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Coherent muon–electron conversion in muonic atoms

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Abstract

Transition rates for coherent muon-electron conversion in muonic atoms, $\mu N \to eN$, are computed for various types of muon number violating interactions. Attention is paid to relativistic atomic effects, Coulomb distortion, finite nuclear size, and nucleon distributions. Discrepancies with previously published results are pointed out and explained. Results are presented for several elements of current and future experimental interest.

1 Introduction

In the standard model (SM) of electroweak interactions, leptons of different flavors do not mix because of vanishing neutrino masses. On the other hand, lepton flavor violation is predicted in various extensions of the SM. Hence, its observation would be direct evidence of physics beyond the SM.

Because the muon is a relatively stable particle which can be abundantly produced, muon–number violating processes are of particular interest. Most recently, the following processes have been studied experimentally (for a review see e.g. [1, 2]): $\mu \to e\gamma$, muonium-antimuonium oscillations, and, the subject of the present work, muon–electron conversion in the field of a nucleus. Muon number violation has also been searched for in kaon, tau, and Z decays.

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A particularly sensitive search for muon number violation is the coherent conversion in a muonic atom, formed by muon and a nucleus. Here by coherent conversion we understand a process $\mu^- N \to e^- N$ in which the nucleus N remains in its initial state (up to recoil effects). The rate of the coherent conversion is enhanced with respect to processes with a nuclear excitation by a factor of the order of the number of nucleons.

Since the original pioneering papers on the conversion theory [3, 4, 5], there have been several theoretical efforts intended to provide an accurate description of coherent muon conversion. There are two kinds of theoretical issues: first, the short distance effects which are responsible for the muon number violation (presumably caused by some "new physics"); second, the long distance atomic physics of the muonic atom, which also determines the transition rate.

The first group of problems has been studied in many extensions of the standard model (for a review see [6, 7, 8]). In particular, in ref. [9] the rate of the coherent conversion $\mu^- N \to e^- N$ was calculated in a variety of gauge models. It was pointed out in that paper that in a large class of models the conversion can be much more probable than the decay $\mu \to e\gamma$. For example, there can be logarithmic enhancements of the form factors leading to conversion which are absent in the decay rate. Such logarithmic effects were also recently discussed in [10].

Weinberg and Feinberg [3] focused on an electromagnetic mechanism of transferring the energy yield to the nucleus. The structure of this electromagnetic interaction is richer than for the $\mu \to e\gamma$ decay since the photon need not be on mass shell. Therefore, it is possible that conversion can occur even if $\mu \to e\gamma$ is forbidden for some reason. The matrix element for conversion contains monopole terms which do not contribute to the decay $\mu \to e\gamma$, because the longitudinal polarization states are possible only for virtual photons. In addition, there may be conversion amplitudes other than photon mediated processes. This makes the conversion on nuclei a particularly interesting process to study.

The early theoretical studies of muon conversion into electrons on nuclei performed in [3, 4, 5] are valid mainly for conversion on light nuclei. The developments before the year 1978 have been summarized in [6]. In heavier atoms new effects become important: relativistic components of the muon wave function, Coulomb distortion of the outgoing electron, and the finite nuclear size. These were first addressed in [11]. Nuclear effects were also analyzed, albeit in a non-relativistic approximation, in [12] and, more recently, in [13, 14].

Recently, we have undertaken a new calculation of the full relativistic atomic physics aspect of the conversion. In this talk our main results are summarized. The details of the calculation and a more extensive analysis will be given in a forthcoming paper [15].

2 General description

The muon–electron transition can occur via various mechanisms (see e.g. [6, 16, 17]). To keep the description fairly general, it is convenient to write down a low energy effective

Hamiltonian for the $\mu \to e$ transition:

$$\begin{aligned}
\mathcal{H} &= -\bar{e}\hat{O}\mu + h.c., && (1)\\
\hat{O} &= -\sqrt{4\pi\alpha}\left[\gamma_\alpha\left(f_{E0} - f_{M0}\gamma_5\right)\frac{q^2}{m^2} + i\sigma_{\alpha\beta}\frac{q^\beta}{m}\left(f_{M1} + f_{E1}\gamma_5\right)\right]A^\alpha(q) && (2)\\
&+ \frac{G_F}{\sqrt{2}}\gamma_\alpha(a - b\gamma_5)J^\alpha, &&\\
J^\alpha &= \bar{u}\gamma^\alpha u + c_d\bar{d}\gamma^\alpha d. && (3)
\end{aligned}$$

In this equation $m = m_\mu$ is muon mass and $a, b, f_{E0,1}, f_{M0,1}$ are dimensionless coupling constants.

