HEAVY QUARK EXPANSION AND INCLUSIVE DECAYS

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Some of the recent developments in the theory and applications of the heavy quark expansion for inclusive processes are reviewed. The topics concern semileptonic and nonleptonic decays, lifetimes and the semileptonic branching fraction.

1 Introduction

The heavy mass expansion has become the standard tool in the theoretical description of heavy quark processes, since in many cases it reduces the model dependence in a significant way and often allows for purely QCD-based calculations [1]. The heavy mass limit of QCD may be formulated as an effective theory, the so called Heavy Quark Effective Theory (HQET) [2]. HQET exhibits additional symmetries beyond the ones known from QCD [3], and these symmetries restrict the nonperturbative aspects of heavy quark processes, leading to model independent relations valid in the heavy mass limit. Furthermore, the framework of effective theory allows us to calculate radiative corrections to the heavy quark limit and to access the structure of the recoil (i.e. the $1/m_Q$) corrections.

In order to set up an $1/m_Q$ expansion for inclusive decays HQET is combined with the method of the operator product expansion (OPE) [4].The approach applied in this case resembles very much the description of deep inelastic scattering (DIS). While in DIS the large scale is the momentum transfer, one can rewrite the expressions in the case of heavy quark decays in such a way that the heavy quark mass m_Q is the relevant large scale for the (OPE).

The OPE yields an infinite sum of operators with increasing dimension as the first step in obtaining a $1/m_Q$ expansion. The dimensions of the operators are compensated by inverse powers of a large scale, which is in general of the order of the heavy mass scale. The decay probability is then given as forward matrix elements of these operators between the state of the decaying heavy hadron; these matrix elements still have a mass dependence, which then may be extracted in terms of a $1/m_Q$ expansion using HQET as for exclusive decays.

In the following we shall first outline the theoretical background for inclusive decays and then apply the method to semileptonic and nonleptonic processes.

2 Theoretical Background

Weak decays of bottom and charm quarks are usually described in terms of an effective Hamiltonian $\mathcal{H}_{eff}$ derived form the full electroweak theory in the limit where all relevant scales are much smaller than the mass of the weak bosons. Dealing with $\Delta B = \pm 1$ processes this Hamiltonian contains only a single heavy quark operator. Expressed in terms of $\mathcal{H}_{eff}$ the inclusive decay rate becomes

$$\begin{aligned}\Gamma \propto \quad & \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X|\mathcal{H}_{eff}|B(P_B)\rangle|^2 \\ &= \int d^4x \, \langle B(P_B)|\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)|B(P_B)\rangle \\ &= 2 \text{ Im} \int d^4x \, \langle B(P_B)|T\{\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)\}|B(P_B)\rangle .\end{aligned} \tag{1}$$

where $|B(P_B)\rangle$ is the decaying heavy meson moving with momentum P_B and $|X\rangle$ is the final state with momentum P_X, and a sum over $|X\rangle$ is performed to obtain the inclusive rate.

The matrix element appearing in (1) contains a large scale, namely the mass of the heavy quark. The first step towards a $1/m_Q$ expansion is to make this large scale explicit. This may be achieved by a phase redefinition of the heavy quark field removing the large part of the heavy quark momentum $m_Q v$

$$Q_v(x) = e^{im_Q(v\cdot x)} Q(x) \tag{2}$$

This corresponds to the standard splitting of the heavy quark momentum p_Q into a large piece and a residual part $p_Q = m_Q v + k$; hence the space time dependence of the field Q_v is related to the residual momentum k by Fourier transformation.

Inserting this into (1) yields

$$\Gamma \propto 2 \text{ Im} \int d^4x \, e^{-im_Q vx} \, \langle H(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}|H(v)\rangle \tag{3}$$

where

$$\widetilde{\mathcal{H}}_{eff} = \mathcal{H}_{eff}(Q \longrightarrow Q_v) \tag{4}$$

is the effective Hamiltonian with the replacement $Q \to Q_v$.

