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Heavy Quark Effective Field Theory

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1 Introduction

1.1 Overview

The standard model (SM) of elementary particle physics [1] has turned out to be a overwhelmingly successful description of the phenomena in high energy physics. Certain aspects of it, namely the couplings of the quarks and leptons to the neutral weak boson, have been tested with very high precision and confirmed the SM even at the level of loop corrections.

Despite of this success many people are convinced that the SM cannot be the final theory. In particular the Higgs sector is not considered satisfactory, since one of its main ingredients, the neutral scalar Higgs boson has not yet been seen. Related to the structure of the Higgs sector is the way the quarks and leptons acquire their masses and the mixing of quark flavours, parametrized by a 3×3 unitary matrix, the so-called Cabibbo Kobayashi Maskawa (CKM) matrix [2].

The CKM matrix in the SM encodes also the effect of CP violation which has been observed up to now only in the system of neutral K mesons. Alternative models, such as the superweak model proposed by Wolfenstein [3] have been formulated and have not yet been ruled out definitively. In other words, the CKM sector of the SM has not been tested at a level of precision that can be compared to what is known about the Z_0 couplings form the LEP experiments.

In order to test this part of the SM one has to study charged current or flavour changing neutral current (FCNC) processes of quarks. The SM makes predictions for the processes involving quarks, however, experimentally only hadrons are observable. In order to connect the parameters of the SM with the data one has to deal with the bound state problem of strong interactions, which has not yet been solved starting form the fundamental entities of QCD, with the possible exception of lattice gauge theory which recently has produced first results on weak form factors [4].

In the past five years considerable progress has been made towards a QCD-based and model independent description of hadrons containing a heavy-flavour quark, where heavy means that the mass of the quark m_Q is large compared to some scale $\bar{\Lambda}$ determined by the light degrees of freedom, e.g. the constituent quark mass of a light quark or the scale Λ_{QCD} entering through the running of the strong coupling α_s. This progress has been achieved by the use of the heavy mass limit for the heavy quark, in which it is replaced by a static source of a colour field. This limit of QCD leads to a well-defined field theory, the so-called Heavy Quark Effective Theory (HQET).

The infinite mass limit from QCD has been known for some time [5], but the main observation, which triggered the enormous development of this field, is that in the infinite mass limit two additional symmetries appear which are not present in full QCD [6]. Since then the implications of the heavy mass limit and HQET have been extensively studied in innumerable publications, and the development of the field is documented in more or less extensive reviews [7].

The two additional symmetries of the heavy mass limit have important phenomenological applications; they lead to model-independent relations between form factors describing e.g. exclusive weak decays. The origin of the new symmetries is quite simple. The first symmetry is a heavy-flavour symmetry, which is due to the fact that the interaction of the quarks with the gluons is flavour-blind and in the heavy mass limit all heavy quarks act as a static source of colour. Formally this corresponds to an $SU(2)$ symmetry relating b and c quarks moving with the same velocity. The second symmetry is the spin symmetry of the heavy quark. The interaction of the heavy quark spin with the "chromomagnetic" field is inversely proportional to the heavy mass and hence vanishes in the infinite mass limit. As a consequence, the rotations for the heavy quark spin become an $SU(2)$ symmetry, which holds for a fixed velocity of the heavy quark.

Corrections to the limit $m_Q \to \infty$ may be studied systematically in the framework of HQET. The corrections are given as power series expansions in two small parameters. The first small parameter is the strong coupling constant, taken at the scale of the heavy quark $\alpha_s(m_Q)$. This type of correction may be calculated systematically using perturbation theory in HQET. The second type of correction is characterized by the small parameter $\bar{\Lambda}/m_Q$, where $\bar{\Lambda}$ is a scale of the light QCD degrees of freedom, e.g. $\bar{\Lambda} \sim m_{hadron} - m_Q$. In the effective theory approach this type of corrections enter through operators of higher dimension, the matrix elements of which have to be parametrized in general by additional form factors.