The part of the Hamiltonian containing the photon field A_α describes the transition of a muon into an electron and an on–shell photon, $\mu \to e\gamma$, whose rate is $\Gamma(\mu \to e\gamma) = \alpha m\left(|f_{M1}|^2 + |f_{E1}|^2\right)/2$, which gives the branching ratio $Br(\mu \to e\gamma) = \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\bar{\nu}_e\nu_\mu) = 96\pi^3\alpha\left(|f_{M1}|^2 + |f_{E1}|^2\right)/\left(G_F^2 m^4\right)$. Due to gauge invariance, the transition vector and axial currents (terms proportional to f_{E0}, f_{M0}) cannot contribute to the transition to an on–shell photon [3]. We explicitly account for this by factoring out q^2/m^2. We further assume that all effective coupling constants introduced in the Hamiltonian are slowly varying functions of all external parameters (photon off–shellness, muon off–shellness etc.) and therefore can be considered as constants in the course of the calculation.

To calculate the transition rate for the coherent conversion $\mu N \to eN$ we first average the effective Hamiltonian over the nucleus. The result of this average depends on muon and electron wave functions and the field of the nucleus:

$$H_{\rm int} = \langle N|\mathcal{H}|N\rangle, \qquad \langle N|N\rangle = 1. \tag{4}$$

To calculate a matrix element of an arbitrary operator $\hat{Q}$ between two nuclei, we use the following approximation:

$$\langle N|\hat{Q}|N\rangle = \int \mathrm{d}^3r\left(Z\rho_p(r)\langle p|Q(r)|p\rangle + (A - Z)\rho_n(r)\langle n|Q(r)|n\rangle\right).$$

In the above equation Z is the number of protons in the nucleus N and $(A - Z)$ is the number of neutrons. Also, $|p\rangle$ and $|n\rangle$ denote states of a single proton and neutron respectively, with densities normalized as follows:

$$\int \mathrm{d}^3r\rho_p(r) = 1, \qquad \int \mathrm{d}^3r\rho_n(r) = 1. \tag{5}$$

To obtain matrix elements of the Hamiltonian, one should calculate the matrix element of the quark currents between two nucleons. A typical momentum transfered by the current is of the order of the muon mass, a small quantity compared to the mass of the nucleus. The time component of the soft current counts the number of constituent quarks in the nucleon. We therefore obtain

$$\begin{aligned}
\langle p|\bar{u}\gamma^0 u + c_d\bar{d}\gamma^0 d|p\rangle &= 2 + c_d,\\
\langle n|\bar{u}\gamma^0 u + c_d\bar{d}\gamma^0 d|n\rangle &= 1 + 2c_d. && (6)
\end{aligned}$$

The matrix element of spatial components of the current is proportional to the velocities of the constituents and is negligible in the present problem.

Calculating the matrix element of the Hamiltonian with respect to the nucleus states, we arrive at an effective Hamiltonian for the coherent $\mu \to e$ conversion in the field of the nucleus:

$$
\begin{aligned}
H_{\rm int} &= H_1 + H_2 + H_3, \\
H_1 &= e \int \mathrm{d}^3 r \overline{\Psi}_e(r)\gamma_0 \left(f_{E0} - f_{M0}\gamma_5\right) \Psi_\mu(r) 4\pi Z \rho_p(r) \\
H_2 &= e \int \mathrm{d}^3 r \overline{\Psi}_e(r) i\sigma_{\alpha\beta} \left(f_{M1} + f_{E1}\gamma_5\right) \Psi_\mu(r) F_{\alpha\beta}(r) \\
H_3 &= \frac{G_F}{\sqrt{2}} \int \mathrm{d}^3 r \overline{\Psi}_e(r)\gamma_0 (a - b\gamma_5) \Psi_\mu(r) \\
&\quad \times \Big(Z(2 + c_d)\rho_p(r) + (A - Z)(1 + 2c_d)\rho_n(r) \Big)
\end{aligned}
\tag{7}
$$

