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Using muons to probe for new physics[1]

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Abstract

Searches for new physics using muons are reviewed. Particular attention is paid to muon number non-conserving processes, like the decay $\mu \to e\gamma$ and muon–electron conversion in muonic atoms. Also, experimental determinations and theoretical predictions for the muon anomalous magnetic moment are reviewed.

Muons were first observed in cosmic ray detectors in 1936 [1, 2]. However, for several following years they were being confused with pions, strongly interacting mesons predicted by Yukawa. It was not until 1947 that the identity of the muon as a weekly interacting particle was demonstrated [3]. Evidence for the existence of two kinds of charged particles with similar masses was provided by a cosmic ray experiment by Powell and collaborators [4] (for the early history of the muon physics see [5, 6]).

Muons have proved to be extremely useful both in fundamental studies and as a tool in applied science. They are relatively long-living (lifetime 2.2 μs) and can be produced in abundance so that intense muon beams can be obtained. In the latter aspect we expect future improvements by several orders of magnitude which may lead to construction of muon colliders, a new generation of particle accelerators.

After 50 years of rich and varied muon physics programs it is appropriate to review what has been achieved so far and what are the future prospects. This talk is but a modest contribution towards this goal. I will focus on two aspects of muon physics, the anomalous magnetic moment measurements and muon number non-conservation searches.

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1 Muon number non-conservation

Lepton number conservation is an empirical law incorporated into the standard model. Most ideas about a more fundamental theory predict violations either of individual (electron, muon, tau) numbers or of the total lepton number. Reactions in which muon number is violated, such as $\mu \to e\gamma$ or muon–electron conversion in the field of a nucleus, are a sensitive probe of "new physics" scenarios. Table 1 lists examples of experimental tests of muon number conservation in reactions in which a muon is present in the initial state. Other experimental searches are performed in meson decays, such as $K_L \to \mu e$, $K_L \to \pi^0 \mu e$, $K_L \to \mu^\pm \mu^\pm e^\mp e^\mp$, $K^+ \to \pi^+ \mu^+ e^-$, and $\pi^0 \to \mu e$ [7]. Exotic Z boson decays such as $Z \to \mu e$ have been searched for at LEP; bounds obtained there are several orders of magnitude weaker than in low energy experiments. However, some couplings like $Z\mu^\mp\tau^\pm$ can be better constrained by the LEP experiments.

Table 1: Experimental searches for muon number non-conservation

Reaction	Current Bound	Ongoing efforts	Proposals
$R(\mu^- N \to e^- N)$	$< 7 \times 10^{-13}$	$\sim 2 \times 10^{-14}$	10^{-16}
	SINDRUM II (Ti)	SINDRUM II (Au, Ti)	MECO (BNL)
$B(\mu^+ \to e^+ e^- e^+)$	$< 1 \times 10^{-12}$	—	—
	SINDRUM [8]	—	—
$B(\mu^+ \to e^+\gamma)$	$< 4.2 \times 10^{-11}$	$\sim 7 \times 10^{-13}$	10^{-14}
	MEGA: 1993 data	MEGA: optimistic goal	PSI
$R(\mu^+ e^- \to \mu^- e^+)$	$< 7.9 \times 10^{-9}$	10^{-11}	
	PSI [9]	PSI [10]	
$B(\mu \to e\gamma\gamma)$	$< 7.2 \times 10^{-11}$		
$R(\mu^- N_1 \to e^+ N_2)$	$< 8.9 \times 10^{-11}$	$\sim 10^{-12}$	
	SINDRUM II (Ti/Ca)	SINDRUM II [11]	

1.1 The decay $\mu \to e\gamma$

In the first years of muon physics the decay $\mu \to e\gamma$ was considered as a candidate for the dominant decay channel. However, it has soon been demonstrated that the neutral component of the final state in the normal muon decay is not a photon. At the end of the 1950s a hypothesis of an intermediate vector boson (IVB) was proposed. It was then thought that $\mu \to e\gamma$ might arise as a loop effect of an IVB. Lack of experimental evidence for such a decay led to the hypothesis of two neutrinos carrying electron and muon flavor quantum numbers, respectively. This was confirmed in a famous experiment in Brookhaven in 1962. One can say that the experimental bounds on $\mu \to e\gamma$ led to the concept of families, a cornerstone of the standard model.