This relation exhibits the similarity between the cross-section calculation in DIS and the present approach to total rates. In DIS there appears a large scale, which is the momentum transfer to the leptons, while here the mass of the heavy quark appears as a large scale.

Analogous to DIS, the next step is to perform an OPE of the product of the two Hamiltonians. After the phase redefinition (2) the remaining matrix element does not involve large momenta of the order of the heavy quark mass any more and hence a short-distance expansion becomes useful if the mass m_Q is large compared to the scale $\bar{\Lambda}$ determining the matrix element. The OPE for the total rate takes the general form

$$\int d^4x\, e^{im_Q vx} \langle H(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}|H(v)\rangle \qquad (5)$$
$$= \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n \hat{C}_{n+3}(\mu)\langle H(v)|\mathcal{O}_{n+3}|H(v)\rangle_\mu,$$

where $\mathcal{O}_n$ are operators of dimension n, with their matrix elements renormalized at scale μ, and $\hat{C}_n$ are the corresponding Wilson coefficients. These coefficients encode the short distance physics related to the heavy quark mass scale and may be calculated in perturbation theory. All long distance contributions connected to the hadronic scale $\bar{\Lambda}$ are contained in the matrix elements of the operators $\mathcal{O}_{n+3}$.

Still the matrix elements of $\mathcal{O}_{n+3}$ are not independent of the heavy quark mass scale, but this mass dependence may be expanded in powers of $1/m_Q$ by means of HQET. In this way the mass dependence of the total decay rate may be accessed completely within an expansion in $1/m_Q$.

The lowest-order term of the OPE are the dimension-3 operators. To leading order in the $1/m_Q$ expansion only one operator appears, any other operator may be related to it up to terms of order $1/m_Q^2$. Usually one chooses $\mathcal{O}_3 = \bar{Q}_v Q_v$ of which the relevant forward matrix element is normalized, since it may be related to the conserved current $\bar{Q}\gamma_\mu Q$.

Thus the leading contribution can always be evaluated without hadronic uncertainties and generically yields the free quark decay rate. This has been previously used as a model for inclusive decays, but now it turns out to be the first term in a systematic $1/m_Q$ expansion of total rates.

Any dimension-four operator may be related to the operator

$$\mathcal{O}_4 \propto \bar{Q}_v \Gamma D_\mu Q_v, \qquad (6)$$

the matrix elements of which vanish by the equations of motion. This statement is equivalent to Luke's theorem [5] derived in the context of exclusive decays.

The first non-trivial non-perturbative contribution comes from dimension-5 operators and are of order $1/m_Q^2$. Their order of magnitude for B decays is small, since $\bar{\Lambda}^2/m_Q^2 \sim (2-3)\%$.

For mesonic decays there are only the two parameters λ_1 and λ_2 defined by the matrix elements between 0^- states

$$2m_B\lambda_1 = \langle H(v)|\bar{h}_v(iD)^2 h_v|H(v)\rangle \tag{7}$$
$$2m_B\lambda_2 = \langle H(v)|\bar{h}_v\sigma_{\mu\nu}iD^\mu iD^\nu h_v|H(v)\rangle \tag{8}$$

The second parameter λ_2 may be related to the mass splitting in the ground state spin symmetry multiplet

$$\lambda_2 = 0.12\mathrm{GeV}^2 \tag{9}$$

We note in passing that the operator defining λ_2 has non-vanishing anomalous dimension and hence λ_2 in fact depends on the scale; the value quoted in (9) is $\lambda_2(m_b)$.

A determination of λ_1 is much more problematic. For instance, λ_1 may be related to the pole masses of the heavy b and c quarks as

$$m_b - m_c = \bar{M}_B - \bar{M}_D + \lambda_1\left(\frac{1}{2\bar{M}_B} - \frac{1}{2\bar{M}_D}\right) + \mathcal{O}(1/m_Q^3) \tag{10}$$

where $\bar{M}_{B/D}$ are spin averaged masses of the ground state spin symmetry doublet in the B and the D system respectively. Relation (10) connects λ_1 with the quark pole masses, which are known to be affected by renormalon ambiguities [6]; it has been pointed out recently that also λ_1 suffers from such a problem [7].

