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Calculation of the semileptonic decay width of the Λ_b baryon

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Abstract

We extend the approach based on the light-cone expansion and the heavy quark effective theory to the inclusive semileptonic decay of an unpolarized b-flavored hadron. It is applied to calculate the semileptonic decay width of the Λ_b baryon and its ratio to that of the B meson. We obtain $\Gamma_{\rm SL}(\Lambda_b) = 51 \pm 9\ |V_{cb}|^2 \mathrm{ps}^{-1}$ and $\Gamma_{\rm SL}(\Lambda_b)/\Gamma_{\rm SL}(B) = 1.052 \pm 0.006$. From the latter, the semileptonic branching fraction for Λ_b is predicted to be $(8.7 \pm 0.7)\%$.

I. INTRODUCTION

Recent measurements [1] have shown that the lifetime of the Λ_b baryon is notably shorter than the lifetime of the $\bar{B}^0$ meson:

$$\frac{\tau(\Lambda_b)}{\tau(\bar{B}^0)} = 0.79 \pm 0.06\,, \tag{1.1}$$

suggesting that corrections to the simple spectator decay picture are significantly different for the Λ_b baryon and B mesons. Studies of the semileptonic branching fractions[1] for the different b-flavored hadrons are also useful to probe the decay dynamics. The semileptonic branching fraction for B mesons has been measured at the $\Upsilon(4S)$ resonance to be [2]

$$BR_{\rm SL}(B) = (10.43 \pm 0.24)\%\,, \tag{1.2}$$

which is an average over the $\bar{B}^0$ and B^-. Recently, the first measurement of the ratio $R_{\Lambda\ell} = BR(b - \text{baryon} \to \Lambda\ell X)/BR(b - \text{baryon} \to \Lambda X)$ was reported by the OPAL collaboration. They measure [3]

$$R_{\Lambda\ell} = (7.0 \pm 1.2 \pm 0.7)\%, \tag{1.3}$$

which should be a good approximation to the average b baryon semileptonic branching fraction. These measurements have shown an interesting difference in the Λ_b and B semileptonic branching fractions.

In this paper we calculate the semileptonic decay width of the Λ_b baryon and its ratio relative to that of the B meson[2]. The last value can then be converted into a value for the semileptonic branching fraction for the Λ_b baryon by using the measured B and Λ_b lifetimes and the semileptonic branching fraction for B mesons. For this purpose we must account for the nonperturbative QCD effects in the underlying weak decays.

An approach based on the light-cone expansion and the heavy quark effective theory has been developed [4 – 6] to incorporate nonperturbative QCD effects in inclusive semileptonic B meson decays. This approach provides a foundation for the parton model [7] for inclusive B decays and furthermore improves it by including QCD corrections in a systematic way. This approach accounts correctly for the phase-space effects and produces a smooth electron spectrum, which is consistent with the experimental data [6]. The calculation of the semileptonic decay width for the B meson has shown [5] that the kinematically enhanced nonperturbative contributions play numerically an important role.

In this paper we extend the results of [4 – 6] and compute the inclusive semileptonic decay rate for an unpolarized hadron H_b containing a b quark. The heavy hadron decays involve two large scales: the heavy hadron mass at the hadron level and the heavy quark mass at

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[1] The semileptonic branching fraction for the b-flavored hadron H_b refers to the branching fraction for the inclusive decay $H_b \to \ell^+\nu$anything, where ℓ indicates e or μ mode.

[2] In the text, the notation B refers to $\bar{B}^0$ and B^- mesons.

the parton level, which are much greater than the QCD scale. Both of them are useful for circumventing nonperturbative QCD effects. Because of the heaviness of the decaying hadron, extended regions of phase space in the inclusive semileptonic decays of heavy hadrons involve large momentum transfer squared. Therefore, the decay dynamics is dominated by the light-cone distance. The light-cone dominance attributes the nonperturbative QCD effects to a single distribution function. On the other hand, the mass of the heavy quark provides a large limit to construct an effective theory describing the heavy quark interacting with the gluons in the heavy hadron. This so-called heavy quark effective theory (HQET) [8] has new (approximate) symmetries that were not manifest in the QCD Lagrangian and sets a framework for parametrizing nonperturbative effects, which relates various phenomena (e.g., the spectroscopy and weak decays of hadrons containing a single heavy quark) to a commen set of parameters, so that it has great predictive power. Sum rules for the distribution function can be derived by virtue of the operator product expansion and the HQET method [9 – 11], which constrain the shape of it. Consequently, we can account for the nonperturbative QCD effects with theoretical uncertainties under control.