More or less parallel to the development of heavy quark symmetry and HQET, which is well suited for exclusive decays, the heavy mass expansion has been applied also to inclusive decays [8]-[15]. The heavy quark mass sets a scale that is large compared to Λ_{QCD}, and one may use a similar setup as in deep inelastic scattering for the inclusive decays; in particular, the operator product expansion for the inclusive decays yields an expansion in inverse powers of the heavy quark mass. In this way one may not only study total rates, but also differential distributions such as the lepton energy spectra in inclusive semileptonic decays.

This theoretical progress has a large impact on the determination of the CKM matrix, in particular on the matrix elements V_{ub} and V_{cb}. In the near future these entries will be measured from semileptonic B decays with at least controllable theoretical uncertainties, since the $1/m_Q$ expansion makes definite predictions and the uncertainties may be estimated in the framework of the $1/m_Q$ expansion and QCD perturbation theory. This is a considerable progress compared to a few years ago, since the QCD bound state problem had to be modelled, and in this way uncontrollable systematic uncertainties entered the game.

This review is intended to summarize the development over the past few years of heavy mass expansion methods based on QCD. The first part is devoted to the theoretical framework and its basis in QCD. The second part deals with applications, in particular with the possibility to extract CKM matrix elements form heavy flavour decays. Finally, the application of the $1/m_Q$ expansion to inclusive decays is consiedered and the lifetimes of heavy hadrons are considered in this framework.

1.2 Effective Field Theory

The concept of effective theory [16] has become a very important tool in theoretical physics; it is widely used in particle physics phenomenology, in particular in the field of weak interaction.

The presently known spectrum of particles spans a mass range of severel orders of magnitude, ranging from the eV region up to the mass of the top quark of about 170 GeV, and new accelerators will be able te explore masses in the TeV regime. Due to this large variation of mass scales a frequently occurring situation in particle physics phenomenology is that a calculation involves very disparate mass scales. The standard example are weak decays of leptons and hadrons, the masses of which lie in the MeV or GeV range and which set the typical scales involved in this type of process. On the other hand, weak interactions are mediated by bosons with masses of about 100 GeV and thus these processes involve also mass scales much larger than the mass of the decaying particle.

Intuitively it is clear that in such a situation a description should be possible which does not explicitly involve the degrees of freedom associated with the large mass scale. In other words, one should switch to an effective theory in which the heavy degrees of freedom have been removed.

In general this is possible due to the so called decoupling theorem [17] which states that in general a heavy particle decouples from the lighter ones in the limit in which its mass tends to infinity; the only remnant of the heavy particle will be on one hand a logarithmic dependence of the coupling parameters of the effective low energy theory on the mass of the heavy particle and terms suppressed by inverse powers of the heavy mass on the other hand.

Technically this corresponds to an expansion of the Greens functions of the full theory (i.e. the one still containing the heavy particle) in inverse powers of the heavy mass up to logarithms, which may be accounted for by renormalization group techniques. This expansion of the Greens functions may be obtained from a Lagrangian field theory, which is called an effective theory.

The leading term in the heavy mass expansion corresponds in general to dimension four contributions to the effective theory Lagrangian and thus defines a renormalizable theory. From these pieces of the Lagrangian one obtains the leading term in the heavy mass expansion of the full theory Greens functions; higher order terms in this expansion may be included by adding higher dimensional operators (i.e. dimension five operators and higher) to the effective Lagrangian. Up to some given order in the heavy mass expansion only a finite number of insertions of these higher dimensional operators is needed and any finite number of insertions of operators of dimension five or higher may be renormalized and hence the apearance of these terms in the effective theory does not spoil the renormalizability of the effective theory up to a finite order in the heavy mass expansion.

