $\widetilde{P}^{(s)}$:

$$
\Delta\widetilde{P}^{(s)} = \Delta\widetilde{P}^{(d)} \equiv |\Delta P|\,e^{i\delta_{\Delta P}}, \tag{41}
$$

and $\widetilde{R}_b \equiv R_b/(1-\lambda^2/2)$. The quantity

$$
R_{SU(3)} = \frac{M_B^2 - M_\pi^2}{M_B^2 - M_K^2}\,\frac{F_{B\pi}(M_K^2;0^+)}{F_{BK}(M_K^2;0^+)} \tag{42}
$$

describes factorizable $SU(3)$ breaking, where $F_{B\pi}(M_K^2;0^+)$ and $F_{BK}(M_K^2;0^+)$ are form factors parametrizing the hadronic quark-current matrix elements $\langle\pi|(\bar{b}d)_{\mathrm{V-A}}|B\rangle$ and $\langle K|(\bar{b}s)_{\mathrm{V-A}}|B\rangle$, respectively. Using, for example, the model of Bauer, Stech and Wirbel [31], we have $R_{SU(3)} = \mathcal{O}(0.7)$. At present, there is unfortunately no reliable approach available to deal with non-factorizable $SU(3)$ breaking. Since already the factorizable corrections are significant, we expect that non-factorizable $SU(3)$ breaking may also lead to sizeable effects.

Let us assume for the moment that H is found experimentally to be different from 1. Introducing then the quantity

$$
h \equiv \left|\frac{H - R_t\,\cos\beta}{1-H}\right| \tag{43}
$$

and keeping the strong phase $\delta_{\Delta P}$ in (40) as an unknown, free parameter, we arrive at the following range for $|\Delta P|$:

$$
\left|h - \sqrt{h^2 + \left(\frac{H - R_t^2}{1-H}\right)}\right| \leq |\Delta P| \leq h + \sqrt{h^2 + \left(\frac{H - R_t^2}{1-H}\right)}, \tag{44}
$$

where terms of $\mathcal{O}(\lambda^2\,\widetilde{R}_b\,\cos\gamma)$ have been neglected. This range shrinks to

$$
|\Delta P| = \sqrt{\frac{H - R_t^2}{1-H}} \tag{45}
$$

for $h=0$, corresponding to $\cos\beta = H/R_t$. In the special case $H=1$, which has been excluded in the formulae given above, we have on the other hand

$$
|\Delta P| \geq \frac{1}{2}\left|\frac{1-R_t^2}{1-R_t\,\cos\beta}\right|. \tag{46}
$$

In order to constrain $|\Delta P|$ following this approach, both the CKM angle β, which is measured in a clean way through mixing-induced CP violation in $B_d \to J/\psi\, K_{\rm S}$, and the parameter R_t, fixing one side of the unitarity triangle, have to be known. The most promising ways to determine R_t are the ratio of the B_d^0–$\overline{B_d^0}$ and B_s^0–$\overline{B_s^0}$ mixings, and the decay $K^+ \to \pi^+\nu\overline{\nu}$ [34]. We are optimistic that knowledge on these quantities will be available by the time the combined branching ratio $\mathrm{BR}(B^\pm \to K^\pm K)$ will be measured, and look forward to experimental data to see how powerful the bounds on $|\Delta P|$ derived above are realized. In Ref. [13], a different strategy to constrain rescattering effects through the ratio H has been proposed.

Let us next have a closer look at the direct CP-violating asymmetries arising in the decays $B^\pm \to K^\pm K$ and $B^\pm \to \pi^\pm K$, which are defined by

$$a_{\rm CP}(B^+ \to K^+\overline{K^0}) \equiv \frac{\mathrm{BR}(B^+ \to K^+\overline{K^0}) - \mathrm{BR}(B^- \to K^- K^0)}{\mathrm{BR}(B^+ \to K^+\overline{K^0}) + \mathrm{BR}(B^- \to K^- K^0)} \tag{47}$$

$$a_{\rm CP}(B^+ \to \pi^+ K^0) \equiv \frac{\mathrm{BR}(B^+ \to \pi^+ K^0) - \mathrm{BR}(B^- \to \pi^-\overline{K^0})}{\mathrm{BR}(B^+ \to \pi^+ K^0) + \mathrm{BR}(B^- \to \pi^-\overline{K^0})}. \tag{48}$$