Once one defines a procedure how to extract a value for λ_1 from experiment or other theoretical considerations one may quote various values, which cover quite a large range

$$0.10 \leq -\lambda_1 \leq 0.66 \tag{11}$$

For details of the various methods we refer the reader to a recent review [1]. In any case, the estimate of the order of magnitude of the leading non-perturbative corrections is confirmed by more sophisticated methods.

The higher order terms of the $1/m_Q$ expansion have been investigated to some extent. In general, the number of independent operators is increasing drastically and a complete analysis becomes difficult. However, some of the $1/m_Q^3$ terms are believed to play a role in the lifetime patterns of heavy hadrons and we shall return to this in section 4.1.

3 Semileptonic Processes

For semileptonic processes the method of OPE may be applied not only to the full H_{eff}, but already to the product of the two hadronic currents appearing in the rate. In this way one may also obtain differential distributions, e.g. the distribution of the leptonic q^2, the energy spectrum of the charged lepton as well as the hadronic invariant mass spectrum of the final state.

The total rate is obtained by integrating these differential distributions, thereby averaging the partonic result for the correlator of the two hadronic currents with a weight given by the appropriate phase space. Hence in semileptonic processe only the concept of "global duality" is used, i.e. the assumption that sufficiently averaged ("smeared") partonic results equal the corresponding hadronic ones. For this reason it is generally believed that the results obtained for semileptonic processes are on firm theoretical grounds.

In the following we shall quote the results obtained for semileptonic processes and discuss the implications for the determination of CKM matrix elements.

3.1 Total Rates and the Determination of V_{cb}

Including the first non-trivial non-perturbative corrections one obtains for the semileptonic decay $B \to X_c \ell \bar{\nu}_\ell$ to leading order α_s

$$\Gamma(B \to X_c \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[\left(1 + \frac{\lambda_1}{2m_c^2}\right) z_0 \left(\frac{m_c^2}{m_b^2}\right) - \frac{9\lambda_2}{2m_c^2} z_2 \left(\frac{m_c^2}{m_b^2}\right) \right], \tag{12}$$

where the two z_j are phase-space functions

$$\begin{aligned} z_0(x) &= 1 - 8x + 8x^3 - x^4 - 12x^2 \log x, \\ z_2(x) &= 1 - \frac{8}{3}x - 8x^2 + 8x^3 + \frac{5}{3}x^4 + 4x^2 \log x. \end{aligned} \tag{13}$$

The corresponding expression for the decay $B \to X_u \ell \bar{\nu}_\ell$ is obtained in the limit $m_c \to 0$ and the replacement $V_{cb} \to V_{ub}$; it reads

$$\Gamma(B \to X_u \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left[1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2}\right]. \tag{14}$$

Note that the non-perturbative corrections are indeed small, and consequently the main corrections are the radiative corrections governed by $\alpha_s(m_Q)$, which are found to be of the order of 10% [8].

The inclusive decay $B \to X \ell \bar{\nu}_\ell$ is dominated by the decays involving charm in the final state and hence the total rate for $B \to X \ell \bar{\nu}_\ell$ or more precisely $B \to X_c \ell \bar{\nu}_\ell$ may be used to extract V_{cb}. On the first sight this seems to be a bad idea, since superficially the rate depends on the fifth power of the heavy quark mass which is not accurately known, and in addition radiative corrections turned out to be large, thereby inducing a sizable scale dependence.

However, it has been pointed out [9], that one may still obtain V_{cb} with relatively small theoretical uncertainties, mainly for two reasons.

