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# $|V_{cb}|$ from semileptonic $B$ decays with two-loop QCD accuracy[1]

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**Abstract**

Recent progress in calculating $\mathcal{O}(\alpha_s^2)$ corrections to inclusive and exclusive semileptonic $b \to c$ transitions is reviewed. The impact of these corrections on the $|V_{cb}|$ determination from both inclusive and exclusive $B$ decays is discussed.

## 1 Introduction

Semileptonic decays of $B$ mesons provide an opportunity to measure the Cabibbo-Kobayashi-Maskawa (CKM) matrix parameter $|V_{cb}|$ with minimal theoretical uncertainties (for a recent review, see e.g. [1]). Its knowledge is important for testing the unitarity of the CKM matrix and for better understanding of the origin of CP violation.

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The value recommended by the Particle Data Group last year is $|V_{cb}| = 0.036$ to $0.046$ [2]. A lot of efforts have since been made to reduce this range. A recent summary of the experimental results [3] gives $|V_{cb}| = 0.0382 \pm 0.0019_{\rm exp}$ and the theoretical uncertainty is of a similar size as the experimental error. This improvement has been made possible in part by calculations of the second order ($\sim \alpha_s^2$) QCD corrections to the $b \to c$ transitions [4, 5, 6, 7, 8]. In this talk these results are briefly reviewed.

There are two main methods of the experimental determination of $|V_{cb}|$, usually referred to as exclusive and inclusive. In the former the end point $\omega = 1$ of the differential distribution $\mathrm{d}\Gamma/\mathrm{d}\omega$ of the decay $B \to D^* l\nu$ ($\omega$ being the product of the $B$ and $D^*$ four-velocities) is measured where the maximal information from heavy quark symmetry can be used [9]. The inclusive method relies on the determination of the total semileptonic width in the decays of $B$ mesons into charmed hadrons. In order to extract $|V_{cb}|$ from the data the perturbative corrections must be accounted for. Below we review the recent progress in their evaluation and present the resulting values of $|V_{cb}|$.

Much of the recent progress in the heavy quark physics is due to the Heavy Quark Expansion. In that framework the nonperturbative effects are described using the operator product expansion (OPE). A correct treatment of perturbative corrections in a manner consistent with the OPE is an interesting theoretical problem. In the case of $b \to c$ transitions it is of practical importance for a consistent extraction of the $|V_{cb}|$ from both inclusive and exclusive $B$ decays.

Also, we mention that neither two-photon nor two-gluon corrections to fermion decays had been known until recently. Such effects are important not only for the $b \to c$ transitions but also for $b \to u$ and for the muon decay.

## 2 Exclusive method

The differential decay rate for $B \to D^* l\nu_l$ can be written as

$$\frac{\mathrm{d}\Gamma\left(B \to D^* l\nu_l\right)}{\mathrm{d}\omega} = f(m_B, m_D^*, \omega)|V_{cb}|^2\, \mathcal{F}^2_{B\to D^*}(\omega), \tag{1}$$

where $f$ is a known function (see e.g. [9]) of $\omega$ and measurable particle masses. $\mathcal{F}_{B\to D^*}$ is the form factor describing the transition between the two hadronic states; its theoretical determination is most reliable at the zero recoil point $\omega = 1$. At that kinematical point the zero recoil sum rules for heavy flavor transitions [10] predict the value of $\mathcal{F}_{B\to D^*}$:

$$|\mathcal{F}_{B\to D^*}| = \left[\xi_A(\mu) - \frac{\xi_\pi(\mu)}{m_c^2}\mu_\pi^2(\mu) - \frac{\xi_G(\mu)}{m_c^2}\mu_G^2(\mu) - \sum_{\epsilon<\mu}|\mathcal{F}_{\rm exc}|^2\right]^{1/2} + \mathcal{O}\left(1/m^3\right).$$

The coefficient functions $\xi_A$, $\xi_\pi$ and $\xi_G$ can be calculated perturbatively. On the other hand, $\mu_\pi^2 = \langle B|\bar{b}\left(i\vec{D}\right)^2 b|B\rangle/2M_B$ and $\mu_G^2 = -\langle B|\bar{b}\left(\vec{\sigma}\vec{B}\right) b|B\rangle/2M_B$ are non–perturbative matrix elements of the kinetic and chromomagnetic operators which appear in the OPE in the order $\Lambda^2_{\rm QCD}/m_Q^2$.

The structure and the order of magnitude of the non–perturbative contributions to the form factor have been understood since a long time [10]; the accuracy of the theoretical predictions was to a large extent limited by unknown perturbative effects. In order to find their magnitude a complete calculation of $\mathcal{O}(\alpha_s^2)$ corrections to $\xi_A(\mu)$ was undertaken.

To find $\xi_A(\mu)$ with $\mathcal{O}(\alpha_s^2)$ accuracy one should calculate the $\mathcal{O}(\alpha_s^2)$ corrections to the elastic $b \to c$ transition at zero recoil, inelastic contribution with additional gluons in the final state, the $\mathcal{O}(\alpha_s)$ correction to the Wilson coefficient $\xi_\pi$ of the kinetic operator in the sum rules and also the two–loop power mixing of the kinetic operator with the unit operator [6].