Using (14) and (15), as well as the $SU(3)$ flavour symmetry as in (40), it is a straightforward exercise to derive the relation

$$\frac{a_{\rm CP}(B^+ \to \pi^+ K^0)}{a_{\rm CP}(B^+ \to K^+\overline{K^0})} = -\,R_{SU(3)}^2\,\frac{\mathrm{BR}(B^\pm \to K^\pm K)}{\mathrm{BR}(B^\pm \to \pi^\pm K)}\,\frac{(1-\lambda^2/2)\,R_b\,\sin\gamma}{R_t\,\sin\beta}\,. \tag{49}$$

Since the unitarity of the CKM matrix implies

$$(1-\lambda^2/2)\,R_b\,\sin\gamma \;=\; R_t\,\sin\beta\,, \tag{50}$$

where a discussion of the $(1-\lambda^2/2)$ factor can be found in [35], we arrive at the simple relation

$$\frac{a_{\rm CP}(B^+ \to \pi^+ K^0)}{a_{\rm CP}(B^+ \to K^+\overline{K^0})} \;=\; -\,R_{SU(3)}^2\,\frac{\mathrm{BR}(B^\pm \to K^\pm K)}{\mathrm{BR}(B^\pm \to \pi^\pm K)}\,, \tag{51}$$

which is quite remarkable and is nicely satisfied by the results given in [24].

As soon as either $a_{\rm CP}(B^+ \to \pi^+ K^0)$ or $a_{\rm CP}(B^+ \to K^+\overline{K^0})$ can be measured, we will be in a position not just to constrain, but to determine $|\Delta P|$ and $\delta_{\Delta P}$ from H and these asymmetries, if β and R_t are again used as an additional input. Moreover, it is possible to determine $R_{SU(3)}$ with the help of relation (51), providing interesting insights into $SU(3)$ breaking. Using (31)–(34), it is even possible to include the rescattering effects *completely* in the generalized bounds on the CKM angle γ that are implied by the minimal value of R given in (31). Interestingly, the final-state interaction effects may lead to a significant enhancement of the combined $B^\pm \to K^\pm K$ branching ratio from the "short-distance" value of $\mathcal{O}(10^{-6})$ to the 10^{-5} level, so that it should be possible to measure this mode at future B-factories, if rescattering effects are in fact large [15, 16].

A similar comment applies to the decay $B_d \to K^0\overline{K^0}$. If it was possible to measure the direct and mixing-induced CP asymmetries arising in this channel, also interesting experimental insights into the dynamics of penguin processes could be obtained. These CP-violating observables require, however, a time-dependent measurement, and can be obtained from the time-dependent CP asymmetry [32]

$$a_{\rm CP}(B_d\to K^0\overline{K^0};t)\equiv\frac{{\rm BR}(B^0_d(t)\to K^0\overline{K^0})-{\rm BR}(\overline{B^0_d}(t)\to K^0\overline{K^0})}{{\rm BR}(B^0_d(t)\to K^0\overline{K^0})+{\rm BR}(\overline{B^0_d}(t)\to K^0\overline{K^0})}=$$

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to K^0\overline{K^0})\,\cos(\Delta M_d\,t)+\mathcal{A}_{\rm CP}^{\rm mix-ind}(B_d\to K^0\overline{K^0})\,\sin(\Delta M_d\,t),\tag{52}$$

where

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to K^0\overline{K^0})=\frac{1-\left|\xi^{(d)}_{K^0\overline{K^0}}\right|^2}{1+\left|\xi^{(d)}_{K^0\overline{K^0}}\right|^2},\quad \mathcal{A}_{\rm CP}^{\rm mix-ind}(B_d\to K^0\overline{K^0})=\frac{2\,{\rm Im}\,\xi^{(d)}_{K^0\overline{K^0}}}{1+\left|\xi^{(d)}_{K^0\overline{K^0}}\right|^2}\tag{53}$$

with

$$\xi^{(d)}_{K^0\overline{K^0}}=-\,e^{-i\,2\,\beta}\,\frac{\overline{P^{(d)}}}{P^{(d)}}\,.\tag{54}$$

Using the penguin amplitude (15), we obtain

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to K^0\overline{K^0})\;=\;\frac{2\,R_t\,|\Delta P^{(d)}|\,\sin\delta^{(d)}_{\Delta P}\,\sin\beta}{R_t^2-2\,R_t\,|\Delta P^{(d)}|\,\cos\delta^{(d)}_{\Delta P}\,\cos\beta+|\Delta P^{(d)}|^2}\tag{55}$$

$$\mathcal{A}_{\rm CP}^{\rm mix-ind}(B_d\to K^0\overline{K^0})\;=\;\frac{-\,2\,|\Delta P^{(d)}|\left[R_t\,\cos\delta^{(d)}_{\Delta P}-|\Delta P^{(d)}|\,\cos\beta\right]\sin\beta}{R_t^2-2\,R_t\,|\Delta P^{(d)}|\,\cos\delta^{(d)}_{\Delta P}\,\cos\beta+|\Delta P^{(d)}|^2}\,.\tag{56}$$