First due to the phase space function appearing in (12) the dependence on m_b is not really m_b^5 but somemwhat milder

$$\Gamma(B \to X_c \ell \bar{\nu}_\ell) \propto m_b^5 z_0 \left(\frac{m_c^2}{m_b^2}\right) \approx (m_b - m_c)^{3.2} (m_b + m_c)^{1.8} \qquad (15)$$

where this is valid in the vicinity of a value of the mass ratio $m_c/m_b = 0.3$. Using the fact that the difference of the quark masses is much better known that their individual values (see (10)) one may reduce the uncertainties due to the quark mass dependences.

Second the radiative corrections and the definition of the mass are strongly correlated [10], i.e. the size of the radiative corrections depends on the choice of the heavy quark mass. Neglecting non-perturbative contributions, the radiative corrections may in general be written as

$$\Gamma(B \to X_c \ell \bar{\nu}_\ell) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 z_0 \left(\frac{m_c^2}{m_b^2}\right) \left(1 + a_1 \frac{\alpha_s}{\pi} + a_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \cdots\right) \quad (16)$$

where the exact value of a_2 depends on the renormalization scheme used. We shall not go into that much detail here and only give the orders of magnitude.

If one chooses e.g. to take as the heavy quark mass the pole mass one finds $a_1 \approx 1.7$ and $a_2 = \mathcal{O}(-10)$ and hence large radiative corrections where even higher order terms become to be important. However, using a running mass (such as the $\overline{MS}$-mass) normalized at some small scale $\mu \approx 1$ GeV, the perturbative corrections are found to be much better behaved: $a_1 \approx 1$ and $a_2 \approx 1$.

Based on this a fairly reliable and model independent extraction of V_{cb} is possible. In fact, it has been argued that this method may be at least as reliable as the determination of V_{cb} based on exclusive decays. While in exclusive $B \to D^*$ transitions the corrections are parametrically of the order $1/m_c^2$ they are in the inclusive decay only of order $1/m_b^2$ and hence may be substantially smaller. This is, however, compensated by the uncertainty in m_b on which the inclusive rate depends, while the leading terms for the exclusive rate depends only on the meson masses.

Two groups have estimated the theoretical uncertainties involved in the extraction ov V_{cb} from inclusive decays. Bigi, Shifman and Uraltsev find (see e.g. their review in ref[1])

$$
\begin{aligned}
|V_{cb}| &= 0.0419 \left(\frac{Br_{sl}}{10.5\%}\right)^{1/2} \left(\frac{1.55 \text{ ps}}{\tau_B}\right)^{1/2} \\
&\quad \times \left(1 - 0.012 \frac{-\lambda_1 - 0.5 \text{ GeV}^2}{0.1 \text{ GeV}^2}\right) (1 \pm 0.015 \pm 0.01 \pm 0.012)
\end{aligned}
\tag{17}
$$

where the uncertainties are due to higher orders in perturbation theory, to the value of m_b and to orders $1/m_Q^3$ and higher.

A more pessimistic estimate of the theoretical uncertainties has been given by Neubert who assigns a larger uncertainty to the value of the b quark mass. He finds[12]

$$
|V_{cb}| = (0.040 \pm 0.004) \left(\frac{Br_{sl}}{10.8\%}\right)^{1/2}
\tag{18}
$$

where in this uncertainty all sources have been combined.

Even with the more pessimistic point of view (18) a model independent determination of V_{cb} is feasible from inclusive decays, with theoretical uncertainties comparable to the determination from $B \to D^* \ell \bar{\nu}_\ell$. The fact that the value of V_{cb} extracted in these two ways coincides with good accuracy is very reassuring.

3.2 Differential Distributions and V_{ub}

As pointed out above one may as well calculate differential distributions using the OPE and HQET. In this case the relevant scale for the OPE is not the heavy quark mass, but rather the variable $Q = m_Q v - q$, where q is the momentum transferred to the leptons. Note that to leading order, Q^2 is the invariant mass of the final state hadrons.