The construction of the effective theory Lagrangian proceeds in three steps. After having identified the massive degree of freedom this degree of freedom is removed by integrating it out from the functional integral of the full theory. This step in general leads to a nonlocal action functional; the nonlocality is related to the mass of the heavy particle which has been integrated out in the previous step and which may be replaced at scales much lower than the mass of the heavy particle by local interactions. Technically this step is an operator product expansion of the nonlocal terms into a series of operators with increasing dimension; the dimension of the effective Lagrangian is four and thus these higher dimensional operators have to have dimensional coupling constants, the scale of which is set by the heavy mass. The terms with dimension four have dimensionless couplings and define the renormalizable piece of the effective Lagrangian; in other words, this part is used to calculate the renormalization group flow in the effective theory.

Alternatively, one may proceed slightly differently in constructing the effective theory Lagrangian and obtain it from matching the full to the effective theory. As a first step one writes down all possible operators up to a given dimension which involve only the light degrees of freedom and which are allowed by the symmetries of the system. The effective theory Lagrangian is then written as a sum of these operators with coefficients which have to be determined from comparing the effective Lagrangian with the one of the full theory.

In the conventional sense one would say that an effective theory is not renormalizable due to the higher dimensional operators. However, up to a finite order in the expansion in inverse powers of the heavy mass one has only a finite number of insertions of this type of operators in the Greens functions, and for this case the renormalization of these contributions is known once the renormalizable piece, i.e. the dimension four operators are known. In this sense the concept of renormalizability becomes irrelevant for effective theories.

The comparison of the full theory with the effective theory is called matching. At some scale μ of the order of the heavy mass the Greens functions of the full and the effective theory are compared, thereby fixing the coupling constants of the effective theory in terms of the parameters of the full one. Changing the scale μ will induce changes in the Greens functions in such a way that physical quantities remain independent of the scale. This in general involves also appropriate changes of the couplings in the effective theory, turning them into "running" quantities depending on the scale μ. The running, i.e. the renormalization group flow, is determined by the renormalization group equations.

The choice of a scale may also be interpreted as a seperation of long and short distance contributions: The effect of physics originating from scales above μ is parametrized in the couplings while any effect from scale below μ is still explicit in the dynamics described by the Lagrangian. Of course, any observable quantity has to be independent of the arbitrary choice of μ, which is assured by the renormalization group equations.

At that point one may rise the question, why the effective theory description has any advantage over performing the calculations in the full theory. First of all, there are examples where the full theory is not known or where the matching to the effective theory is technically not feasible (such as chiral perturbation theory and QCD). In these cases it is obvious that one directly performs the calculations in the effective theory. But even in cases where the matching may be performed explicitly (such as weak interactions or Heavy Quark Effective Theory) it may be advantageous to switch to an effective theory description, mainly for two reasons. Firstly the strong interactions become weak at large scales and the matching and running may be done perturbatively. The renormalization group then automatically takes care of large logarithms appearing in perturbation theory and resums them properly. Secondly, it may happen that in the effective theory additional symmetries appear which are not present in the full theory. This is the case for Heavy Quark Effective Theory where these additional symmetries restrict the non-perturbative input quite severely.

2 Theoretical Background

In this first part of the review the theoretical background of Heavy Quark Effective Theory (HQET) is explained. The first section deals with the Lagrangian aof HQET and the corresponding expansion of the fields. The second section deals with one of the major achievements of the infinite mass limit, namely the additional symmetries that appear as $m_Q \to \infty$. Finally, in the third section the corrections to the limiting behaviour are discussed and some sample calculations are considered.

2.1 $1/m_Q$ Expansion of QCD

2.1.1 Lagrangian and fields

Different approaches have been suggested to perform the infinite mass limit of QCD. Here we shall discuss only two of them in some detail. The first one [18] is the one which is along the lines of the effective theory discussion of the introduction; here the starting point will be the generating functional of Greens functions of full QCD, which will be represented as a functional integral. After identifying the heavy degrees of freedom these may be integrated out explicitly, leading to a non-local action, which in the second step is expanded into local terms, which define the Lagrangian of the effective theory.