In the above equations the fields $\Psi_e(r)$ and $\Psi_\mu(r)$ stand for the second quantized operators, and the electromagnetic tensor $F_{\alpha\beta}$ is understood as a classical electric field produced by the nucleus. The matrix element of $H_{\rm int}$ taken between appropriate initial and final states will give the amplitude for the coherent conversion.

In our case the initial state is the muon in the $1S$ orbit around the nucleus; the final state is an electron with the energy equal to the energy of the initial muon. The corrections due to the nucleus recoil are small and we do not consider them here.

In their pioneering work on muon–electron conversion [3] Weinberg and Feinberg performed an approximate calculation of the coherent conversion rate. Using simple estimates, we would like to show when complete treatment of the problem requires going beyond the approximations used in Ref. [3]. For this purpose we describe the scales relevant for the problem. The wave function of the muon bound in the lowest orbit is characterized by the Bohr radius $a_B = (\alpha Z m)^{-1}$. The radius of the nucleus R_N scales like $m_\mu R_N \sim (Z/4)^{1/3}$. Evidently, the nucleus can be considered as point–like only if the Bohr radius is much larger than the radius of the nucleus $a_B \gg R_N$. This implies $Z \ll 60$. On the other hand, the relativistic corrections are governed by the parameter $Z\alpha$. Therefore, for high Z elements such as Pb, it is not clear a priori if the non-relativistic treatment of the muon bound state is sufficient. Moreover, another physical effect is governed by the same parameter $Z\alpha$. Consider the case when $\mu \to e$ conversion occurs due to an exchange of the photon with the nucleus. In this case the kinematics of the decay dictates that the virtuality of the photon is determined by the mass of the decaying muon. The process can be considered point–like, if this scale is much less than the Bohr radius. This implies that the photon–mediated conversion can be considered as a point–like process for $Z\alpha \ll 1$. Therefore, for heavy nuclei such as Pb the consideration of the conversion process as point like is no longer valid. The appearance of the part H_2 in the effective Hamiltonian $H_{\rm int}$ reflects this observation: one notes that the $\mu \to e$ transition current couples to the electric field of the nucleus, not to the proton density directly.

Clearly, for light nuclei one can rely on the hierarchy of the scales and perform an approximate calculation of the rate. In this case the conversion rate will be proportional to the square of the muon wave function at the origin and the square of the nucleus form-factor. Such an approximation was used in Ref. [3]. When the charge of the nucleus grows, all scales relevant to the problem become comparable and the above approximations cannot be trusted. In this case a correct treatment of the problem requires solving the Dirac equation for both muon and electron wave functions in the field of the nucleus.

To describe proton and neutron distributions in the nucleus we use two–parametric Fermi functions:

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+e^{(r-r_{p(n)})/a_{p(n)}}}, \qquad \int d^3 r \rho_{p(n)}(r) = 1. \tag{8}$$

In the above equation $r_{p(n)} \sim A^{1/3}$ is the radius of the proton (neutron) fraction of the nucleus and $a_{p(n)}$ is the thickness of the boundary of the proton (neutron) fraction. Precise values of these parameters depend on the nucleus and can be found in tables [18, 19].

The transition rate for the coherent conversion is given by:

$$\omega(\mu^- N \to e^- N) = \omega_{conv} = \sum_{\lambda_f} |H^{fi}_{\rm int}|^2. \tag{9}$$

Here the sum goes over all quantum numbers which the electron in the final state can have in addition to energy. The muon bound state wave function is normalized to unity. The electron wave function of the continuous spectrum is normalized such that:

$$\int {\rm d}^3 r \psi^*_{E',\lambda'}(r)\psi_{E,\lambda}(r) = \delta_{\lambda,\lambda'} 2\pi \delta(E'-E) \tag{10}$$

where λ denotes a set of all discrete quantum numbers which electron obeys in addition to energy.