If we look at expressions (55) and (56), we observe that these observables depend on the three variables $|\Delta P^{(d)}|/R_t$, $\delta^{(d)}_{\Delta P}$ and β. Interestingly, the quantity $|\Delta P^{(d)}|$ parametrizing penguin topologies with internal up and charm quarks enters in combination with R_t. Since β can be measured through $B_d\to J/\psi\,K_{\rm S}$, the former two "unknowns" can be determined from $\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to K^0\overline{K^0})$ and $\mathcal{A}_{\rm CP}^{\rm mix-ind}(B_d\to K^0\overline{K^0})$. Note that we do not have to use any flavour symmetries to accomplish this task.

In contrast to $B^+\to K^+\overline{K^0}$, the decay $B^0_d\to K^0\overline{K^0}$ does not receive an annihilation amplitude corresponding to $A^{(s)}$, but contributions from "penguin annihilation" topologies. Comparing $\mathcal{A}_{\rm CP}^{\rm dir}(B_d\to K^0\overline{K^0})$ with $a_{\rm CP}(B^+\to K^+\overline{K^0})$, we have a probe for the importance of these contributions. Another important role in this respect is played by the decay $B^0_d\to K^+K^-$, as we have already noted. If the $A^{(s)}$ amplitude and the "penguin annihilation" contributions turn out to be small, a comparison of the values for $|\Delta P^{(d)}|$ and $\delta^{(d)}_{\Delta P}$ obtained this way with those from the $SU(3)$ approach discussed above, using H and direct CP violation in $B^\pm\to K^\pm K$ or $B^\pm\to\pi^\pm K$, we have an experimental probe for $SU(3)$ breaking. On the other hand, using in addition to the direct and mixing-induced CP asymmetries again the $SU(3)$ flavour symmetry and the observable H specified in (40), we have two options:

i) if we use R_t as an input, β can be extracted simultaneously with $|\Delta P|$ and $\delta_{\Delta P}$;

ii) if we use β as an input, R_t can be extracted simultaneously with $|\Delta P|$ and $\delta_{\Delta P}$.

Following the strategies proposed in this section, it should be possible to obtain interesting insights into the dynamics of rescattering and penguin processes, and in particular to take them into account in the generalized bounds on the CKM angle γ, which are implied by $R_{\min}$ given in (31). Other strategies to accomplish this task have recently been proposed in [15, 16], where also the uncertainties related to electroweak penguins [3, 12] and their control through experimental data are discussed.

5 A Brief Look at New Physics

Let us assume in this section that B_d^0–$\overline{B_d^0}$ mixing receives significant contributions from physics beyond the Standard Model, so that the B_d^0–$\overline{B_d^0}$ mixing phase is no longer given by 2β, but by a general phase (for the notation, see for instance [36])

$$\phi_{\rm M}^{(d)} = 2\beta + 2\phi_{\rm new}^{(d)} \,. \tag{57}$$

Since it is unlikely that the $B_d \to J/\psi\, K_{\rm S}$ decay amplitude is affected sizeably by new-physics contributions [37, 38], this mode allows us to determine $\phi_{\rm M}^{(d)}$ from its mixing-induced CP asymmetry as in the case of the Standard Model analysis.

If $B_d \to K^0\overline{K^0}$ is still governed by the Standard Model diagrams, its mixing-induced CP asymmetry (56) is modified as

$$\mathcal{A}_{\rm CP}^{\rm mix-ind}(B_d \to K^0\overline{K^0}) = \tag{58}$$
$$\frac{R_t^2\,\sin 2\phi_{\rm new}^{(d)} - 2\,R_t\,|\Delta P^{(d)}|\,\cos\delta_{\Delta P}^{(d)}\,\sin\left(\beta + 2\phi_{\rm new}^{(d)}\right) + |\Delta P^{(d)}|^2 \sin\left(2\beta + 2\phi_{\rm new}^{(d)}\right)}{R_t^2 - 2\,R_t\,|\Delta P^{(d)}|\,\cos\delta_{\Delta P}^{(d)}\,\cos\beta + |\Delta P^{(d)}|^2}\,,$$