This paper is organized in the following manner. We describe in Sec. II the formalism showing how the light-cone dominance attributes the nonperturbative QCD effects on the inclusive semileptonic decays to a single distribution function. The semilpetonic decay width for an unpolarized H_b hadron is expressed in terms of the distribution function. The properties of the distribution function is discussed in Sec. III. Two sum rules for the distribution function are obtained using the operator product expansion and the heavy quark effective theory. The general formulas for the inclusive semileptonic decay of the b-flavored hadron are applied in Sec. IV to the Λ_b baryon. Section V contains a conclusion.

II. FORMALISM

Consider the inclusive semileptonic decay of an unpolarized hadron H_b containing a b quark

$$H_b \to X_q \ell \bar{\nu}_\ell, \;\; \ell = e \text{ or } \mu, \tag{2.1}$$

where X_q is any possible hadronic final state containing a charm quark ($q = c$) or an up quark ($q = u$). The decay is induced by weak interactions and the decay rate is given by

$$d\Gamma = \frac{G_F^2 \left|V_{qb}\right|^2}{(2\pi)^5 P_0} L^{\mu\nu} W_{\mu\nu} \frac{d^3 k_\ell}{2E_\ell} \frac{d^3 k_\nu}{2E_\nu}, \tag{2.2}$$

where P denotes the momentum of the hadron H_b and $k_{\ell(\nu)}$ and $E_{\ell(\nu)}$ are the momentum and energy of the electron (antineutrino), respectively. V_{qb} are the elements of the Cabibbo-Kabayashi-Maskawa (CKM) matrix. $L_{\mu\nu}$ is the leptonic tensor, which results from the summation over the spins of the charged lepton and antineutrino

$$L^{\mu\nu} = 2(k_\ell^\mu k_\nu^\nu + k_\nu^\mu k_\ell^\nu - g^{\mu\nu} k_\ell \cdot k_\nu + i\varepsilon^{\mu\nu}{}_{\alpha\beta} k_\ell^\alpha k_\nu^\beta). \tag{2.3}$$

$W_{\mu\nu}$ is the hadronic tensor, which incorporates all the messy complexity of nonperturbative QCD effects for the inclusive process. It can be expressed in terms of a current commutator between the H_b hadron states:

$$W_{\mu\nu} = -\frac{1}{2\pi}\int d^4y e^{iq\cdot y}\frac{1}{2J+1}\sum_s \langle H_b(P,s)\left|[j_\mu(y), j_\nu^\dagger(0)]\right| H_b(P,s)\rangle, \tag{2.4}$$

where q stands for the momentum transfer to the lepton pair, $q = k_\ell + k_\nu$, and $j_\mu(y) = \bar{q}(y)\gamma_\mu(1-\gamma_5)b(y)$ is the weak current. The spin-J hadron state $|H_b(P,s)\rangle$ satisfies the standard covariant normalization $\langle H_b(P,s)|H_b(P,s)\rangle = 2P_0(2\pi)^3\delta^3(\mathbf{0})$. In general, the hadronic tensor can be decomposed in terms of scalars $W_a(q^2,\, q\cdot P)$, $a = 1,\ldots,5$, as follows :

$$W_{\mu\nu} = -g_{\mu\nu}W_1 + \frac{P_\mu P_\nu}{M^2}W_2 - i\varepsilon_{\mu\nu\alpha\beta}\frac{P^\alpha q^\beta}{M^2}W_3 + \frac{q_\mu q_\nu}{M^2}W_4 + \frac{P_\mu q_\nu + q_\mu P_\nu}{M^2}W_5\,, \tag{2.5}$$

where M is the mass of the hadron H_b.