The experimental situation in $\mu \to e\gamma$ searches until the end of 1972, as well as earlier references, can be found in [12]. More recent summary has been given in [13]. Currently the best upper bound on the branching ratio $\mathrm{BR}(\mu \to e\gamma) < 4.2 \times 10^{-11}$ (at 90% C.L.) comes from the experiment MEGA [14], based on data collected up to 1993. After more recent MEGA data have been analyzed, this bound will improve.

The search for $\mu^+ \to e^+\gamma$ in the contemporary experiments is based on looking for simultaneous, collinear, back-to-back photon and positron with energies equal half the muon mass, 52.8 MeV. There are two important sources of background. A radiative decay of the muon, $\mu^+ \to e^+\nu_e\bar{\nu}_\mu\gamma$, has the same signature if the neutrinos carry away little energy; this is the so-called physics background. In experiments with high rates of muons there may occur also accidental background, with an energetic electron coming from a normal decay of one muon, and a photon from a radiative decay of another one. Recently, a new idea has been put forward to reduce both backgrounds [15, 16], based on a careful study of angular distributions of photons and electrons in normal and radiative decays of polarized muons. In the decay $\mu^+ \to e^+_{R,L}\gamma$, e^+_R has an angular distribution $1 - P\cos\theta_e$ (P is the degree of muon polarization and θ_e is the angle between the direction of the muon spin and the positron track), while for e^+_L it is $1 + P\cos\theta_e$. On the other hand, the highest energy positrons in the normal decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$, as well as the highest energy photons in the radiative decay, follow the distribution law $1 + P\cos\theta_{e,\gamma}$.

By looking for the positrons emitted in the direction antiparallel to the muon spin one suppresses both the physics and the accidental backgrounds for $\mu^+ \to e^+_R\gamma$. On the other hand, by searching for photons emitted in that direction one can suppress the accidental background for $\mu^+ \to e^+_L\gamma$.

If polarized muons are used, it seems feasible to search for $\mu^+ \to e^+\gamma$ with the sensitivity of about 10^{-14}. At this level one can perform an interesting test of supersymmetric grand unification models [17, 18, 19]. In addition to suppressing the backgrounds, experiments with polarized muons could reveal the chirality of the produced electron, which could help in determining the underlying mechanism for this exotic decay. A proposal is currently under preparation for a new $\mu^+ \to e^+\gamma$ search at Paul Scherrer Institute (PSI).

1.2 Conversion of muons into electrons on nuclei

1.2.1 Muon–electron conversion: theory

Theory of a muon conversion into electrons in the field of a nucleus was first studied by Weinberg and Feinberg [20]. They focused on an electromagnetic mechanism of transfering the energy yield to the nucleus. The structure of this electromagnetic interaction is richer than for the $\mu \to e\gamma$ decay since the photon need not be on mass shell. Therefore, it is possible that the conversion can occur even if $\mu \to e\gamma$ is exactly forbidden for some reason. For example, the matrix element for the conversion contains monopole terms which do not contribute to the decay $\mu \to e\gamma$; this is because the longitudinal polarization states are possible only for virtual photons. In addition, there may be other mechanisms of the conversion apart from the photon mediated processes. This makes the conversion on nuclei a particularly interesting process to study.

The early theoretical studies of the muon conversion into electrons on nuclei performed in [20, 21, 22] were valid mainly for conversion on light nuclei. The developments before the year 1978 have been summarized in [23]. In heavier atoms new effects become important: relativistic components of the muon wave function, Coulomb distortion of the outgoing electron, and the finite nuclear size. These were addressed in [24]. Nuclear effects were also analyzed, albeit in a non-relativistic approximation, in [25] and, more recently, in [26, 27].

There are two aspects of theoretical nature in describing the muon–electron conversion, characterized by different distance scales. The short distance effects which are responsible for the muon number violation may be caused only by some as yet unknown "new physics". However, the rate of the transition depends also on the long distance atomic physics of the muonic atom.

The first group of problems has been studied in many extensions of the standard model (for a review see [23, 28, 29]). In particular, in ref. [30] the rate of the coherent conversion $\mu^- N \to e^- N$ was calculated in a variety of gauge models. It was pointed out in that paper that in a large class of models the conversion can be much more probable than the decay $\mu \to e\gamma$. This is because of the logarithmic enhancement of the form factors leading to the conversion but absent in the decay rate. Such logarithmic effects were also recently discussed in [31].