The most technically demanding part of these calculations is the form factor $\eta_A$ which describes the perturbative renormalization of the axial current in the elastic $b \to c$ transition at the zero recoil point to $\mathcal{O}(\alpha_s^2)$ accuracy. This problem has been solved by two different methods. First, in Ref. [4], the result was obtained by expanding the Feynman diagrams in powers of $(m_b - m_c)/m_b$. Later, in Ref. [5] an exact analytical expression for $\eta_A$ has been obtained, confirming the results of [4].

Other corrections necessary to evaluate $\xi_A(\mu)$ have been recently calculated in Ref. [6]. In an analysis of these results it is convenient to separate the $\mathcal{O}(\alpha_s^2)$ corrections into the part proportional to $\beta_0\alpha_s^2$ (so called BLM corrections [11], first calculated for the $\eta_A$ function in [12]) and genuine $\mathcal{O}(\alpha_s^2)$ corrections. The complete resummation of the BLM corrections for $\xi_A(\mu)$ was performed in [16].

The magnitude of the genuine $\mathcal{O}(\alpha_s^2)$ corrections is an indicator of our ability to control the higher order effects, because, in contrast to BLM corrections, it is difficult to go beyond the second order of perturbation theory at present. The size of the last calculated term can be used to estimate the uncalculated higher order corrections. In [6] we demonstrated that for any reasonable choice of the scale parameter $\mu$ in the sum rules, consistent with the $1/m_c$ expansion, the $\mathcal{O}(\alpha_s^2)$ non-BLM corrections contribute at the level of less than 0.5% to the coefficient function $\xi_A(\mu)$.

The second order non-BLM QCD corrections have been used to estimate the uncertainty due to the remaining uncalculated higher order effects. A recent study [1] gives

$$\mathcal{F}_{B\to D^*} \simeq 0.91 \; - \; 0.013\,\frac{\mu_\pi^2 - 0.5\,\mathrm{GeV}^2}{0.1\,\mathrm{GeV}^2} \; \pm \; 0.02_{\mathrm{excit}} \; \pm \; 0.01_{\mathrm{pert}} \; \pm \; 0.025_{1/m^3},$$

which is finally quoted as $\mathcal{F}_{B\to D^*} = 0.91 \pm 0.06$.

With the recent experimental value [3] $\mathcal{F}_{B\to D^*}(1)|V_{cb}| = (34.3\pm1.6)\cdot10^{-3}$ we find

$$|V_{cb}| = (37.7 \pm 1.8_{\mathrm{exp}} \pm 2.5_{\mathrm{theor}}) \cdot 10^{-3}. \tag{2}$$

A more optimistic estimate of the theoretical errors in $\mathcal{F}_{B\to D^*}$ is given in [13]; it leads to the theoretical uncertainty in $|V_{cb}|$ of $\pm1.2\cdot10^{-3}$ (see, however, a discussion in [1]).

# 3 Inclusive method

In this method $|V_{cb}|$ is determined from the inclusive semileptonic decay width of the $B$ meson, $\Gamma_{\mathrm{sl}}(B \to X_c l \nu_l)$. Applying the OPE to the decay width one finds that the non–perturbative corrections in this case are suppressed by at least two powers of the $b$–quark mass. Their magnitude is estimated to be of the order of 5% [14, 1].

With the non–perturbative corrections under control, the size of the perturbative corrections was subject of some discussions in the literature. The necessity of performing a complete calculation of the $\mathcal{O}(\alpha_s^2)$ corrections to the semileptonic decay width of the $b$ quark has been repeatedly emphasized in recent years.

Computing complete $\mathcal{O}(\alpha_s^2)$ correction to $\Gamma_{\mathrm{sl}}$ remains a daunting task at present. In comparison with zero recoil calculations described in the previous section, the main difficulties are: an additional kinematical variable describing the invariant mass of the leptons and the presence of real radiation of one and two gluons. In view of these difficulties it is necessary to find a way of estimating the $\mathcal{O}(\alpha_s^2)$ effects with sufficient accuracy.

To illustrate our idea we write the semileptonic decay width $\Gamma(b \to c l \nu_l)$ as $\Gamma_{\rm sl} = \int_0^{(m_b-m_c)^2} {\rm d}q^2 \left({\rm d}\Gamma_{\rm sl}/{\rm d}q^2\right)$, where $q^2$ is the invariant mass of the lepton pair. The upper boundary of this integral corresponds to the zero recoil point, where the $\mathcal{O}(\alpha_s^2)$ QCD correction is already known [4, 5]. If we compute the correction at the other boundary ($q^2 = 0$) we might be able to estimate the total correction. More precisely, what we really need is a deviation of the corrections from the BLM result, which is known for all $q^2$ [15, 17].

