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Non-Abelian Dipole Radiation and the Heavy Quark Expansion

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Abstract

Dipole radiation in QCD is derived to the second order in α_s. A power-like evolution of the spin-singlet heavy quark operators is obtained to the same accuracy. We give an $\mathcal{O}(\alpha_s^2)$ relation between the short-distance low-scale running heavy quark mass and the $\overline{\rm MS}$ mass. We discuss the properties of an effective QCD coupling $\alpha_s^{(d)}(E)$ which governs the dipole radiation. We argue that this coupling is advantageous for describing heavy quark physics.

Theoretical description of heavy flavor decays benefits from a strong hierarchy between the mass of the decaying quark and the typical scale of the strong interactions, $m_{b(c)} \gg \Lambda_{\rm QCD}$. The current level of precision requires an accurate treatment of nonperturbative effects even in beauty decays. A consistent genuinely QCD-based framework for this is provided by the heavy quark expansion (HQE) which combines the Wilson operator product expansion (OPE) with the nonrelativistic expansion. It allows a simultaneous precise description of both perturbative and nonperturbative effects.

An important class of applications of the HQE are semileptonic weak transitions between b and c quarks, in which both initial and final quarks are heavy. The most informative predictions can be made for the transition amplitudes in the so-called small velocity (SV) limit [1], when the velocities of heavy hadrons in initial and final states are small. The heavy quark in this case plays a role of a slowly moving source of the color Coulomb field which affects the light degrees of freedom in the hadrons. This physical picture is formalized in

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the Wilsonian approach by integrating out high-momentum degrees of freedom of full QCD and resorting to an effective low-energy theory.

A peculiarity of the effective theory for heavy flavor hadrons is its essentially Minkowskian nature. Any process in which the velocity of the heavy quark Q changes involves gluon radiation with energy and momentum in the whole range up to the quark masses. On the other hand, when the energy loss is small compared to m_Q, the radiation off the heavy colored particles is almost a classical effect and therefore a universal process-independent description is possible. In the small velocity limit it is the familiar dipole radiation, with some peculiarities due to the non-Abelian nature of QCD.

Even though the dipole radiation has the most obvious manifestation in the SV processes, it is relevant in many other situations, from the zero-recoil transitions to inclusive decay widths. The reason is that the OPE ensures certain universality of the effects of soft physics originating at the momentum scale well below m_Q for all genuinely short-distance observables.

In this letter we derive the non-Abelian dipole radiation to second order in α_s and use it to obtain the power evolution of a number of spin-singlet heavy quark operators to order α_s^2. In particular, a gauge-invariant relation between a short-distance low-scale running heavy quark mass and the $\overline{\rm MS}$ mass $\bar m_Q(m_Q)$ is given, which becomes a practical necessity since the two-loop accuracy in the processes with heavy quarks is becoming the state of the art.

The radiation by a heavy charged particle when its velocity changes is well known from classical electrodynamics. It is obtained from the Liénard–Wiechert retarded potentials $A_\mu(\vec r,\, t)$ at $\vec r \to \infty$ and has the form

$$\frac{1}{\omega}\frac{{\rm d}I(\omega)}{{\rm d}\omega} \;=\; \frac{\alpha}{\pi}\left(\frac{1}{|\vec v\,|}\ln\frac{1+|\vec v\,|}{1-|\vec v\,|}-2\right)\frac{1}{\omega} \;=\; \frac{2}{3}\,\frac{\alpha}{\pi}\frac{\vec v^{\,2}}{\omega} \;+\; \mathcal{O}(\vec v^{\,4})\;, \tag{1}$$

where ω is frequency, $I(\omega)$ intensity, and α is the fine structure constant. We will be interested only in the dipole term $\propto \vec v^{\,2}$ in QCD, and do not consider multipole radiation proportional to higher powers of $\vec v$.

In QED the same relation holds, with $1/\omega\; {\rm d}I/{\rm d}\omega$ giving the probability of radiating soft photon(s) with energy ω. Moreover, there are no higher-order corrections in $\alpha_{\rm em}$ provided $\omega \ll m$. The effects of the photon interaction with vacuum fluctuations are also suppressed, by powers of ω/m_e, and the soft radiation in QED is governed by $\alpha(0)$.