The atomic physics aspects were studied in [24, 25, 26]. Probably the most complete study to date is given in [24], where relativistic wave functions, Coulomb distortion, and finite nuclear size effects are taken into account in the analysis of the coherent conversion. In many other calculations various aspects of the coherent conversion have been considered, but none of those calculations covered all potentially important effects to the extent the paper [24] did. For example, in [25, 26] the nuclear effects were analyzed but only in a nonrelativistic approximation.

The atomic physics aspects have recently been addressed again [32]. In that study we pay special attention to relativistic effects and Coulomb distortion in the wave functions, as well as parameters of nucleon distributions in various nuclei.

1.2.2 Experiments

The experimental searches for the conversion have a long history, going back to the pioneering cosmic ray experiment by Lagarrigue and Peyrou [33]. The progress made in the following 45 years and the future prospects are summarized in Table 2.

The most recent experimental progress has been made in a series of measurements by the SINDRUM II Collaboration. The SINDRUM II detector is characterized by a large solid angle and good momentum resolution obtained by measuring at least one turn of the helical trajectories of the decay electrons [13]. The limiting factor is the muon stop

Table 2: History of searches for muon conversion into electrons on nuclei

Year	Muon source	Target	Upper bound	Ref.
1952	Cosmic rays	Cu, Sn	4×10^{-2}	[33]
1955	Nevis cycl.	Cu	5×10^{-4}	[34]
1961	Berkeley synchroc.	Cu	4×10^{-6}	[35]
1961-62	CERN synchroc.	Cu	2.2×10^{-7}	[36, 37]
1972	Virginia SREL synchroc.	Cu	1.6×10^{-8}	[38]
1977	SIN	S	7×10^{-11}	[39]
1984	TRIUMF	Pb	4.9×10^{-10}	[40]
		Ti	4.6×10^{-12}	
1992	PSI	Pb	4.6×10^{-11}	[41]
1993-97		Ti	7×10^{-13}	[42]
1998-99 (?)	PSI	Ti, Au (?)	$\sim 10^{-14}$	[42]
2000 (?)	AGS	Al	$< 10^{-16}$	[45]

rate. In order to further increase the sensitivity of the search, a new concept has been developed for the muon source. This year the spectrometer is expected to start working with a different pion beam at PSI. Muons are obtained from pions decaying in the Pion Muon Converter (PMC), which is an 8 meter long superconducting solenoid. In order to reduce the so called prompt background (high energy electrons from pion decays) pions are prevented from entering the spectrometer; this is ensured by a beam stopper at the end of PMC, where pions which do not decay are absorbed. These improvements are expected to permit measurements of $R_{\mu e}$ at the 10^{-14} level.

The ideas for a conversion search with sensitivity below 10^{-16} were put forward by Lobashev and Dzhilkibaev [43] and resulted in a proposal of an experiment in the Moscow Meson Factory [44]. This experiment has not been performed at MMF nor at other places where the proposal was submitted (TRIUMF, LAMPF, PSI). Recently it has been realized that the Alternating Gradient Synchrotron (AGS) facility in Brookhaven may be an ideal place for this project. After 1999, AGS becomes an injector for the Relativistic Heavy Ion Collider (RHIC). With this task requiring mere 2 hours of AGS time, 22 hours a day will remain for other physics programs. Very recently a proposal for an experiment called MECO has been submitted to BNL [45]. It would use the AGS proton beam for producing high intensity secondary muon beam which would permit a muon conversion search with a sensitivity better than 10^{-16}.

In addition to allowing a search for the muon conversion with more than 2 orders of magnitude better sensitivity than previous searches, MECO would serve as a demonstration facility for several elements of technology needed at a muon collider. In particular, the muon source yield of 10^{11} muons per second would be about 4 orders of magnitude more intense that what has been constructed so far. This would be a major step towards the $10^{13}\mu/s$ intensity required by a muon collider. These intense muon sources require large capture solenoids; the MECO solenoid would produce a 3 Tesla field.