while the form of the direct CP asymmetry is still given by (55). Note that the "true" value of R_t enters in (58). Although there are some models for new physics with a significant contribution to both B_d^0–$\overline{B_d^0}$ and B_s^0–$\overline{B_s^0}$ mixings, where their ratio still gives the "true" value of R_t, this need not be the case in general. In [39], a "model-independent" construction of the unitarity triangle was proposed, which holds for rather general scenarios of physics beyond the Standard Model. The key assumption is that new physics contributes significantly only to B_d^0–$\overline{B_d^0}$ mixing, as has also been done in this section. In particular, $\phi_{\rm new}^{(d)}$ and the "true" values of R_t and β can be determined this way.

An important feature of (58) is that the new-physics phase $\phi_{\rm new}^{(d)}$ does not enter in the combination $\beta + \phi_{\rm new}^{(d)}$ proportional to $\phi_{\rm M}^{(d)}$, since β shows up also in the decay amplitude (15). If $B^\pm \to \pi^\pm K$ is also governed by the Standard Model contributions – large CP violation in this mode, i.e. much larger than $\mathcal{O}(10\%)$, would indicate that this assumption does not hold – the expression (40) for the observable H remains unchanged as well. Consequently, using the $SU(3)$ flavour symmetry, $\phi_{\rm M}^{(d)}$ and the "true" value of R_t (determined as sketched above) as input parameters, the observables H, $\mathcal{A}_{\rm CP}^{\rm mix-ind}(B_d \to K^0\overline{K^0})$

and direct CP violation in either $B_{u,d} \to K\overline{K}$ or $B^\pm \to \pi^\pm K$ allow the simultaneous determination of $|\Delta P|$, $\delta_{\Delta P}$, β and $\phi_{\rm new}^{(d)}$. Comparing the values of β and $\phi_{\rm new}^{(d)}$ obtained this way with those from the "model-independent" approach [39], we may find indications for new physics if a significant disagreement should show up. If the experimental probes for the annihilation amplitude $A^{(s)}$ and the "penguin annihilation" topologies discussed in the previous section show that these contributions play in fact a minor role, the most plausible interpretation of such a disagreement would be a new-physics contribution to the "penguin-induced" $B_d \to K^0\overline{K^0}$ decay amplitude, although it could in principle also originate from new-physics effects in the "tree-dominated" decay $B_d \to J/\psi K_{\rm S}$. An unambiguous indication for the latter scenario would e.g. be sizeable direct CP violation in $B_d \to J/\psi K_{\rm S}$.

6 Conclusions

In summary, we have performed an analysis of $B \to \pi K$ decays in the presence of rescattering processes. We may distinguish between two kinds of such final-state interaction effects. The first one is related to rescattering processes of the kind $B^+ \to \{\overline{D^0}D_s^+\} \to \pi^+K^0$ and can be considered as a long-distance contribution to penguin topologies with internal charm quarks. Such topologies may affect the branching ratios for the decays $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ considerably. On the other hand, the second class, which is due to rescattering effects of the kind $B^+ \to \{\pi^0K^+\} \to \pi^+K^0$, is related to penguin topologies with internal up quark exchanges and to annihilation processes, and does not affect the branching ratios significantly. However, it may lead to a sizeable CP-violating weak phase in the $B^+ \to \pi^+K^0$ decay amplitude, and could thereby induce CP asymmetries at the level of 10% [11]–[14]. The corresponding rescattering effects represent an important limitation of the theoretical accuracy of the "naïve" bounds on the CKM angle γ derived in [2].

Moreover, we have derived isospin relations among the $B^+ \to \pi^+K^0$ and $B_d^0 \to \pi^-K^+$ decay amplitudes by defining "tree" and "penguin" amplitudes in a proper way. These relations play a key role to probe γ and allow the derivation of generalized bounds on this CKM angle. Due to a subtlety in implementing the $SU(2)$ isospin symmetry of strong interactions, the "tree" amplitude defined this way receives not only colour-allowed "tree" contributions, but also contributions from penguin and annihilation topologies, and may therefore be shifted significantly from its "factorized" value. Interestingly, the small present central value $R = 0.65$, which has recently been measured by the CLEO collaboration, may already indicate that this is actually the case.