Due to gluon self-interaction the dipole radiation in non-Abelian theory is different. Our main interest lies in the domain where ω is large compared to $\Lambda_{\rm QCD}$ (the nonperturbative multipole expansion of the color-singlet systems was discussed in [2]). In this regime perturbative calculations can be performed.

Below we introduce some standard notations used for heavy quarks. Let us consider a process of scattering of a color-singlet "weak" current J with momentum q on a heavy quark Q in the SV kinematics. For simplicity, the initial quark is assumed to rest. The initial Q and final state $\tilde Q$ quarks can have arbitrary masses; however, both masses must be large, so that the nonrelativistic expansion can be applied. The SV limit $\vec v = \vec q/m_{\tilde Q}$, $|\vec v\,| \ll 1$, is kept by adjusting $\vec q$ appropriately. For simplicity we consider the case of equal masses, $m_{\tilde Q} = m_Q$, although nothing depends on this assumption. The current J must have a non-vanishing tree level nonrelativistic limit (e.g., $J_0^\dagger = \bar Q\gamma_0\tilde Q$ or $J_S^\dagger = \bar Q\tilde Q$), otherwise it can be arbitrary.

Inclusive processes of the scattering of heavy quarks are described by an appropriate structure function $W(q_0, \vec{q})$ which is a sum of all transition probabilities induced by J into final states with momentum $\vec{q}$ and energy $m_Q + q_0$. The optical theorem relates it to the discontinuity of the forward transition amplitude $T(q_0, \vec{q})$ at physical values of q_0 :

$$T(q_0, \vec{q}) \;=\; \frac{i}{2m_Q}\int d^4x\, \mathrm{e}^{\,-iqx}\, \langle Q|T\, J(x)J^\dagger(0)|Q\rangle\;,\;\; W(q_0, \vec{q}) = 2\,\mathrm{Im}\,T(q_0, \vec{q})\;. \tag{2}$$

In the heavy quark limit the spin degrees of freedom become irrelevant, and we assume averaging over spin states (also color, for perturbative calculations). The structure functions have a threshold $q_0^{\rm min} \;=\; \sqrt{\vec{q}^{\,2}+m_Q^2} - m_Q \simeq m_Q\vec{v}^{\,2}/2$ corresponding to elastic transitions. The variable $\omega \;=\; q_0 - q_0^{\rm min}$ measures the hadron excitation energy in the final state.

The nonrelativistic expansion and the perturbative treatment are both justified if $\Lambda_{\rm QCD} \ll \omega \ll m_Q$, and this hierarchy is assumed in what follows. The nonperturbative aspects will be addressed later. In perturbative calculations we can use quark states instead of actual heavy hadrons, as was tacitly assumed above. The structure function W takes the following form in the heavy quark limit:

$$W(\omega, \vec{v}) \;=\; N\,\delta(\omega) + \frac{2\vec{v}^{\,2}}{3}\frac{d(\omega)}{\omega} + \mathcal{O}\left(\vec{v}^{\,4}\right)\;. \tag{3}$$

At $\vec{v}=0$ only the elastic peak is present. The dipole radiation is described by $d(\omega)$ and N is a (velocity-dependent) factor depending on the current.

Motivated by the dipole radiation in QED, we define the dipole coupling $\alpha_s^{(d)}$ by projecting Eq. (3) on its second term:

$$C_F\frac{\alpha_s^{(d)}(\omega)}{\pi\omega} \;=\; \lim_{\vec{v}\to 0}\;\lim_{m_Q\to\infty}\,\frac{3}{2\vec{v}^{\,2}}\frac{W(\omega, \vec{v})}{\int_0^\omega\, W(\omega', \vec{v})\,\mathrm{d}\omega'}\;. \tag{4}$$

Here ω is assumed to be positive. The denominator in the last ratio eliminates the overall normalization of the effective nonrelativistic current. In this form the heavy quark limit $m_Q \to \infty$ yields a finite result. The normalization integral includes the elastic peak, which makes it infrared safe at arbitrary $\vec{v}$. On the other hand, the exact upper limit does not matter in the SV kinematics, since it affects the ratio only by corrections $\sim \vec{v}^{\,2}$. In this way, the inelastic SV structure function determines the effective QCD running coupling [1] driving the dipole radiation of gluons. It is a dimensionless function of the ratio $\Lambda_{\rm QCD}/\omega$.