The second way [19] is along the lines of the textbook way to obtain the non-relativistic limit of the Dirac equation by transforming away the couplings to the small components of the spinor. These transformations, the so-called Foldy Wouthuysen transformations, yield a different Lagrangian and also a different expansion for the fields. However, it became clear that these two approaches are completely equivalent, since they yield the same physical amplitudes, and which representation should be used depends on the problem under consideretion.

The first way to formulate the heavy mass limit is by integrating out heavy degrees of freedom from the functional integral of QCD Greens functions [18]. This integration may in fact be done explicitly, since for the case at hand it amounts to a Gaussian functional integration. Formulating the heavy mass limit in this way clearly exhibits that it corresponds to an effective theory in the usual sense. We start from the generating functional of the QCD Greens functions

$$Z(\eta,\bar{\eta},\lambda) = \int [dQ][d\bar{Q}][d\phi_\lambda] \exp\Big\{ iS + iS_\lambda + i\int d^4x \, (\bar{\eta}Q + \bar{Q}\eta + \phi_\lambda \lambda) \Big\}, \tag{1}$$

where $\phi_\lambda = q,\ A_\mu^a$ denotes the light degrees of freedom (light quarks q and gluons A_μ) with the action S_λ, while S denotes the piece of the action for the heavy quark Q including its coupling to the gluons

$$S = \int d^4x \, \bar{Q}(i\not{D} - m_Q)Q. \tag{2}$$

where

$$D_\mu = \partial_\mu + igA_\mu, \tag{3}$$

is the covariant derivative of QCD. We have introduced source terms η for the heavy quark and λ for the light degrees of freedom.

We shall consider hadrons containing a single heavy quark, and we assume that this heavy hadron moves with a velocity v

$$v = \frac{p_{\text{hadron}}}{m_{\text{hadron}}}, \quad v^2 = 1, \quad v_0 > 0 \tag{4}$$

This velocity vector may be used to split the heavy quark field Q into an "upper" component ϕ and a "lower" one χ

$$\phi_v = \frac{1}{2}(1+\not{v})Q, \ \ \not{v}\phi_v = \phi_v\,, \tag{5}$$

$$\chi_v = \frac{1}{2}(1-\not{v})Q, \ \ \not{v}\chi_v = -\chi_v\,, \tag{6}$$

and to define a decomposition of the covariant derivative into a "longitudinal" and a "transverse" ($\perp$) part

$$D_\mu = v_\mu(v\cdot D) + D_\mu^\perp\,, \quad \Big\{\not{D}^\perp, \not{v}\Big\} = 0. \tag{7}$$

Using (5-7) the action (2) of the heavy quark field takes the form

$$S = \int d^4x \left[\bar{\phi}_v\{i(v\cdot D) - m_Q\}\phi_v - \bar{\chi}_v\{i(v\cdot D) + m_Q\}\chi_v + \bar{\phi}_v i\not{D}^\perp \chi_v + \bar{\chi}_v i\not{D}^\perp \phi_v \right]. \tag{8}$$

The heavy quark in the meson is very close to being on shell, and thus the space time dependence of the heavy quark field is mainly that of a free particle moving with velocity v. This suggests a reparametrization of the fields by removing the space time dependence of a solution of the free Dirac equation. We shall chose the "particle-type" parametrization corresponding to the "positive energy solution" of the Dirac equation

$$\phi_v = e^{-im_Q(v\cdot x)} h_v\,, \qquad \chi_v = e^{-im_Q(v\cdot x)} H_v, \tag{9}$$

such that the space time dependence of the remaining fields h_v and H_v is determined by the residual momentum $k = p - m_Q v$, which is due to binding effects of the heavy quark inside the heavy hadron, and which is a "small" quantity of order Λ_{QCD}.