Instead of performing another attempt to "calculate" rescattering effects – a realistic theoretical treatment is unfortunately out of reach at present – we advocate to use experimental data to obtain insights into this phenomenon. In this respect, the decays $B^\pm \to K^\pm K$ and $B^\pm \to \pi^\pm K$ are of particular interest. Using the $SU(3)$ flavour symmetry, β and R_t as an input, the combined branching ratios for these modes imply a range for $|\Delta P|$. Measuring moreover direct CP violation in these decays – the corresponding

CP asymmetries can interestingly be related to the combined branching ratios with the help of the $SU(3)$ flavour symmetry – both $|\Delta P|$ and $\delta_{\Delta P}$ can be determined. Following these lines, the rescattering processes can be taken into account completely in the generalized bounds on γ derived in [15]. A different strategy using the $B^\pm \to K^\pm K$ and $B^\pm \to \pi^\pm K$ observables to accomplish this goal was proposed in [15, 16].

In order to obtain experimental insights into penguin processes, the decay $B_d \to K^0\overline{K^0}$, which exhibits in contrast to the other decays considered in this paper mixing-induced CP violation, plays also an important role. Combining the mixing-induced and direct CP-violating observables of this mode with each other, the knowledge of β allows the determination of $|\Delta P^{(d)}|/R_t$ and $\delta^{(d)}_{\Delta P}$ without using any flavour symmetry arguments. Moreover, the importance of annihilation and penguin annihilation topologies can be probed this way. If these contributions should turn out to be of minor importance, $B_d \to K^0\overline{K^0}$ may not only shed light on the dynamics of penguin decays, but also on new-physics contributions to the $B_d \to K^0\overline{K^0}$ decay amplitude.

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Appendix: Isospin Decomposition

This appendix is devoted to the explicit derivation of the isospin decomposition of the $B^+ \to \pi^+ K^0$ and $B^0_d \to \pi^- K^+$ decay amplitudes. Let us to this end write the low energy effective Hamiltonian (4) as

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}}\left[\lambda_u^{(s)} H_u + \lambda_c^{(s)} H_c + \lambda_t^{(s)} H_{\rm P}\right], \tag{59}$$

where

$$H_u = C_1(\mu) Q_1^u + C_2(\mu) Q_2^u = H_u^0 + H_u^1 \tag{60}$$

$$H_c = C_1(\mu) Q_1^c + C_2(\mu) Q_2^c \tag{61}$$

$$H_{\rm P} = -\sum_{i=3}^{6} C_i(\mu) Q_i \tag{62}$$

with

$$H_u^0 \equiv \frac{1}{2}\left[C_1(\mu)\left(Q_1^u + Q_1^d\right) + C_2(\mu)\left(Q_2^u + Q_2^d\right)\right] \tag{63}$$

$$H_u^1 \equiv \frac{1}{2}\left[C_1(\mu)\left(Q_1^u - Q_1^d\right) + C_2(\mu)\left(Q_2^u - Q_2^d\right)\right]. \tag{64}$$

While H_c and $H_{\rm P}$ correspond to $|I, I_3\rangle = |0,0\rangle$ isospin configurations, in the case of H_u both $|0,0\rangle$ and $|1,0\rangle$ pieces are present, which are described by H_u^0 and H_u^1, respectively.

As is well-known, the B^+ and B_d^0 mesons form an isospin doublet, i.e.

$$\left|B^+\right\rangle = \left|\frac{1}{2},+\frac{1}{2}\right\rangle, \quad \left|B_d^0\right\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle, \tag{65}$$

whereas the isospin decomposition of the π^+K^0, π^-K^+ final states is given by

$$\left|\pi^+K^0\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},+\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2},+\frac{1}{2}\right\rangle \tag{66}$$

$$\left|\pi^-K^+\right\rangle = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle, \tag{67}$$

and contains both $I=1/2$ and $I=3/2$ components.

Taking into account that

$$|1,0\rangle \otimes \left|\frac{1}{2},+\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},+\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2},+\frac{1}{2}\right\rangle \tag{68}$$

$$|1,0\rangle \otimes \left|\frac{1}{2},-\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle \tag{69}$$

and using moreover the Wigner–Eckart theorem, we arrive at

$$\left\langle K^0\pi^+\right| H_u \left|B^+\right\rangle = \sqrt{2}\left[\frac{1}{3}\left(M_u^{1\prime} - M_u^1\right) + \sqrt{\frac{1}{3}}M_u^0\right] \tag{70}$$

$$\left\langle K^+\pi^-\right| H_u \left|B_d^0\right\rangle = \sqrt{2}\left[\frac{1}{3}\left(M_u^{1\prime} - M_u^1\right) - \sqrt{\frac{1}{3}}M_u^0\right], \tag{71}$$