The OPE and factorization of the infrared effects ensures that $\alpha_s^{(d)}(\omega)$ is universal: it does not depend on the choice of the weak current J probing the heavy quark structure, or on the ratio of the quark masses. Also, there is no dependence on the type of the initial heavy hadron as soon as the onset of the quark-hadron duality is passed. This property always holds perturbatively. The coupling $\alpha_s^{(d)}$ obeys the usual renormalization group (RG) equation

[1] S. Brodsky suggested to call it "radiation charge". A more precise term could be also a "dipole radiation charge", to distinguish from soft radiation in the ultrarelativistic case.

with the standard first two terms in the β-function. Because of its physical definition, the evolution of $\alpha_s^{(d)}$ is properly defined when heavy flavor thresholds are passed.

In practice one needs to know the relation between $\alpha_s^{(d)}$ and the standard reference coupling $\alpha_s^{\overline{\rm MS}}$. We calculated it to the second order in α_s^2 using the technique of Ref. [3]:

$$\frac{\alpha_s^{(d)}(\mu)}{\pi} = \frac{\alpha_s^{\overline{\rm MS}}(\mu)}{\pi} + \left[\left(\frac{5}{3} - \ln 2\right)\frac{\beta_0}{2} - C_A\left(\frac{\pi^2}{6} - \frac{13}{12}\right)\right]\left(\frac{\alpha_s}{\pi}\right)^2 \;, \tag{5}$$

where $C_A = N_c$ for the SU(N_c) gauge group and $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$. The term $\sim C_A$ in eq. (5) represents the so-called genuine (non-BLM) second-order effect; it has purely non-Abelian origin. The BLM-type terms [4] which account for the effects of running of the QCD coupling in the leading-order calculations are readily computed to any order in this case (the results of [5] can be directly used for such calculation). The term $\beta_0^2\alpha_s^3$ and the expression for the case of massive flavors are given in [6]. Even in the Abelian theory with light flavors $\alpha_s^{(d)}(\mu)$ deviates from the V-scheme coupling $\alpha_s^{(V)}(\mu)$ [4], their commensurate scales are somewhat different. This is a purely kinematic effect; $\alpha_s^{(d)}(\mu)$ incorporates integration over the invariant mass of the light fermion pair in the interval from 0 to μ, while in $\alpha_s^{(V)}$ the fermion loop enters at a fixed spacelike momentum.

A similar effective coupling for emission of soft gluons in the ultrarelativistic case $|\vec{v}| \to 1$ has been also calculated to two loops [7]; its exact definition, however, is less direct. It differs from $\alpha_s^{(d)}$ in the second-order terms, although the difference is small numerically.

A construction of the HQE in QCD requires a proper definition of the basic objects of the effective theory, in the first place the heavy quark mass, kinetic operator and all other composite operators. A consistent approach is possible if an explicit separation scale μ between the low- and high-momentum domains is introduced. All operators then depend on μ. A normalization-point–independent pole mass of the heavy quark, although appearing in purely perturbative calculations at a given order, cannot be completely defined when nonperturbative effects are addressed. This applies also to the parameter $\overline{\Lambda}$ measuring the difference between the heavy hadron mass M_{H_Q} and m_Q in the heavy quark limit. Instead, one must use in the OPE the short-distance mass $m_Q(\mu)$ and, correspondingly, $\overline{\Lambda}(\mu) = \lim_{m_Q\to\infty} M_{H_Q} - m_Q(\mu)$ with $\Lambda_{\rm QCD} \ll \mu \ll m_Q$ [8]. The low-scale short-distance mass can be defined in different ways. We recall, however, that using the standard $\overline{\rm MS}$ mass if $\mu \ll m_Q$ is unnatural and should be avoided [9].