The technical details of the calculations at $q^2 = 0$, also called the point of maximal recoil, are explained in detail in Refs. [7, 8]. Here we sketch the main ideas of that calculation. Since an exact calculation is not yet feasible, we construct an expansion in the velocity of the final quark. If the final ($c$) quark is not much lighter than the $b$ quark this expansion converges fairly well. The expansion parameter is $\delta \equiv (m_b - m_c)/m_b$. Both the Feynman amplitudes and the phase space can be represented as series in powers and logarithms of $\delta$.

In case of two–loop virtual corrections as well as in the emission of two real gluons, the expansion in $\delta$ is a Taylor expansion. The situation is different in the case of the single gluon radiation in diagrams where there is in addition one virtual gluon loop. Due to on–shell singularities of the one–loop diagrams, the Taylor expansion breaks down; in order to deal with such diagrams we employ the recently developed method of "eikonal expansions" [18].

Let us now summarize the numerical results of these calculations [7]. For the purpose of this discussion we use the mass ratio of the $c$ and $b$ quarks $m_c/m_b = 0.3$. It is again useful to separate the BLM and non–BLM corrections. The complete resummation of the leading BLM corrections to $\Gamma_{\rm sl}$ was performed in [17].

The $\mathcal{O}(\alpha_s^2)$ non–BLM corrections calculated in [7] turn out to be quite small. At $q^2_{\rm max} = (m_b - m_c)^2$ (zero recoil) the non–BLM contributions give a $-0.1\%$ correction to ${\rm d}\Gamma_{\rm sl}/{\rm d}q^2$. At the maximal recoil boundary $q^2 = 0$ the non–BLM correction constitutes about 1%.

There are good reasons to assume that the magnitude of the non–BLM corrections increases with the increase in the phase space available for the real radiation [7]. Based on this observation we expect that the largest discrepancy between the BLM prediction and the complete correction occurs at the lower end of the $q^2$ distribution, i.e. for $q^2 = 0$.

Therefore, taking the value of the non–BLM corrections at $q^2 = 0$ as an upper bound, we conclude that the non–BLM piece of the $\mathcal{O}(\alpha_s^2)$ correction does not exceed the value of 1% for any $q^2$.

We note that the accuracy of the $|V_{cb}|$ determination from the inclusive decays was sometimes questioned because of the unknown higher orders corrections. The uncertainty related to perturbative effects and to the quark mass values was thought to be as large as 10% in $|V_{cb}|$. Our results show that the non–BLM effects are unlikely to cause such large effects.

As for the BLM effects, considered in [17], we note that the actual impact of all–orders BLM corrections and the values of quark masses which should be used in the calculations of $\Gamma_{\rm sl}$ should be considered simultaneously, because the uncertainties in both effects tend to compensate each other (see [1] for a recent discussion). A thoughtful choice of the normalization scale for the quark masses [19] significantly reduces higher order perturbative corrections and the related uncertainty.

We quote finally the value of $|V_{cb}|$ obtained from inclusive measurements:

$$\begin{aligned} |V_{cb}|\ (\Upsilon(4S)) &= (40.6 \pm 1.2_{\rm exp})\,(1 \pm 0.05_{\rm th}) \cdot 10^{-3}, \\ |V_{cb}|\ (Z) &= (43.1 \pm 0.6_{\rm exp})\,(1 \pm 0.05_{\rm th}) \cdot 10^{-3}. \end{aligned} \tag{3}$$

To derive this value, we used experimental results as discussed recently in [3]. We give separately two values of $|V_{cb}|$, as obtained from the measurements at the $Z$ and at the $\Upsilon(4S)$ resonances.

For the theoretical uncertainties, we used the estimate presented in [1], Eq. (8.5), where we added all sources of the theoretical uncertainties linearly. The theoretical uncertainty attributed in [1] to uncalculated higher order perturbative effects is consistent with our estimates of the $\mathcal{O}(\alpha_s^2)$ corrections.

# 4 Conclusions

In this talk we have summarized the recent results on $\mathcal{O}(\alpha_s^2)$ corrections to $b \to c$ transition. At the moment, complete $\mathcal{O}(\alpha_s^2)$ corrections are known for the exclusive method and a reliable estimate exists for the inclusive method for $|V_{cb}|$ determination from semileptonic $B$ decays.

Explicit calculations show that the $\mathcal{O}(\alpha_s^2)$ corrections for both inclusive and exclusive transitions are dominated by BLM corrections. The genuine two-loop corrections contribute less than 1% to the differential decay rates. With BLM effects resummed for both inclusive and exclusive decays, this implies that no significant uncertainty in $|V_{cb}|$ determination should be attributed to uncalculated higher order effects.

Thus, an important obstacle in the theoretical predictions for the $|V_{cb}|$ determination from semileptonic $B$ decays has been eliminated. In this situation further improvement of theoretical predictions seems only possible with more accurate estimates of the non–perturbative effects (in particular in exclusive decays) and more precise determination of the input parameters, such as quark masses and numerical values of the non–perturbative matrix elements.

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