Expressed in these fields the action of the heavy quark becomes

$$S = \int d^4x \left[\bar{h}_v i(v \cdot D) h_v - \bar{H}_v \{ i(v \cdot D) + 2m_Q \} H_v + \bar{h}_v i \not{D}^{\perp} H_v + \bar{H}_v i \not{D}^{\perp} h_v \right], \tag{10}$$

The term containing the sources is also rewritten in terms of the fields h_v and H_v

$$\int d^4x \, (\bar{\eta}\psi + \bar{\psi}\eta) = \int d^4x \, (\bar{\rho}_v h_v + \bar{h}_v \rho_v + \bar{R}_v H_v + \bar{H}_v R_v), \tag{11}$$

where ρ_v and R_v are now source terms for the upper component field h_v and the lower componenent part H_v respectively.

In terms of the new variables the generating functional reads

$$\begin{aligned} Z(\rho_v, \bar{\rho}_v, R_v, \bar{R}_v, \lambda) &= \int [dh_v][d\bar{h}_v][dH_v][d\bar{H}_v][d\phi_\lambda] \\ &\exp \left\{ iS + S_\lambda + i \int d^4x \, (\bar{\rho}_v h_v + \bar{h}_v \rho_v + \bar{R}_v H_v + \bar{H}_v R_v + \phi_\lambda \lambda) \right\}, \end{aligned} \tag{12}$$

where the action S for the heavy quark is given in eq.(10).

From (10) it is obvious that the heavy degree of freedom is the lower component field H_v, since it has a mass term $2m_Q$, while the upper component field h_v is a massless field describing the static heavy quark. In the heavy mass limit only the Greens functions involving the field h_v have to be calculated, and hence we integrate over H_v in the functional integral (12) with the sources of the lower component field R_v and $\bar{R}_v$ set to zero. This can be done explicitly, since it is a Gaussian integration

$$Z(\rho_v, \bar{\rho}_v, \lambda) = \int [dh_v][d\bar{h}_v][d\lambda] \Delta \exp \left\{ iS + S_\lambda + i \int d^4x \, (\bar{\rho}_v^+ h_v^+ + \bar{h}_v^+ \rho_v^+ + \phi_\lambda \lambda) \right\}, \tag{13}$$

where now the action functional for the heavy quark becomes a non-local object

$$S = \int d^4x \left[\bar{h}_v^+ i(v \cdot D) h_v^+ - \bar{h}_v^+ \not{D}^{\perp} \left(\frac{1}{i(v \cdot D) + 2m_Q - i\epsilon} \right) \not{D}^{\perp} h_v^+ \right]. \tag{14}$$

This Gaussian integration corresponds to the replacement

$$H_v = \left(\frac{1}{2m_Q + ivD} \right) i \not{D}_{\perp} h_v \tag{15}$$

for the lower component field. Furthermore, the Gaussian integration yields a determinant Δ. In the full theory one may also perform this Gaussian integration, and the determinant obtained contains all the closed loops of quarks. After renormalization of the full theory their contribution starts at order $1/m^2$ with an Uehling potential like term. In the effective theory one may take the determinant Δ to be a constant, if the terms of order $1/m_Q^2$ and higher comming from the closed heavy quark loops are included by matching to the full theory. Since we shall discuss only the leading term of the $1/m_Q$ expansion in this section, we may drop the determinant in what follows.