A physically appropriate gauge-invariant scheme suggested in [5,9,10] is based on the SV sum rules relating the moments of the SV structure functions to the local heavy-quark operators [11]. The normalization point is introduced as an upper cutoff in the integral over excitation energy. For example, for a heavy hadron H_Q one defines

$$\begin{aligned}
\overline{\Lambda}(\mu) \;&\equiv\; \lim_{m_Q\to\infty}\left[M_{H_Q} - m_Q(\mu)\right] = \lim_{\vec{v}\to 0}\,\lim_{m_Q\to\infty}\,\frac{2}{\vec{v}^{\,2}}\frac{\int_0^\mu\,\omega\,W(\omega,\,\vec{v}\,)\,{\rm d}\omega}{\int_0^\mu\,W(\omega,\,\vec{v}\,)\,{\rm d}\omega}\;,\\
\mu_\pi^2(\mu) \;&\equiv\; \frac{\langle H_Q|\bar{Q}(i\vec{D}\,)^2 Q|H_Q\rangle_\mu}{\langle H_Q|\bar{Q}Q|H_Q\rangle_\mu} = \lim_{\vec{v}\to 0}\,\lim_{m_Q\to\infty}\,\frac{3}{\vec{v}^{\,2}}\frac{\int_0^\mu\,\omega^2\,W(\omega,\,\vec{v})\,{\rm d}\omega}{\int_0^\mu\,W(\omega,\,\vec{v})\,{\rm d}\omega}\;,
\end{aligned} \tag{6}$$

etc. These operator relations suggest that the perturbative evolution is described by one and the same function, $\alpha_s^{(d)}(\mu)$ of Eq. (4):

$$\frac{\mathrm{d}\overline{\Lambda}(\mu)}{\mathrm{d}\mu} \;=\; \frac{4}{3}C_F\frac{\alpha_s^{(d)}(\mu)}{\pi}\,, \qquad \frac{\mathrm{d}\mu_\pi^2(\mu)}{\mathrm{d}\mu^2} \;=\; C_F\frac{\alpha_s^{(d)}(\mu)}{\pi}\,. \tag{7}$$

The same α_s^2 running of μ_π^2 was obtained in [12] from the zero-recoil transitions.

A similar μ-dependence holds for the renormalized slope of the Isgur-Wise function when it is defined in the same physical way [9] based on the Bjorken sum rule:

$$\mu\frac{\mathrm{d}\left[\varrho^2(\mu)-\frac{1}{4}\right]}{\mathrm{d}\mu} \;=\; \frac{4}{3}\,\frac{2\alpha_s^{(d)}(\mu)}{3\pi}\,. \tag{8}$$

Due to short-distance effects, the observable transition form factors do not have a literal heavy quark limit at $\vec{v}\neq 0$ but vanish, and require factoring out the perturbative suppression due to gluon radiation, to yield the effective μ-dependent IW function $\xi(v^2;\,\mu)$. In analogy with Eqs. (6), it is convenient to define $\xi(v^2;\,\mu)$ for small v as

$$\xi(v^2;\,\mu) \;=\; \lim_{m_Q\to\infty}\frac{F(v)}{\left[\frac{1}{2\pi}\int_0^\mu\; W(\omega,\,\vec{v}\,)\,\mathrm{d}\omega\right]^{1/2}}\,, \qquad \varrho^2(\mu) = -2\frac{\mathrm{d}\xi(v^2;\,\mu)}{\mathrm{d}v^2}\bigg|_{v=0}\,,$$

where F is the form factor, say, of the vector current $\bar{Q}\gamma_0 Q$ and W is the corresponding structure function. This is the scheme suggested in [9] for which Eq. (8) holds.

The RG equations (7) are proportional to $\alpha_s^{(d)}$. Hence, the OPE shows that the first-order running is not renormalized in the Abelian theory without light fermions to *all orders* in the coupling [5], since in such a theory $\alpha_s^{(d)}$ identically coincides with $\alpha(0)$.