We can express the decay rates in terms of the five hadronic structure functions W_a, $a = 1,\ldots,5$. The differential decay rate for the process (2.1) in the rest frame of the H_b hadron is

$$\frac{d^3\Gamma}{dE_\ell dq^2 dq_0} = \frac{G_F^2|V_{qb}|^2}{16\pi^3 M}\left[W_1 q^2 + W_2(2E_\ell q_0 - 2E_\ell^2 - \frac{q^2}{2}) + W_3\frac{q^2}{M}(q_0 - 2E_\ell)\right]. \tag{2.6}$$

The structure functions W_4 and W_5 do not appear above because their contribution is proportional to the square of the charged-lepton mass and we ignore the lepton masses.

Now we employ the light-cone dominance to simplify the expression for the hadronic tensor. We proceed along the lines of [4 – 6]. It is well known that integrals like the one in Eq. (2.4) are dominated by distances where

$$0 \le y^2 \le \frac{1}{q^2}\,. \tag{2.7}$$

For inclusive semileptonic decays (2.1), q^2 is timelike and varies in the physical range

$$0 \le q^2 \le (M - M_{X_{min}})^2\,, \tag{2.8}$$

where $M_{X_{min}}$ is the minimum value of the invariant mass of the hadronic final state. Because of the heaviness of the hadron H_b, for extended regions of phase space the momentum transfer squared is much larger than the QCD scale $\Lambda_{\rm QCD}$. Therefore, the integral of Eq. (2.4) is dominated by the light-cone distances in the space-time structure. This allows to replace the commutator of the two currents with its singularity on the light cone times an opreator bilocal in the b quark fields, whose on-light-cone matrix element between H_b states is also dominant.

The light-cone dominance leads to the expression of the hadronic tensor in terms of a distribution function

$$W_{\mu\nu} = 4(S_{\mu\alpha\nu\beta} - i\varepsilon_{\mu\alpha\nu\beta})\int d\xi f_{H_b}(\xi)\varepsilon(\xi P_0 - q_0)\delta\left[(\xi P - q)^2 - m_q^2\right](\xi P - q)^\alpha P^\beta\,, \tag{2.9}$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha} - g_{\mu\nu}g_{\alpha\beta}$ and m_q is the mass of the final quark q. The distribution function is defined by

$$f_{H_b}(\xi) = \frac{1}{4\pi M^2}\int d(y\cdot P)e^{i\xi y\cdot P}\frac{1}{2J+1}\sum_s \langle H_b(P,s)|\bar{b}(0)\not{P}(1-\gamma_5)b(y)|H_b(P,s)\rangle|_{y^2=0}\,. \tag{2.10}$$

It should be emphasized that the form of the distribution function is not identical for different hadrons. The light-cone dominance implies that the five hadronic structure functions can be written in terms of a single distribution function:

$$W_1 = 2[f_{H_b}(\xi_+) + f_{H_b}(\xi_-)]\,, \tag{2.11}$$

$$W_2 = \frac{8}{\xi_+ - \xi_-}[\xi_+ f_{H_b}(\xi_+) - \xi_- f_{H_b}(\xi_-)]\,, \tag{2.12}$$

$$W_3 = -\frac{4}{\xi_+ - \xi_-}[f_{H_b}(\xi_+) - f_{H_b}(\xi_-)]\,, \tag{2.13}$$

$$W_4 = 0\,, \tag{2.14}$$

$$W_5 = W_3\,, \tag{2.15}$$

where

$$\xi_\pm = \frac{q\cdot P \pm \sqrt{(q\cdot P)^2 - M^2(q^2 - m_q^2)}}{M^2}\,. \tag{2.16}$$

The differential decay rate Eq. (2.6) becomes

$$\frac{d^3\Gamma}{dE_\ell dq^2 dq_0} = \frac{G_F^2|V_{qb}|^2}{4\pi^3 M}\frac{q_0 - E_\ell}{\sqrt{\mathbf{q}^2 + m_q^2}}\left\{f_{H_b}(\xi_+)(2\xi_+ E_\ell M - q^2) - (\xi_+ \to \xi_-)\right\}\,. \tag{2.17}$$

Integrating over the phase space, we obtain the total semileptonic decay width for the unpolarized hadron H_b

$$\Gamma = \frac{G_F^2 M^5|V_{qb}|^2}{192\pi^3}\int_r^1 d\xi_+\,\xi_+^5 f_{H_b}(\xi_+)\left(1 - 8\frac{r^2}{\xi_+^2} + 8\frac{r^6}{\xi_+^6} - \frac{r^8}{\xi_+^8} - 24\frac{r^4}{\xi_+^4}\ln\frac{r}{\xi_+}\right), \tag{2.18}$$

where $r = m_q/M$ and we have ignored the $f_{H_b}(\xi_-)$ term whose contribution is negligibly small. Notice that the physical hadron mass M instead of the b quark mass enters Eq. (2.18), which enhances the decay width with respect to the free quark decay [5].