The non-locality of the action functional is connected to the large scale set by the heavy quark mass, and the non-local terms may be expanded in terms of an infinite series of local operators, which come with increasing powers of $1/m_Q$. In the context of a field theory this corresponds to a short distance expansion and hence these operators have to be renormalized. The tree level relations may be read off from the geometric series expansion of the non-local term in (14). In this way we obtain the expansion of the field and the Lagrangian

$$\begin{aligned} Q(x) &= e^{-im_Q vx}\left[1+\left(\frac{1}{2m_Q+ivD}\right)i\not{D}_\perp\right]h_v \qquad (16)\\ &= e^{-im_Q vx}\left[1+\frac{1}{2m_Q}\not{D}_\perp+\left(\frac{1}{2m_Q}\right)^2(-ivD)\not{D}_\perp+\cdots\right]h_v \\ \mathcal{L} &= \bar{h}_v(ivD)h_v+\bar{h}_v i\not{D}_\perp\left(\frac{1}{2m_Q+ivD}\right)i\not{D}_\perp h_v \qquad (17)\\ &= \bar{h}_v(ivD)h_v+\frac{1}{2m_Q}\bar{h}_v(i\not{D}_\perp)^2 ih_v+\left(\frac{1}{2m_Q}\right)^2\bar{h}_v(i\not{D}_\perp)(-ivD)(i\not{D}_\perp)h_v+\cdots \end{aligned}$$

A Greens function with an operator insertion is treated in a similar way; the net effect of this is that the heavy quark fields in the operator insertion are replaced by the expansion (16).

The second way we shall consider here is the traditional one which is based on the Foldy Wouthuysen transformations [19]. The basic idea is to find a transformation

$$Q(x) = e^{-iS}Q'(x), \qquad (18)$$

which is unitary in spinor space and which decouples the large and small component fields in the Lagrangian. Defining the large and small component fields in the same way as above, one writes

$$\begin{aligned} h^{(+)}(x) &= \frac{1+\not{v}}{2}e^{im_Q vx}\left(\cdots e^{-\frac{1}{2m_Q^3}\mathcal{O}_3}e^{-\frac{1}{2m_Q^2}\mathcal{O}_2}e^{-\frac{1}{2m_Q}\mathcal{O}_1}\right)Q(x) \qquad (19)\\ h^{(-)}(x) &= \frac{1-\not{v}}{2}e^{-im_Q vx}\left(\cdots e^{-\frac{1}{2m_Q^3}\mathcal{O}_3}e^{-\frac{1}{2m_Q^2}\mathcal{O}_2}e^{-\frac{1}{2m_Q}\mathcal{O}_1}\right)Q(x) \end{aligned}$$

and determines the operators $\mathcal{O}_i$ in such a way that (to given order in $1/m_Q$ no coupling terms between $h^{(+)}$ and $h^{(-)}$ appear in the action (2). This may be done recursively and one obtains for the first few operators

$$\mathcal{O}_1 = i\not{D}_\perp \qquad (20)$$

$$\mathcal{O}_2 = \frac{1}{2}(i\not{D}_\perp\not{v}(ivD)+\not{v}(ivD)\not{D}\perp) \qquad (21)$$

$$\mathcal{O}_3 = \frac{1}{3}(i\not{D}\perp)^3+\frac{1}{4}(i\not{D}_\perp)(ivD)^2+\frac{1}{2}(ivD)(i\not{D}_\perp)(ivD)\frac{1}{4}(ivD)^2(i\not{D}_\perp)) \qquad (22)$$

Expanding (19) in a power series in $1/m_Q$ one finds for the fields

$$Q(x) = e^{-im_Q v\cdot x}\left[1+\frac{1}{2m_Q}(i\not{D}_\perp)+\frac{1}{4m_Q^2}\left(v\cdot D\not{D}_\perp-\frac{1}{2}\not{D}_\perp^2\right)+\cdots\right]h_v(x) \qquad (23)$$

where $h_v(x) = h^{(+)}(x)+h^{(-)}(x)$.