2 Anomalous magnetic moment of the muon

The anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, can in principle be measured directly from the difference of precesion frequencies of the momentum and spin directions, ω_a. This method has been used by three experiments carried out at CERN and is also being used by the new experiment E821 at Brookhaven. Results obtained in past measurements and the accuracy expected to be reached at E821 are summarized in Table 3. Since the magnetic field is calibrated using NMR probes, the actual experiments measure the ratio of the difference frequency ω_a and the proton spin precesion frequency ω_p. In order to determine a_μ the ratio of the proton and muon magnetic moments must also be measured [46].

Table 3: Measurements of the muon anomalous magnetic moment

Laboratory	Value	Ref.
Columbia (Nevis)	$< 5 \times 10^{-2}$	[47]
CERN	$(1162 \pm 5) \times 10^{-6}$	[48]
CERN	$(116616 \pm 31) \times 10^{-8}$	[49]
CERN	$(1165923 \pm 8.4) \times 10^{-9}$	[50, 7]
BNL	$(? \pm 4 - 2) \times 10^{-10}$	[51, 52]

a_μ provides both a sensitive test of quantum effects in the standard model and a window to potential "new physics" effects. The current experimental value is in good agreement with theoretical expectations and already constrains physics beyond the standard model such as supersymmetry and supergravity [53, 54, 55, 56, 57], dynamical or loop muon mass generation [58], compositeness [59, 60], leptoquarks [61], two-Higgs-doublet extensions of the standard model [62] etc.

The experiment E821 at Brookhaven National Laboratory which has recently began is expected to reduce the uncertainty in $a_\mu^{\rm exp}$ to roughly $\pm 40 \times 10^{-11}$, with one month of dedicated running. With subsequent longer dedicated runs it could statistically approach the anticipated systematic uncertainty of about $\pm 10 - 20 \times 10^{-11}$ [63]. At those levels, both electroweak one and two loop effects become important and "new physics" at the multi-TeV scale is probed. Indeed, generic muon mass generating mechanisms (via perturbative or dynamical loops [58]) lead to $\Delta a_\mu \approx m_\mu^2/\Lambda^2$, where Λ is the scale of "new physics". At $\pm 40 \times 10^{-11}$ sensitivity, $\Lambda \approx 5$ TeV is being explored.

To fully exploit the anticipated experimental improvement, the standard model prediction for a_μ must be known with comparable precision. That requires detailed studies of very high order QED loops, hadronic effects, and electroweak contributions through two loop order. The contributions to a_μ are traditionally divided into

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm Hadronic} + a_\mu^{\rm EW} \tag{1}$$

QED loops have been computed analytically to the sixth order and to an even higher order numerically [64, 65, 66, 67, 68, 69, 70]

$$\begin{aligned} a_\mu^{\rm QED} &= \frac{\alpha}{2\pi} + 0.765857381(51)\left(\frac{\alpha}{\pi}\right)^2 + 24.050531(40)\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad +126.02(42)\left(\frac{\alpha}{\pi}\right)^4 + 930(170)\left(\frac{\alpha}{\pi}\right)^5 \end{aligned} \tag{2}$$

Employing $\alpha = 1/137.03599944(57)$ obtained from the electron $g_e - 2$, implies [69]

$$a_\mu^{\rm QED} = 116584706(2) \times 10^{-11} \tag{3}$$

The uncertainty is well within the $\pm 20 - 40 \times 10^{-11}$ goal.

Hadronic vacuum polarization corrections to a_μ enter at $\mathcal{O}(\alpha/\pi)^2$. They can be evaluated via a dispersion relation using $e^+e^- \to hadrons$ data and perturbative QCD (for the very high energy regime). Recent analysis of e^+e^- data [71] and hadronic τ decays [72] (including hadronic corrections in two-loop QED diagrams [73, 74]) gives

$$a_\mu^{\rm Hadronic}(\text{vac. pol.}) = 6911(100) \times 10^{-11} \tag{4}$$

The accuracy of the estimate of the hadronic contribution has not yet reached the desired level. Ongoing improvements in $e^+e^- \to hadrons$ measurements at low energies along with additional theoretical input should significantly lower the uncertainty in (4).