III. PROPERTIES OF THE DISTRIBUTION FUNCTION

Several important properties of the distribution function can be derived from field theory. First of all, because of the conservation of the b quantum number, the distribution function is normalized to unity:

$$\int d\xi f_{H_b}(\xi) = \frac{1}{2M^2}P^\mu \frac{1}{2J+1}\sum_s \langle H_b(P,s)\left|\bar{b}(0)\gamma_\mu(1-\gamma_5)b(0)\right| H_b(P,s)\rangle = 1. \tag{3.1}$$

Consider next $f_{H_b}(\xi)$ in the rest frame of the H_b hadron. In this frame,

$$f_{H_b}(\xi) = \frac{1}{2\pi}\int dy_0 e^{iM\xi y_0}\frac{1}{2J+1}\sum_s \langle H_b(P,s)|b^\dagger(0)P_L b(y_0)|H_b(P,s)\rangle, \tag{3.2}$$

where the left-handed projection operator $P_L = (1-\gamma_5)/2$. Inserting a complete set of hadronic states between quark fields and translating the y_0 dependence out of these fields, one gets

$$f_{H_b}(\xi) = \sum_m \delta(M - \xi M - p^0_m)\frac{1}{2J+1}\sum_s \left|\langle m\left|b_L(0)\right|H_b(P,s)\rangle\right|^2, \tag{3.3}$$

where $b_L = P_L b$. Therefore $f_{H_b}(\xi)$ obeys positivity. The hadronic state $|m\rangle$ with momentum p_m is physical and must have $0 \le p^0_m \le M$, thus $f_{H_b}(\xi) = 0$ for $\xi \le 0$ or $\xi \ge 1$. Therefore, the support of the distribution function reads $0 \le \xi \le 1$. These results, deduced in the H_b rest frame, hold in arbitrary frame due to Lorentz invariance. Furthermore, Eq. (3.3) gives a probabilistic interpretation of the distribution function, namely $f_{H_b}(\xi)$ is the probability of finding a b quark with a momentum ξP inside the unpolarized hadron H_b. In the limit $f_{H_b}(\xi) = \delta(\xi - m_b/M)$, the free quark decay is reproduced [e.g., Eq. (2.18) reduces to the free quark semileptonic decay width].

Since the H_b hadron contains a single heavy quark, i.e. the b quark, the heavy quark effective theory can be applied. The HQET is successful in describing various nature of heavy hadrons. More properties of the distribution function can be deduced exploiting the techniques of the operator product expansion and the HQET.

Since the b quark is very heavy within the H_b hadron one can extract the large space-time dependence

$$b(y) = e^{-im_b v\cdot y} b_v(y), \tag{3.4}$$

where v is the velocity of the initial hadron H_b, defined by $v = P/M$. In order to estimate the matrix element of the bilocal operator we must reduce it to local ones. To this end we make a Taylor expansion of the field in a gauge-covariant form. This leads to an operator product expansion

$$\bar b(0)\gamma^\beta(1-\gamma_5)b(y) = e^{-im_b v\cdot y}\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}y_{\mu_1}\cdots y_{\mu_n}\bar b_v(0)\gamma^\beta(1-\gamma_5)\mathcal{S}[k^{\mu_1}\cdots k^{\mu_n}]b_v(0), \tag{3.5}$$

where $k_\mu = iD_\mu = i(\partial_\mu - ig_s A_\mu)$ and $\mathcal{S}$ denotes a symmetrization. The Lorentz structure allows to express the matrix element of the local operator on the right-hand side of Eq. (3.5) between the spin-averaged H_b hadron states in terms of the H_b-hadron momentum:

$$\begin{aligned}&\frac{1}{2J+1}\sum_s\langle H_b(P,s)|\bar b_v(0)\gamma^\beta(1-\gamma_5)\mathcal{S}[k^{\mu_1}\cdots k^{\mu_n}]b_v(0)|H_b(P,s)\rangle = \\ &2(C_{n0}P^\beta P^{\mu_1}\cdots P^{\mu_n} + \sum_{i=1}^{n} M^2 C_{ni} g^{\beta\mu_i}P^{\mu_1}\cdots P^{\mu_{i-1}}P^{\mu_{i+1}}\cdots P^{\mu_n}) \\ &+ \text{terms with } g^{\mu_i\mu_j}.\end{aligned} \tag{3.6}$$