Analogous to DIS the hadronic tensor depends on the Lorentz-invariants $\sqrt{Q^2}$ and vQ and one has to distinguish various regions of phase space depending on the values of these variables.

In the bulk part of phase space both variables are of order m_Q and one may perform the OPE in the way outlined above. In this case one obtains the nonperturbative corrections expressed in terms of λ_1 and λ_2 as for the total rates. As an example we quote the result for the lepton energy spectrum for $B \to X_u \ell \bar{\nu}_\ell$

$$
\begin{aligned}
\frac{d\Gamma}{dy} &= \frac{G_F^2 \, |V_{ub}|^2 \, m_b^5}{192\pi^3} \left[\left(2y^2(3-2y) + \frac{10y^2}{3} \frac{\lambda_1}{m_b^2} + 2y(6+5y) \frac{\lambda_2}{m_b^2} \right) \Theta(1-y) \right. \\
&\quad \left. - \frac{\lambda_1 + 33\lambda_2}{3m_b^2} \delta(1-y) - \frac{\lambda_1}{3m_b^2} \delta'(1-y) \right]
\end{aligned}
\tag{19}
$$

where y is the normalized energy of the charged lepton $y = 2E_\ell/m_b$.

This function is well behaved except close to the endpoint of the spectrum, where δ-function-like singularities appear. This problem may be traced back to the fact that close to the endpoint the OPE breaks down.

In order to cure this problem we again distinguish two cases. We define the endpoint region in which Q^2 becomes small although the components of Q are still of order m_Q: $Q^2 = \mathcal{O}(\Lambda_{QCD} m_Q)$ and $vQ = \mathcal{O}(m_Q)$. In this region the OPE needs to be modified and instead of only two parameters one has to introduce a "shape function" to parametrize the nonperturbative input. Without going into details, the spectrum for $B \to X_u \ell \bar{\nu}_\ell$ is written as

$$\begin{aligned} \frac{d\Gamma}{dE_\ell} &= \frac{G_F^2 \,|\, V_{qb}|^2}{12\pi^3} E_\ell^2 \int dk_+ \, f(k_+) \, \Theta \Big[m_b^* - 2\, E_\ell \Big] \\ & \quad \times \Big\{ 3 m_b^{*2} - 4 m_b^* \, E_\ell \Big\} . \end{aligned} \tag{20}$$

where the "shape function" $f(k_+)$ is given by the formal expression

$$f(k_+) = \langle B(v) | \, \bar{h}_v \, \delta(k_+ - iD_+) \, h_v \, | B(v) \rangle \tag{21}$$

and m^* is an "effective mass" $m^* = m_b + k_+$. The first few moments A_n of the structure function

$$A_n = \int \mathrm{d}k_+ \, k_+^n \, f(k_+)$$

are known: $A_0 = 1$ is the normalization of $f(k_+)$, $A_1 = 0$ due to the equations of motion of the heavy quark and $A_2 = -\lambda_1/3$.

Closer to the endpoint we enter the resonance region, where $Q^2 \sim \Lambda_{QCD}^2$ and $vQ \sim \Lambda_{QCD}$. Here the rate is dominated by single states or resonances and the appropriate description would be to sum over the lowest few resonances.

The semileptonic $b \to u$ transitions may be used to extract V_{ub}. However, the total inclusive semileptonic rate is dominated by the charm contribution and one has to look at differential rates to access V_{ub} by studying phase space regions where no charm decays may occur. Traditionally people have used the lepton energy looking at large energies beyond the charm endpoint. Recently a new method has been proposed which uses the hadronic invariant mass spectrum, where the part between m_π and m_D can only be due to $b \to u$ transitions. On the other hand, one may calculate this spectrum using the OPE and HQET. It turns out[13,14] that for a reasonable ansatz[14] for the shape function the part of the spectrum which extends beyond m_D is small, of the order of 20%. A useful quantity is the normalized integral of the spectrum up