The result in (4) must be supplemented by hadronic light by light amplitudes (which are of three loop origin) [75]. The recent estimates [76, 77] agree with each other within error bars; here we employ the value obtained in [77]

$$a_\mu^{\rm Hadronic}(\text{light by light}) = -79(15) \times 10^{-11} \tag{5}$$

Combining (4) and (5) leads to the total hadronic contribution

$$a_\mu^{\rm Hadronic} = 6832(101) \times 10^{-11} \tag{6}$$

The main objective of the new experiment at BNL is to examine the electroweak contributions to a_μ. At the one loop level, the standard model predicts [78, 79, 80, 81, 82]

$$\begin{aligned} a_\mu^{\rm EW}(1\,\text{loop}) &= \frac{5}{3}\frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left[1 + \frac{1}{5}(1-4s_W^2)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_{W,H}^2}\right)\right] \\ &\approx 195 \times 10^{-11} \end{aligned} \tag{7}$$

where $G_\mu = 1.16639(1) \times 10^{-5}$ GeV^{-2}, and the weak mixing angle $\sin^2\theta_W \equiv s_W^2 = 1 - M_W^2/M_Z^2 = 0.224$. We can safely neglect the $\mathcal{O}\left(m_\mu^2/M_{W,H}^2\right)$ terms in (7).

The one loop result in (7) is about five to ten times the anticipated experimental error. Naively, one might expect higher order (2 loop) electroweak contributions to be of relative $\mathcal{O}(\alpha/\pi)$ and hence negligible; however, that is not the case. Kukhto, Kuraev, Schiller, and Silagadze (KKSS) [83] have shown that some two loop electroweak contributions can be quite large and must be included in any serious theoretical estimate of $a_\mu^{\rm EW}$ or future confrontation with experiment.

The two loop electroweak contributions to $a_\mu^{\rm EW}$ naturally divide into so-called fermion and boson parts

$$a_\mu^{\rm EW} = a_\mu^{\rm EW}(1\,\text{loop}) + a_\mu^{\rm EW}(2\,\text{loop; ferm.}) + a_\mu^{\rm EW}(2\,\text{loop; bos.}) \tag{8}$$

The $a_\mu^{\rm EW}(2\,\text{loop; ferm.})$ includes all two loop electroweak corrections which contain closed fermion loops while all other contributions are lumped into $a_\mu^{\rm EW}(2\,\text{loop; bos.})$. The fermionic correction $a_\mu^{\rm EW}(2\,\text{loop; ferm.})$ was calculated in [84, 85]. For $M_{\rm Higgs} \approx 250$ GeV it reduces $a_\mu^{\rm EW}$ by 11.8%. More recently that effort has been completed by computing the bosonic contribution, $a_\mu^{\rm EW}(2\,\text{loop; bos.})$ [86]. Combining these results leads to a total reduction of $a_\mu^{\rm EW}$ by a factor $(1 - 97\alpha/\pi) \approx 0.77$ and the new electroweak prediction

$$a_\mu^{\rm EW} = 151(4) \times 10^{-11} \tag{9}$$

The assigned error of $\pm 4 \times 10^{-11}$ is due to uncertainties in M_H and quark two loop effects. It also allows for possible three loop (or higher) electroweak contributions.

The present theoretical prediction for the muon anomalous magnetic moment is

$$a_\mu^{\rm theory} = 116591689(101) \times 10^{-11} \tag{10}$$

with extremely small QED and EW uncertainties. What remains is to reduce the hadronic uncertainty by a factor of 3 (or more) via improved $e^+e^- \to$ *hadrons* data and additional theoretical input. Then, one can fully exploit the anticipated improvement in $a_\mu^{\rm exp}$ from E821 at Brookhaven.

3 Conclusions

I have reviewed two aspects of muon physics: searches for muon number non-conservation and measurements of its anomalous magnetic moment. The new measurement of $g - 2$ at Brookhaven at the level of accuracy improved by a factor of 20 or more will test the quantum corrections in the standard model and impose bounds on possible new physics. New ideas for background suppression in $\mu \to e\gamma$ searches might permit a stringent test of supersymmetric grand unification models. Most exciting is the proposal for searching for muon conversion into electrons in the nuclear field at the level of 10^{-16} or lower. Such an experiment would test many proposed extensions of the standard model at an unprecedented level of precision. With these new ideas muon research remains one of the most interesting areas in particle physics.

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