It is convenient to use the expression for the wave functions as suggested in [20]:

$$\psi^m_k = \begin{pmatrix} g^{(k)}(r)\chi^m_k \\ i f^{(k)}(r)\chi^m_{-k} \end{pmatrix}. \tag{11}$$

In this equation m is the eigenvalue of the operator J_z, $J = L + S$; $-k$ is the eigenvalue of the operator $L\cdot\sigma + 1$ (see e.g. [20]); χ^m_k is an orthonormalized spinor. For the muon wave function we use the lowest energy bound state wave function; this suggests that $j = 1/2$, $m = \pm 1/2$, $k = -1$. We will distinguish muon and electron radial wave functions by a subscript.

A calculation of the explicit expression for the transition rate ω_{conv} is now straightforward. We find

$$\begin{aligned}
\omega_{conv} &= \left| \int {\rm d}r r^2 \left[\tilde{f}_{E0}(r)(g_e^- g_\mu^- + f_e^- f_\mu^-) + \frac{f_{M1}}{m}(g_e^- f_\mu^- + f_e^- g_\mu^-)\frac{dV}{dr}\right]\right|^2 \\
&+ \left| \int {\rm d}r r^2 \left[\tilde{f}_{M0}(r)(g_e^- g_\mu^- + f_e^- f_\mu^-) + \frac{f_{E1}}{m}(g_e^- f_\mu^- + f_e^- g_\mu^-)\frac{dV}{dr}\right]\right|^2, \\
\tilde{f}_{E0}(r) &= -f_{E0}\frac{4\pi Z\alpha}{m^2}\rho_p(r) + \frac{G_F}{\sqrt{2}} a \left[Z(2+c_d)\rho_p(r) + N(1+2c_d)\rho_n(r)\right], \\
\tilde{f}_{M0}(r) &= -f_{M0}\frac{4\pi Z\alpha}{m^2}\rho_p(r) + \frac{G_F}{\sqrt{2}} b \left[Z(2+c_d)\rho_p(r) + N(1+2c_d)\rho_n(r)\right],
\end{aligned} \tag{12}$$

where $V(r) = -eA^0(r)$ is the muon potential energy in the field of the nucleus.

The above equation provides a general expression for the rate of the reaction $\mu^- N \to e^- N$ which we will use for numerical analysis in the next section. The radial wave functions $f^-_{e,\mu}$ and $g^-_{e,\mu}$ are obtained by solving the Dirac equation. Inside the nucleus and close to it these solution are found numerically. At large distances we match the numerical solutions to the exact Coulomb wave functions.

3 Numerical analysis

In general, the following integrals are needed for the description of the transition rate:

$$
\begin{aligned}
I_1^p &= -\frac{4\pi Z\alpha}{m^2}\int \mathrm{d}r r^2 \rho_p(r) g_\mu^- g_e^- , \qquad I_2^p = -\frac{4\pi Z\alpha}{m^2}\int \mathrm{d}r r^2 \rho_p(r) f_\mu^- f_e^- \\
I_1^n &= -\frac{4\pi Z\alpha}{m^2}\int \mathrm{d}r r^2 \rho_n(r) g_\mu^- g_e^- , \qquad I_2^n = -\frac{4\pi Z\alpha}{m^2}\int \mathrm{d}r r^2 \rho_n(r) f_\mu^- f_e^- \\
I_3 &= \frac{1}{m}\int \mathrm{d}r r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r} g_\mu^- f_e^- , \qquad I_4 = \frac{1}{m}\int \mathrm{d}r r^2 \frac{\mathrm{d}V(r)}{\mathrm{d}r} f_\mu^- g_e^- .
\end{aligned} \tag{13}
$$