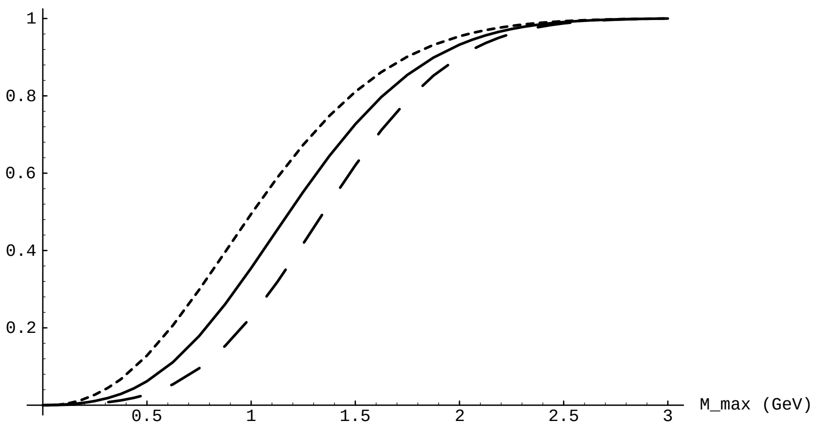

Figure 1: The integrated hadronic invariant mass spectrum as a function on M_{max}. The different curves correspond to $m_b = 4.72$ GeV (long dashed), $m_b = 4.82$ GeV (solid), $m_b = 4.92$ GeV (short dashed). The ansatz for the shape function corresponds to $\lambda_1 = -0.4$ GeV^2 and $\alpha_s = 0.3$. The figure is taken from [14].

to a maximal invariant mass M_{max}

$$\Phi(M_{max}) = \frac{1}{\Gamma(b \to u\ell\bar{\nu}_\ell)} \int_0^{M_{max}} dM_X \frac{d\Gamma}{dM_X}$$

In fig.1 we plot Φ as a function of M_{max} where the different curves correspond to different values of m_b.

Fig.1 indicates that indeed at least 70% of the hadronic invariant mass spectrum lies below m_D and hence method may allow for a model independent determination of V_{ub}. Note that this statement is almost independent of the special ansatz for the shape function, since the integrated quantity Φ depends only on the low moments of the shape function, i.e. on λ_1 [13].

4 Nonleptonic Processes

The method of OPE and HQET may also be used to calculate rates for purely hadronic processes, in particular lifetimes. However, compared to the semileptonic case, where the intergration over the leptonic phase space ensured some "smearing" of the hadronic (or partonic) quantities, the situation for nonleptonic processes is slightly different. Instead of global duality one has to use "local duality" which means that partonic and hadronic quantities are compared point by point. This is potentially dangerous, although in the present application local duality is used far away from the resonance region, where it certainly would fail. Not much can be said about the reliability of duality in general and local duality in particular, so we shall proceed and assume that global as well as local duality may be applied.

4.1 Lifetimes

To leading order in the $1/m_Q$ expansion all lifetime ratios are unity, since the lifetimes are then determined by the free quark decay. If we assume light flavour symmetry for the matrix elements λ_1 and λ_2, the corrections to the ratio between two meson lifetimes has to be of order $1/m_Q^3$ and is hence given by dimension six operators, which we have not yet discussed in the previous sections. Since the value of λ_1 is different in a meson and a baryon and $\lambda_2 = 0$ for a Λ_Q baryon, we expect

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + O(1/m_b^3)\,, \frac{\tau(B_s)}{\tau(B_d)} = 1 + O(1/m_b^3)\,, \frac{\tau(\Lambda_b)}{\tau(B_d)} = 1 + O(1/m_b^2) \quad (22)$$

In order to discuss the lifetime ratios for mesons one needs to consider higher orders in the $1/m_Q$ expansion. Unfortunately, in general the number of dimension six operators that appear at order $1/m_Q^3$ is quite large. However, not all of these operators contribute to lifetime ratios, since they have to be sensitive to the light quark inside the heavy hadron [15,4].