According to [10], one can calculate perturbatively a physical (gauge-invariant) low-scale running mass $m_Q(\mu)$ with $\mu\ll m_Q$ by subtracting, order by order, the infrared part given by the SV sum rules, from the pole mass m_Q^{pole}:

$$m_Q(\mu) = \left[m_Q^{\mathrm{pole}}\right]_{\mathrm{pert}} - \left[\overline{\Lambda}(\mu)\right]_{\mathrm{pert}} - \frac{1}{2m_Q(\mu)}\left[\mu_\pi^2(\mu)\right]_{\mathrm{pert}}\,.$$

This mass determines the nonrelativistic kinetic energy term in the renormalized heavy quark Hamiltonian. The two-loop relation between this $m_Q(\mu)$ and the $\overline{\mathrm{MS}}$ mass is

$$\begin{aligned} m_Q(\mu) = \bar{m}\Bigg\{1 + \frac{4}{3}\frac{\alpha_s(\bar{m})}{\pi}\left(1-\frac{4}{3}\frac{\mu}{\bar{m}}-\frac{\mu^2}{2\bar{m}^2}\right) \\ +\left(\frac{\alpha_s(\bar{m})}{\pi}\right)^2\left[K-\frac{8}{3}+\frac{\mu}{\bar{m}}\left(\frac{8\beta_0}{9}X_1+\frac{8\pi^2}{9}-\frac{52}{9}\right)+\frac{\mu^2}{\bar{m}^2}\left(\frac{\beta_0}{3}X_2+\frac{\pi^2}{3}-\frac{23}{18}\right)\right]\Bigg\} \end{aligned} \tag{9}$$

where

$$K = \frac{\beta_0}{2}\left(\frac{\pi^2}{6}+\frac{71}{48}\right) + \frac{665}{144}+\frac{\pi^2}{18}\left(2\ln 2-\frac{19}{2}\right)-\frac{1}{6}\zeta(3),$$

$$X_1 = \ln\frac{2\mu}{\bar{m}}-\frac{8}{3}, \quad X_2 = \ln\frac{2\mu}{\bar{m}}-\frac{13}{6}$$

and $\bar{m}=\bar{m}(\bar{m})$ is the $\overline{\mathrm{MS}}$ mass normalized at the scale $\bar{m}$. The $\mathcal{O}(\alpha_s^2)$ relation between the pole mass and the $\overline{\mathrm{MS}}$ mass [13] was used here. We neglected small terms $\sim\mu^3/m_Q^2$

which can be incorporated in the same way, if necessary. The next-order BLM correction to Eq. (9) is given in [6].

The nonperturbative effects in the dipole radiation are suppressed by powers of $\Lambda_{\rm QCD}/\omega$. In particular, the actual structure functions in Eqs. (2) refer to a heavy hadron (B meson, Λ_b etc.) but not an isolated heavy quark. Likewise, the complete effective coupling $\alpha_s^{(d)}$ defined in this way, as any other effective charge [14], acquires power-suppressed nonperturbative contributions; they differ, for example, in B and Λ_b even in the heavy quark limit. One can quantify the nonperturbative effects in $\alpha_s^{(d)}(\omega)$ by constructing its $1/\omega$ expansion; the methods used in the OPE are applicable here (see [6] for more details). The nonperturbative effects in $\alpha_s^{(d)}$ are expressed through the expectation values of local heavy quark operators $\bar{Q}O_kQ$.

The first nontrivial operator is the kinetic operator $\bar{Q}(i\vec{D}\,)^2Q$, which, by dimension counting, would lead to $1/\omega^2$ effects. However, it does not contribute to $\alpha_s^{(d)}$. Indeed, we can consider the initial heavy quark moving with the spacelike momentum $|\vec{p}| \ll \omega$. The structure function for such a state is directly related to the one at rest, and is modified only in terms $\sim \vec{p}^{\,2}\vec{v}^{\,4}$ which can be neglected. Since the kinetic expectation value for such a state is non-zero, the corresponding coefficient function must vanish.