The terms with $g^{\mu_i\mu_j}$ can be omitted on the light cone. Substituting Eqs. (3.5) and (3.6) into Eq. (2.10) yields

$$f_{H_b}(\xi) = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}(\sum_{i=0}^{n}C_{ni})\delta^{(n)}(\xi - \frac{m_b}{M}). \tag{3.7}$$

Therefore we obtain the following moment sum rule for the distribution function

$$M_n(m_b/M) = \sum_{i=0}^{n} C_{ni}, \tag{3.8}$$

where the nth-moment about a point $\tilde{\xi}$ of the distribution function is in general defined by

$$M_n(\tilde{\xi}) = \int_0^1 d\xi (\xi - \tilde{\xi})^n f_{H_b}(\xi). \tag{3.9}$$

By definition, $M_0(\tilde{\xi}) = C_{00} = 1$.

We employ the HQET to estimate further expansion coefficients in Eq.(3.6). In this effective theory the QCD b-quark field $b(y)$ is related to its HQET counterpart $h_v(y)$ by means of an expansion in powers of $1/m_b$,

$$b(y) = e^{-im_b v\cdot y}\left[1 + \frac{i\not{D}}{2m_b} + O(\frac{\Lambda_{\rm QCD}^2}{m_b^2})\right] h_v(y). \tag{3.10}$$

The effective Lagrangian takes the form

$$\mathcal{L}_{\rm HQET} = \bar{h}_v iv\cdot D h_v + \bar{h}_v \frac{(iD)^2}{2m_b} h_v + \bar{h}_v \frac{g_s G_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b} h_v + O(\frac{1}{m_b^2}), \tag{3.11}$$

where $g_s G^{\alpha\beta} = i[D^\alpha, D^\beta]$ is the gluon field-strength tensor. Only the first term in Eq. (3.11) remains in the $m_b \to \infty$ limit, which has the heavy quark spin-flavor symmetry. The other two terms give the $1/m_b$ corrections. Following the method of [10] to relate matrix elements of local operators in full QCD to those in the HQET, the expansion coefficients C_{ni} in Eq. (3.6) can be expressed in terms of the HQET parameters. The nonperturbative QCD effects can, in principle, be calculated in a systematic manner. In this formalism the moment $M_n(m_b/M)$ is expected to be of order $(\Lambda_{\rm QCD}/m_b)^n$. A few resulting coefficients of this method are

$$C_{10} = \frac{5m_b}{3M} E_b(H_b) + O(\Lambda_{\rm QCD}^3/m_b^3), \tag{3.12}$$
$$C_{11} = -\frac{2m_b}{3M} E_b(H_b) + O(\Lambda_{\rm QCD}^3/m_b^3), \tag{3.13}$$
$$C_{20} = \frac{2m_b^2}{3M^2} K_b(H_b) + O(\Lambda_{\rm QCD}^3/m_b^3), \tag{3.14}$$
$$C_{21} = C_{22} = 0, \tag{3.15}$$

where the dimensionless HQET parameters

$$K_b(H_b) = -\frac{1}{2M}\langle H_b(P,s)|\bar{h}_v \frac{(iD)^2}{2m_b^2} h_v|H_b(P,s)\rangle, \tag{3.16}$$
$$G_b(H_b) = -\frac{1}{2M}\langle H_b(P,s)|\bar{h}_v \frac{g_s G_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b^2} h_v|H_b(P,s)\rangle, \tag{3.17}$$