These types of spectator effects have been reconsidered recently in [16] and we shall follow this line of argument. Although intuitively these dimension six operators seem to yield only small contributions, their effect is enhanced by a phase space factor. The non-spectator effects typically yield at the partonic level at least a three particle phase space, while the spectator effects have to leading order only a two particle final state. This yields an enhancement of these contributions by a phase space factor $16\pi^2$. Qualitatively, this means that the $1/m_Q$ expansion for the lifetime ratios takes the form

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + \frac{1}{m_b^3}\left[a_0 + \frac{1}{m_b}a_1 + \cdots\right] + \frac{16\pi^2}{m_b^3}\left[b_0 + \frac{1}{m_b}b_1 + \cdots\right] \quad (23)$$

and hence the coefficient b_0 is the leading contribution to the lifetime ratio.

The contribution b_0 can be evaluated in terms of the forward matrix elements of the dimension-six operators involving the spectator quark. In total there are four relevant matrix elements which we parametrize as [16]

$$
\begin{aligned}
\frac{1}{2m_{B_q}}\langle B_q|\bar{b}_L\gamma_\mu q_L\,\bar{q}_L\gamma^\mu b_L|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}B_1\,,\\
\frac{1}{2m_{B_q}}\langle B_q|\bar{b}_R\,q_L\,\bar{q}_L\,b_R|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}B_2\,,\\
\frac{1}{2m_{B_q}}\langle B_q|\bar{b}_L\gamma_\mu t_a q_L\,\bar{q}_L\gamma^\mu t_a b_L|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}\varepsilon_1\,,\\
\frac{1}{2m_{B_q}}\langle B_q|\bar{b}_R\,t_a q_L\,\bar{q}_L\,t_a b_R|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}\varepsilon_2\,. \qquad (24)
\end{aligned}
$$

where $t_a = \lambda_a/2$ are the generators of colour SU(3). Note that if we assume factorization we have $B_i = 1$ and $\varepsilon_i = 0$. Factorization holds exactly in the limit of large N_c and hence one expects $B_i = \mathcal{O}(1)$ and $\varepsilon_i = O(1/N_c)$.

Following Ref.[16] one may now evaluate the lifetime ratio between charged and neutral B mesons in terms of B_i and ε_i

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + k_1 B_1 + k_2 B_2 + k_3\varepsilon_1 + k_4\varepsilon_2\,, \qquad (25)$$

where the k_i can be calculated and depend on the mass ratio m_c/m_b and the Wilson coefficients entering the effective Hamiltonian. These Wilson coefficients depend on the renormalization scale μ which is of order m_b. In table 1 we list the values for k_i for different scales μ.

μ	k_1	k_2	k_3	k_4
$m_b/2$	$+0.044$	0.003	-0.735	0.201
m_b	$+0.020$	0.004	-0.697	0.195
$2m_b$	-0.008	0.007	-0.665	0.189

Table 1: Values of the coefficients k_i for different choices of the renormalization scale μ. The value of m_c^2/m_b^2 is fixed at 0.085. The table is taken from [16].

It is interesting to note that the coefficients k_3 and k_4 of the octett-octett operators are larger by a factor five to ten, and if one expects these matrix elements to be suppressed by a factor $1/N_C$ it is a possible scenario that the nonfactorizable contributions actually dominate the corrections to the lifetime ratio. However, present data is compatible with neglecting these contributions.

A similar analysis maybe performed for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ The result found in [16] is

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.98 + \Delta \tag{26}$$

where Δ is of order $1/m_b^3$ and is most likely only a few percent. Given the experimental value which is considerably lower the potential problem with the Λ_b lifetime still remains.

4.2 The Semileptonic Branching Fraction and Charm Counting

Once one is able to calculate lifetimes one may also directly calculate the semileptonic branching fraction of b hadrons within the $1/m_Q$ expansion.