Tables with the values of these integrals for various elements will be given in [15]. Using those values, the conversion rate is

$$
\omega_{conv} = 3\cdot 10^{23}(\omega^{(1)}_{conv} + \omega^{(2)}_{conv}) \ \mathrm{sec}^{-1} \tag{14}
$$

$$
\begin{aligned}
\omega^{(1)}_{conv} &= \left| f_{E0} I_p - \frac{G_F}{\sqrt{2}}\frac{m^2}{4\pi Z\alpha} a\Big(Z(2+c_d)I_p + N(1+2c_d)I_n\Big) + f_{M1} I_{34}\right|^2 , \\
\omega^{(2)}_{conv} &= \left| f_{M0} I_p - \frac{G_F}{\sqrt{2}}\frac{m^2}{4\pi Z\alpha} b\Big(Z(2+c_d)I_p + N(1+2c_d)I_n\Big) + f_{E1} I_{34}\right|^2 ,
\end{aligned} \tag{15}
$$

where

$$
I_p = -\left(I_1^p + I_2^p\right), \qquad I_n = -\left(I_1^n + I_2^n\right), \qquad I_{34} = I_3 + I_4. \tag{16}
$$

All dimensional parameters in the above equations are expressed in fermi. For proton and neutron distributions we use the two–parameter Fermi distribution (8) with parameters taken from Ref. [18, 19].

For the present application we neglect the effects of the vacuum polarization; such approximation is justified in view of much bigger errors in wave function integrals induced directly by the nuclear distribution uncertainties.

4 Branching ratio for μ-e conversion in a specific model

We consider here one specific model which predicts lepton flavor violation, a supersymmetric grand-unified theory discussed in Ref. [17].

The analysis of Ref. [17] implies that magnetic couplings f_{M1} and f_{E1} (cf. Eq.(2)) are significantly enhanced in a large region of the parameter space of those models in comparison with other couplings in the effective Hamiltonian. Therefore, it is reasonable to neglect all other couplings in the effective Hamiltonian and to analyze the dependence of $\omega_{conv}/\omega_{capt}$ on the choice of target.

With these approximations the ratio of the conversion rate to the capture rate becomes

$$\frac{\omega_{conv}}{\omega_{capt}} = 3\cdot 10^{12}\Big(|f_{E1}|^2+|f_{M1}|^2\Big)B(A,Z), \tag{17}$$

$$\text{with} \qquad B(A,Z)=10^{11}\,\frac{I_{34}^2}{\omega_{capt}/\mathrm{sec}^{-1}}. \tag{18}$$

We compare the above formula with the Weinberg–Feinberg approximation, in which the rate is obtained by replacing $B(A,Z)$ in (17) by $B_{WF}(A,Z)$:

$$B_{WF}(A,Z)=8\alpha^5 m Z_{\mathrm{eff}}^4 Z F_p^2\frac{1}{\omega_{capt}}\frac{1}{3\cdot 10^{12}} \tag{19}$$

In this formula Z_{eff} denotes the "effective Z," obtained by averaging the muon wave function over the nuclear density (see e.g. [12]), and F_p is the formfactor describing the charge distribution, given by

$$F_p=\frac{\int \mathrm{d}^3 r\rho(r)\frac{\sin mr}{mr}}{\int \mathrm{d}^3 r\rho(r)}. \tag{20}$$

In table 1 we show the B and B_{WF} coefficients for three elements. We conclude that in this particular model the ratio of the coherent conversion rate to the capture rate, as described by our $B(A,Z)$, does not change significantly with changing the target. The WF approximation tends to slightly overestimate the conversion rate. In ref. [11] various corrections to the WF approximations have been studied; in that approach B_{WF} should be replaced by $B_S\equiv C_1C_2C_3B_{WF}$, where the correction factors $C_{1,2,3}$ are listed in table III in [11]. We agree with these results for non-photonic mechanisms of conversion. However, for the photonic case we are considering here, our results differ from [11] even stronger than from the WF approximation, as can be seen in table 1. This is because the effect of the non-locality in the interaction with the electric field was not taken into account in [11]. It is especially important for heavy elements.

It is instructive to compare the conversion rate to the branching ratio for $\mu\to e\gamma$ in this model:

$$\frac{Br(\mu\to e\gamma)}{\omega_{conv}/\omega_{capt}}=\frac{96\pi^3\alpha}{G_F^2m^4}\frac{1}{3\cdot 10^{12}B(A,Z)}\approx\frac{428}{B(A,Z)}. \tag{21}$$

This ratio varies from 389 for ^{27}Al to 238 for ^{48}Ti, and increases again for heavy elements to 342 for ^{208}Pb.