and $E_b(H_b) = K_b(H_b) + G_b(H_b)$. The parameter $K_b(H_b)$ corresponding to the second term in the Lagrangian Eq. (3.11) measures the kinetic energy of the b quark inside H_b. The parameter $G_b(H_b)$ corresponding to the third term in the Lagrangian Eq. (3.11) measures the chromomagnetic energy due to the spin coupling between the b quark and the light constituents in H_b. Both of them are expected to be of order $(\Lambda_{\rm QCD}/m_b)^2$. Thus two sum rules for the distribution function can be derived via the moment sum rule (3.8). They dertermine up to order $(\Lambda_{\rm QCD}/m_b)^2$ the mean value μ and the variance σ^2 of the distribution function, which characterize the position of the maximum and its width, respectively:

$$\mu = \frac{m_b}{M}[1 + E_b(H_b)], \tag{3.18}$$

$$\sigma^2 = \left(\frac{m_b}{M}\right)^2 \left[\frac{2K_b(H_b)}{3} - E_b^2(H_b)\right], \tag{3.19}$$

with the definitions

$$\mu \equiv M_1(0) = \tilde{\xi} + M_1(\tilde{\xi}), \tag{3.20}$$
$$\sigma^2 \equiv M_2(\mu) = M_2(\tilde{\xi}) - M_1^2(\tilde{\xi}). \tag{3.21}$$

Therefore, the distribution function $f_{H_b}(\xi)$ is sharply peaked around $\xi = \mu \approx m_b/M$ close to 1 and its width of order $\Lambda_{\rm QCD}/M$ is narrow, in agreement with intuitive expectations.

IV. APPLICATION

As discussed in the last section, the normalization of the distribution function is fixed to be exactly 1 by current conservation and the sum rules (3.18) and (3.19) restrict the position of the maximum and the width of the distribution function, which improve considerably the theoretical accuracy, but otherwise its shape is not determined. In order to calculate the decay rate we shall adopt a distribution function [4 – 6] that satisfies the theoretical constraints of the previous section:

$$f(\xi) = N \frac{\xi(1-\xi)}{(\xi - a)^2 + b^2}\theta(\xi)\theta(1-\xi)\,, \tag{4.1}$$

where N is the normalization constant and a and b two parameters, which depend on the hadron H_b and are related via the sum rules (3.18) and (3.19) to the two HQET parameters $K_b(H_b)$ and $G_b(H_b)$. For $a = m_b/M$ and $b = 0$, this distribution function reduces to a delta function, $\delta(\xi - m_b/M)$, and thus reproduces the free-quark decay model.

The semileptonic decay width of the unpolarized hadron H_b can be calculated by use of Eqs. (2.18) and (4.1) adding the known perturbative order α_s correction [12]. The semileptonic decay width of the B meson has been computed [5] in our approach. A similar calculation can be done for the Λ_b baryon. One needs to know the values of the corresponding parameters.

As mentioned in the introduction, the HQET relates various phenomena with each other. Information on the parameters can be gained with the help of the HQET. Unlike B mesons, the chromomagnetic contribution vanishes for the ground-state baryon Λ_b:

$$G_b(\Lambda_b) = 0, \tag{4.2}$$

since the light constituents inside Λ_b have total spin zero. The difference between the b-quark kinetic energies in the B meson and in the Λ_b baryon can be inferred by using the HQET mass formula [13] . It follows that $K_b(\Lambda_b) - K_b(B) = 0.0002 \pm 0.0006$. For numerical analyses, we therefore assume the approximate equality:

$$K_b(\Lambda_b) \approx K_b(B) = -\frac{\lambda_1}{2m_b^2}\,, \tag{4.3}$$

which is further supported by the calculations of the QCD sum rules for the B meson [14] and for the Λ_b baryon [15]. The difference between the b and c quark masses can also be derived [13] from the observed hadron masses by using the mass formula:

$$m_b - m_c = (\overline{M}_B - \overline{M}_D)\left\{1 - \frac{\lambda_1}{2\overline{M}_B\overline{M}_D} + O(1/m_c^3)\right\}, \tag{4.4}$$

where the spin-averaged meson masses $\overline{M}_B = \frac{1}{4}(M_B + 3M_{B^*}) = 5.31$ GeV and $\overline{M}_D = \frac{1}{4}(M_D + 3M_{D^*}) = 1.97$ GeV. Finally, the remaining theoretical input parameters for our calculation are m_b, λ_1, and the strong coupling constant α_s, which is practically the same set of parameters as in the case of the B meson [5].