where

$$\begin{aligned} M_u^0 &\equiv \left\langle \frac{1}{2},\pm\frac{1}{2}\right| H_u^0 \left|\frac{1}{2},\pm\frac{1}{2}\right\rangle \\ M_u^1 &\equiv \mp\sqrt{3}\left\langle \frac{1}{2},\pm\frac{1}{2}\right| H_u^1 \left|\frac{1}{2},\pm\frac{1}{2}\right\rangle \\ M_u^{1\prime} &\equiv \sqrt{\frac{3}{2}}\left\langle \frac{3}{2},\pm\frac{1}{2}\right| H_u^1 \left|\frac{1}{2},\pm\frac{1}{2}\right\rangle \end{aligned} \tag{72}$$

are "reduced" matrix elements, which carry CP-conserving strong phases in our formalism. Since H_c and $H_{\rm P}$ correspond to isospin singlet configurations, we have

$$\left\langle K^0\pi^+\right| H_c \left|B^+\right\rangle = \sqrt{\frac{2}{3}}M_c^0, \quad \left\langle K^0\pi^+\right| H_{\rm P} \left|B^+\right\rangle = \sqrt{\frac{2}{3}}M_{\rm P}^0 \tag{73}$$

$$\left\langle K^+\pi^-\right| H_c \left|B_d^0\right\rangle = -\sqrt{\frac{2}{3}}M_c^0, \quad \left\langle K^+\pi^-\right| H_{\rm P} \left|B_d^0\right\rangle = -\sqrt{\frac{2}{3}}M_{\rm P}^0, \tag{74}$$

where the reduced matrix elements are defined in analogy to (72).

Combining all these equations, we arrive at

$$A(B^+ \to \pi^+ K^0) \equiv \left\langle K^0\pi^+ \right| \mathcal{H}_{\rm eff} \left| B^+ \right\rangle$$

$$= G_{\rm F}\left[\lambda_u^{(s)}\left\{\frac{1}{3}\left(M_u^{1'} - M_u^1\right) + \sqrt{\frac{1}{3}}M_u^0\right\} + \lambda_c^{(s)}\sqrt{\frac{1}{3}}M_c^0 + \lambda_t^{(s)}\sqrt{\frac{1}{3}}M_{\rm P}^0\right] \equiv P^{(s)} \quad (75)$$

$$A(B_d^0 \to \pi^- K^+) \equiv \left\langle K^+\pi^- \right| \mathcal{H}_{\rm eff} \left| B_d^0 \right\rangle$$

$$= -\,G_{\rm F}\left[\lambda_u^{(s)}\left\{\frac{1}{3}\left(M_u^1 - M_u^{1'}\right) + \sqrt{\frac{1}{3}}M_u^0\right\} + \lambda_c^{(s)}\sqrt{\frac{1}{3}}M_c^0 + \lambda_t^{(s)}\sqrt{\frac{1}{3}}M_{\rm P}^0\right]. \quad (76)$$

Comparing (75) with (9), it is easy to read off the isospin decompositions of $P_u^{(s)}$, $P_c^{(s)}$ and $P_t^{(s)}$, while $T^{(s)}$ has to be defined by

$$T^{(s)} \equiv \lambda_u^{(s)}\, G_{\rm F}\, \frac{2}{3}\left(M_u^1 - M_u^{1'}\right) \quad (77)$$

in order to get the isospin relations (19) and (20), which are at the basis of the bounds on γ derived in [15]. This discussion shows nicely that these relations can be derived by using only isospin arguments, i.e. even without using the terminology of "penguin" and "tree" contributions as is usually done by CP practitioners.

Let us finally note that (11)–(13) and (26), which are expressed in terms of hadronic matrix elements of four-quark operators, can be obtained easily from (75)–(77) by taking into account that (70) and (71) yield

$$\left\langle K^0\pi^+ \right| H_u \left| B^+ \right\rangle + \left\langle K^+\pi^- \right| H_u \left| B_d^0 \right\rangle = \frac{2}{3}\sqrt{2}\left(M_u^{1'} - M_u^1\right) \quad (78)$$

$$\left\langle K^0\pi^+ \right| H_u \left| B^+ \right\rangle - \left\langle K^+\pi^- \right| H_u \left| B_d^0 \right\rangle = 2\sqrt{\frac{2}{3}}\, M_u^0\,, \quad (79)$$

and using moreover (60)–(62).

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