Table 1: Comparison of our results for the coefficient $B(A,Z)$ in the rate formula (17) with the Weinberg-Feinberg approximation $B_{WF}(A,Z)$ and with $B_S(A,Z)$ obtained from ref. [11]. $Z_{\rm eff}$ is taken from [12] and the capture rates ω_{capt} from [21]; F_p is computed using (20).

Element	$B(A,Z)$	$B_{WF}(A,Z)$	$B_S(A,Z)$	$Z_{\rm eff}$	F_p	ω_{capt}, $[10^6/\text{sec}]$
Al	1.1(1)	1.2	1.3	11.62	0.63	0.7
Ti	1.8	2.0	2.2	17.61	0.53	2.6
Pb	1.25(15)	1.6	2.2	33.81	0.15	13.0

5 Bounds on the muon number violating couplings from SINDRUM

Using updated results for the atomic part of the theoretical description of the coherent muon electron conversion, we reconsider the analysis performed in Ref. [22]. In that paper the upper limit for the ratio $\omega_{conv}/\omega_{capt}$ has been reported based on the measurement using ^{208}Pb. These results were combined with previous measurement on ^{48}Ti target to obtain the bounds on muon–number violating coupling constants. In that analysis it was assumed that the effective Hamiltonian does not contain a piece which corresponds to the photon mediated conversion.

As a first step we have to switch to the model and notations of Ref. [22]. The model, considered in Ref. [22], corresponds to the exchange of a heavy vector or scalar boson which mediates $\mu \to e$ transition. For this reason, we should equate $f_{E0,1}$ and $f_{M0,1}$ to zero in our general expression for conversion rate, eq. (12). Also, to switch to their notations, one should substitute $b = a$, $ac_d \to (g_V^0 - g_V^1)/2$, and $a \to (g_V^0 + g_V^1)/2$ in that equation. Constants g_V^0, g_V^1 then parameterize the coupling of the heavy gauge boson to isoscalar and isovector parts of the quark current respectively.

Then, one gets for the conversion rate:

$$\omega_{conv} \sim I_p^2\left[g_V^0\left(1-\frac{N}{A}\xi\right)+g_V^1\left(\frac{(Z-N)}{3A}+\frac{N}{3A}\xi\right)\right]^2. \tag{22}$$

In this equation we use the notation (cf. Eq. (16)): $\xi = I_p/I_n$. Using the bounds on the $\mu \to e$ branching ratios measured with Ti and Pb targets, we derive the new bounds for the coupling constants g_V^0, g_V^1:

$$|g_V^0| < 8 \pm 2 \cdot 10^{-7}, \qquad |g_V^1| < 40 \pm 13 \cdot 10^{-6}. \tag{23}$$

The variation in the boundaries shown above is the estimate of the uncertainty of the result induced by uncertainties in the input nuclear parameters.

These new bounds should be compared with the values

$$|g_V^0| < 3.9 \cdot 10^{-7}, \qquad |g_V^1| < 9.7 \cdot 10^{-6}. \tag{24}$$

which were given in Ref. [22] using the results of Ref. [11].

There are two reasons for such large differences. As we have mentioned already, in Ref. [22] the results of Ref. [11] have been used to interpret experimental data. We note in this respect, that Table I of ref. [11] does not provide correct results – the results for $\omega_{conv}/\omega_{capture}$ quoted there are about a factor of 2 too large. Second, in the case of a Pb target the difference in proton and nucleon distributions becomes quite noticeable. However, the numbers in Table I of [11] were obtained using identical proton and neutron distributions. These two reasons conspire to give quite a large discrepancy.

We note, however, that the values for the bounds quoted above (especially for g_V^1) are very sensitive to the input parameters. This happens, because the bounds are derived from a system of linear equations in which the coefficients of g_V^1 there are much smaller than all other parameters. Therefore even a small variation in the input parameters can generate quite substantial change in the resulting value for g_V^1.

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