For m_b we use

$$m_b = 4.9 \pm 0.2 \text{ GeV}. \tag{4.5}$$

According to the QCD sum rule calculations [14,15], we take

$$\lambda_1 = -(0.5 \pm 0.2) \text{ GeV}^2. \tag{4.6}$$

The strong coupling constant is renormalization scale dependent. We vary the scale over the range of $m_b/2 \le \mu_r \le m_b$ to estimate the theoretical error due to the scale dependence. In addition, using the modified parametrization [5] of the distribution function with two more parameters α and β

$$f(\xi) = N\frac{\xi(1-\xi)^\alpha}{[(\xi-a)^2+b^2]^\beta}\theta(\xi)\theta(1-\xi)\ , \tag{4.7}$$

we find that the values of the semileptonic decay widths for both Λ_b and B are insensitive to the change of the shape of the distribution function if the mean value and the variance of it are kept fixed. This insensitivity diminishes the model dependence. We neglect the mass, lifetime, and semileptonic branching fraction differences between $\bar{B}^0$ and B^- in all the numerical analyses of this work. Because of the small $|V_{ub}/V_{cb}| \sim 0.1$, the contributions due to the $b \to u$ transition are negligible at the present level of accuracy. The CKM matrix element $|V_{cb}|$ cancels in the ratio of the Λ_b and B semileptonic decay widths making the prediction of the ratio very reliable. We obtain the following results for the semileptonic decay width of the Λ_b baryon and its ratio to that of the B meson:

$$\Gamma_{\rm SL}(\Lambda_b) = 51 \pm 9\ |V_{cb}|^2 {\rm ps}^{-1}, \tag{4.8}$$

$$\frac{\Gamma_{\rm SL}(\Lambda_b)}{\Gamma_{\rm SL}(B)} = 1.052 \pm 0.006\,. \tag{4.9}$$

The B meson mass $M_B = 5.28$ GeV [2] and the Λ_b baryon mass $M_{\Lambda_b} = 5.62$ GeV from the ALEPH and CDF average [16] have been used. We find that the semileptonic decay width of the Λ_b baryon is about 5% larger than the semileptonic decay width of the B meson. Using the theoretical value in Eq. (4.9), togather with the measured semileptonic branching fraction for the B meson in Eq. (1.2) and the lifetime ratio of Λ_b and B in Eq. (1.1), the semileptonic branching fraction for the Λ_b is predicted to be

$$BR_{\rm SL}(\Lambda_b) = BR_{\rm SL}(B)\frac{\tau(\Lambda_b)}{\tau(B)}\frac{\Gamma_{\rm SL}(\Lambda_b)}{\Gamma_{\rm SL}(B)} = (8.7 \pm 0.7)\%\,, \tag{4.10}$$

where the error is dominated by the experimental uncertainties on the Λ_b and B lifetimes.

V. CONCLUSIONS

The much more model-independent approach to inclusive semileptonic B meson decays has been extended to the inclusive decays $H_b \to X_q \ell \bar{\nu}_\ell$. We have presented the formula for the semileptonic decay width of an unpolarized hadron containing a b quark. Eq. (2.18) shows that the decay width depends on the mass M of the decaying hadron H_b. An increase in M amounts to an increase in the semileptonic decay width, but it does not go as M^5 partly because of the suppression due to the decrease in the mean value of the distribution function forced simultaneously by the sum rule (3.18). We applied the formulas to compute the semileptonic decay width of the Λ_b baryon and found that it is about 5% larger than the semileptonic decay width of the B meson. This suggests that the increase in the total Λ_b decay width observed in the Λ_b and B lifetime measurements originates mainly in the nonleptonic-decay sector. Our prediction for the semileptonic branching fraction for Λ_b in Eq. (4.10) is compatible with the OPAL result in Eq. (1.3). Pure measurements of the semileptonic branching fraction for Λ_b are eagerly awaited. Once such measurements become available, the theoretical value for the semileptonic decay width of Λ_b obtained in this work can be used to gain an independent determination of the CKM matrix element $|V_{cb}|$. These measurements and more accurate measurements of the Λ_b and B lifetimes will probe the decay dynamics more extensively and deeply.

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