There is one spin-singlet, so-called Darwin operator $O_D = -i\bar{Q}D_kD_0D_kQ$ yielding $1/\omega^3$-suppressed terms (for simplicity, we consider the case of a B meson). The OPE relates its effect [6] to the logarithmic anomalous dimension $\hat{\gamma}_D$ of O_D:

$$\frac{\delta_{\rm D}\alpha_s^{(d)}(\omega)}{\alpha_s^{(d)}(\omega)} = \frac{3\pi\hat{\gamma}_D(\omega)}{8\alpha_s^{(d)}(\omega)}\frac{\rho_D^3(\omega)}{\omega^3} \simeq -\left(\frac{\alpha_s(\mu)}{\alpha_s(\omega)}\right)^{-\frac{13}{2\beta_0}}\frac{39\rho_D^3(\mu)}{32\omega^3} \approx -\left(\frac{0.55\,\mathrm{GeV}}{\omega}\right)^3,$$

where the sum rule for the Darwin term [9,15,16] was used, and

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\rho_D^3(\mu) = \hat{\gamma}_D(\mu)\rho_D^3(\mu), \quad \hat{\gamma}_D(\mu) \simeq -\frac{13\alpha_s^{(d)}(\mu)}{4\pi},$$

$$\rho_D^3(\mu) \;=\; \frac{\langle B|O_D|B\rangle_\mu}{2M_B} \simeq \frac{2\pi\alpha_s}{9}\tilde{f}_B^2M_B \approx 0.1\,\mathrm{GeV}^3 \;\;\text{at}\;\; \alpha_s(\mu)\simeq 1\,.$$

We also employed the factorization estimate of ρ_D^3 [15]; the anomalous dimensions were calculated in [17]. We see that the nonperturbative effects are expected to die out quickly at large energy, $\delta_{\rm np}\alpha_s^{(d)}(\omega) \;\sim\; (\Lambda_{\rm QCD}/\omega)^k\,, \;\; k\geq 3\,.$

As an effective QCD coupling, $\alpha_s^{(d)}$ has an additional advantage: it directly measures the strength of the interaction, whereas such popular couplings as $\alpha_s^{(D)}$ or $\alpha_s^{(R)}$ determine the *deviation* of the corresponding observables, Adler D-function $D(Q^2)$ or $R(e^+e^- \to \mathrm{hadrons})$, from their tree-level value. From this perspective, $\alpha_s^{(d)}$ is closer to the so-called V-scheme coupling $\alpha_s^{(V)}$ related to the heavy quark potential $V(R)$ [18,4,19]. An exact definition of $\alpha_s^{(V)}$ in higher orders is not evident at the moment, however. Emergence of terms $\alpha_s^k\ln\alpha_s$ with $k\geq 4$ in its relation to $\alpha_s^{\overline{\rm MS}}$ [18] may indicate that $\alpha_s^{(V)}$ is not a genuinely short-distance quantity.

We defined $\alpha_s^{(d)}$ directly in Minkowski space. Since the OPE is applicable here, this is not too important. However, the nonperturbative terms related to the asymptotic nature of the power expansion of the OPE which are exponentially suppressed in the Euclidean domain, may yield an oscillating component which is only power-suppressed in $\alpha_s^{(d)}$ (for a recent discussion, see [20]). The coupling $\alpha_s^{(d)}$ may also show some (smeared) resonance structure at the heavy flavor thresholds. To eliminate these features one can easily construct the Euclidean counterpart of $\alpha_s^{(d)}$.

The coupling $\alpha_s^{\overline{\rm MS}}$ proved to be indispensable for complicated multi-loop calculations. However, it is unphysical in some respects and its use as an expansion parameter sometimes obscures the perturbative expansion [21]. At the moment the complete $\mathcal{O}(\alpha_s^2)$ relation between $\alpha_s^{\overline{\rm MS}}$ and $\alpha_s^{(d)}$ is sufficient. A calculation of order α_s^3 seems feasible as well.

Using $\alpha_s^{(d)}$ is advantageous in heavy quark decays and may improve the accuracy of the perturbative estimates. Since the velocity of the charm hadrons is typically small in B decays, $\alpha_s^{(d)}$ directly determines the gluon bremsstrahlung and thus sums the lowest-momentum component of the corrections, where the effects of higher orders in a differently defined α_s could be large.