The total decay rate is composed of three contributions

$$\Gamma_{tot} = \Gamma(b \to c\ell\nu) + \Gamma(b \to c\bar{u}d') + \Gamma(b \to c\bar{c}s') \tag{27}$$

where the prime indicates the CKM rotated down quark fields. From this one can evaluate the semileptonic branching fraction $Br_{sl} = \Gamma(B \to X\ell\bar{\nu}_\ell)/\Gamma_{tot}$ and the number of charm and anticharm quark created per B decay $n_c = 1 + Br(b \to c\bar{c}s) - Br(b \to \text{non charm states})$. Both quantities may be calculated in terms of a $1/m_Q$ expansion, where the nonperturbative contributions are again parametrized in terms of λ_1 and λ_2. However, they turn out to be so small that they can safely be neglected. Furthermore, the radiative corrections to this quantity have been evaluated already up to next-to-leading order [11], which introduces a dependence on the renormalization scale μ.

The naive parton model predicts $Br_{sl} \sim 15\%$ and $n_c \sim 1.2$ Since the nonperturbative corrections are small it was concluded that values for Br_{sl} smaller that 12% are hard to accomodate in theory. However, the radiative corrections (the next to leading order analysis, including the effect of the charm mass, has been performed in [11]) lower the value of Br_{sl} significantly. The present theoretical situation is shown in fig.2 where the shaded aerea represents a reasonable variation of m_c/m_b and the renormalization point μ.

Since the experimental values obtained from the $\Upsilon(4s)$ machines ("low energy data") and from Z_0 decays ("high energy data") are slightly different, two experimental data points appear in fig.2. The experimental situation is reviewed elsewhere in these proceedings [17].

Given the new experimental and theoretical developments the potential problem with the semileptonic branching fraction seems to disappear; only the low energy data are still in disagreement with the theoretical predictions; however, this discrepancy is not very severe.

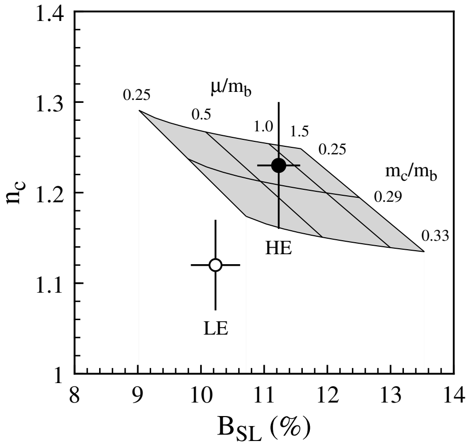

Figure 2: Predictions for Br_{sl} and n_c as a function of m_c/m_b and the renormalization point μ The data points come form the "low energy" and "high energy" measurements (see text). The figure is taken from [12].

5 Conclusions

The $1/m_Q$ expansion has become a widely accepted tool also for inclusive processes. It relies on the method of OPE, which itself is based on the concept of duality. While there is no proof for the assumption of duality, at least global duality consitutes one of the main ingredients of almost all QCD calculations.

In semileptonic processes global duality is assumed and hence the results for this class of processes seems on firm grounds. The total semileptonic rate allows to extract a value of V_{cb} with reasonable theoretical acuracy, which is in good agreement with the value obtained from exclusive decays. The combination ot the two methods has already reduced the relative uncertainty in our knowledge of V_{cb} to less than ten percent.

Differential distributions may be used to extract also V_{ub} from inclusive semileptonic decays. In particular, the hadronic invariant mass distribution is promising, since only a few tens of percent of the rate for $B \to X_u \ell \bar{\nu}_\ell$ is due to invariant masses larger than M_D. Thus a cut on the invariant hadronic mass can eleminate the charm background without affecting the $b \to u$ transitions severely.

The $1/m_Q$ expansion in inclusive nonleptonic processes invokes the concept of local duality, and the reliability of such calculations has been questioned [18]. Still the pattern of b-hadron lifetimes is reproduced at least qualitatively, although at the quantitative level some potential problems, such as the Λ_b lifetime, remain.

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