The value of $\alpha_s^{(d)}(E)$ can in principle be fitted from double-differential distributions in $b \to c\,\ell\nu$ decays, using the definition (4) for any measured combination of the actual semileptonic structure functions of B meson. However, the corrections due to finite m_Q and $\vec{v}$ must be accounted for. The dependence of the average invariant mass of hadrons in the final state on q^2, E_ℓ or other suitable kinematical constraint on the bremsstrahlung energy is expected to be sensitive to $\alpha_s^{(d)}$. Other possibilities to measure it exist in the near-threshold production of $b\bar{b}$ in $\mathrm{e^+e^-}$ annihilation or in top quark physics. All they require, however, additional investigation.

To summarize, we have derived the non-Abelian dipole radiation by a nonrelativistic color particle to the second order in the strong coupling. By virtue of the SV sum rules this determines the two-loop power mixing of the number of spin-singlet effective local heavy quark operators. We have given the two-loop relation between the gauge-invariant short-distance low-scale heavy quark mass $m_Q(\mu)$ suitable for the Wilson OPE and $m_Q(m_Q)$ in the $\overline{\rm MS}$ scheme. We have discussed the properties of the effective dipole radiation coupling $\alpha_s^{(d)}$ which is useful for heavy quark physics. A more detailed discussion can be found in [6].

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REFERENCES

[1] M. Voloshin and M. Shifman, *Yad. Fiz.* **47** (1988) 801 (*Sov. J. Nucl. Phys.* **47** (1988) 511).

[2] M.B. Voloshin, *Nucl. Phys.* **B154** (1979) 365;
K. Gottfried, *Phys. Rev. Lett.* **40** (1978) 598.

[3] A. Czarnecki and K. Melnikov, *Phys. Rev.* **D56** (1997) 7216.

[4] S.J. Brodsky, G. Lepage and P. Mackenzie, *Phys. Rev.* **D28** (1983) 228.

[5] N.G. Uraltsev, *Nucl. Phys.* **B491** (1997) 303.

[6] A. Czarnecki, K. Melnikov and N. Uraltsev, in preparation.

[7] S. Catani and L. Trentadue, *Nucl. Phys.* **B327** (1989) 323.

[8] I. Bigi *et al.*, *Phys. Rev.* **D50** (1994) 2234.

[9] I. Bigi, M. Shifman and N. Uraltsev, *Ann. Rev. Nucl. Part. Sci.* **47** (1997) 591.

[10] I. Bigi *et al.*, *Phys. Rev.* **D56** (1997) 4017.

[11] I. Bigi *et al.*, *Phys. Rev.* **D52** (1995) 196.

[12] A. Czarnecki, K. Melnikov and N. Uraltsev, *Phys. Rev.* **D57** (1998) 1769.

[13] N. Gray *et al.*, *Z. Phys.* **C48** (1990) 673.

[14] G. Grunberg, *Phys.Lett.* **95B** (1980) 70; *Phys. Rev.* **D29** (1984) 2315.

[15] I. Bigi *et al.*, *Int. Journ. Mod. Phys.* **A9** (1994) 2467.

[16] I. Bigi *et al.*, *Phys. Lett.* **B339** (1994) 160;
C.-K. Chow and D. Pirjol, *Phys. Rev.* **D53** (1996) 3998.

[17] M. Voloshin and M. Shifman, *Sov. Phys. JETP* **64** (1986) 698.

[18] T. Appelquist, M. Dine and I. Muzinich, *Phys. Lett.* **B69** (1977) 231; *Phys. Rev.* **D17** (1978) 2074.

[19] M. Peter, *Phys. Rev. Lett.* **78** (1997) 602; *Nucl. Phys.* **B501** (1997) 471.

[20] B. Chibisov *et al.*, *Int. J. Mod. Phys.* **A12** (1997) 2075.

[21] S.J. Brodsky and H.J. Lu, *Phys. Rev.* **D51** (1995) 3652; hep-ph/9506322; hep-ph/9409462.