The diagonalized Lagrangian reads

$$\mathcal{L} = \bar{h}_v(iv\cdot D)h_v+\tilde{K}_1+\tilde{M}_1+\tilde{E}_1+\tilde{K}_2+\tilde{M}_1+\tilde{E}_1+\cdots \qquad (24)$$

where we have defines the abbreviations

$$\tilde{K}_1 = \bar{h}_v \frac{(iD)^2}{2m_Q} h_v, \quad \tilde{M}_1 = \frac{(-i)}{2m_Q} \bar{h}_v \sigma_{\mu\nu} (iD^\mu)(iD^\nu) h_v, \quad \tilde{E}_1 = \bar{h}_v \frac{(ivD)^2}{2m_Q} h_v \tag{25}$$

$$\tilde{K}_2 = \frac{1}{8m_Q^2} \bar{h}_v [(iD^\mu), [(-ivD), (iD_\mu)]] h_v \tag{26}$$

$$\tilde{M}_2 = \frac{(-i)}{8m_Q^2} \bar{h}_v \sigma_{\mu\nu} \{(iD^\mu), [(-ivD), (iD^\nu)]\} h_v, \quad \tilde{E}_2 = \bar{h}_v \frac{(ivD)^3}{2m_Q} h_v$$

The Lagrangian as well as the fields have a different expansion compared to the one given in (16) and 17; the differences starting at order $1/m_Q^2$. In particular, the Lagrangian (24) does not exhibit any explicit time derivative $iv\partial$; the term ivD appears only in commutators where it becomes an electric field or derivatives of it.

On the first sight it seems that one obtains indeed a different answer, depending on how the heavy mass limit is performed. However, as far as physical observables are concerned, all approaches yield the same result. Hence it is a matter of convenience which approach is preferable.

The way terms may be reshuffled in order to obtain the same result for physical observables comes from the fact, that the corrections to the static limit originate from two sources. The first type of $1/m_Q$ corrections are due to the expansion of the field Q, yielding a series of local operators, since the original quark current $\bar{q}\Gamma Q$ was also a local operator. The second type originates from the corrections to the Lagrangian which are treated as perturbations and hence appear as time-ordered products of the current with the corrections terms of the Lagrangian. In the approach where the small component field is integrated out the higher order term in the $1/m_Q$ expansion of the Lagrangian contain explicit factors of (ivD) corresponding in the rest frame to time derivatives.

Inserting such a time derivative into a time-ordered product one obtains a local term, since

$$\int d^4x \langle T \left[\bar{h}_v(x) \mathcal{O}(ivD) h_v(x) \bar{h}_v(0) \Gamma q(0) \right] \rangle \propto \langle \bar{h}_v(o) \mathcal{O} P_+ \Gamma q(0) \rangle \tag{27}$$

where $\mathcal{O}$ is some arbitrary combination of Dirac matrices and derivatives and P_+ is the projection on the upper component. The (nonlocal) contraction of $h_v(x)\bar{h}_v(0)$ is proportional to $1/(ivD)$ which gets cancelled by the corresponding factor in the numerator, resulting in a local contribution.

In this way certain terms of the Lagrangian may be rehuffled into the fields and vice versa. Formally this may be obtained by a field redefinition, since (ivD) is contained in the leading order term of the Lagrangian. In other words, all approaches yield at least on shell the same results and they are equivalent in the sense of field redefinitions.

2.1.2 Reparametrization Invariance and Residual Mass Term

Full QCD has Lorentz invariance as well as chiral symmetry, which ensures that no mass renormalization can occur in the massless limit. We shall discuss how much of these invariances survives once we go to the heavy mass limit and integrate out the small component fields.

Lorentz invariance as it is usually implemented in a quantum field theory is not present any more in the heavy mass limit, since the choice of the vector v corresponds to a choice of a specific coordinate frame. Only if we would transform the vector v "by hand" we would obtain again invariance. Another way of dealing with this problem is to "integrate in" more degrees of freedom by summing over all possible velocities as suggested in [20]

On the other hand, when we started from the full QCD Lagrangian, there was no dependence on the vector v and even when v appeared explicitly in the equations everything was still independent of v as long as all orders in $1/m_Q$ were taken into account. In other words, a dependence on v emerges at that point when we truncate the $1/m_Q$ expansion.

Hence we expect that the $1/m_Q$ expansion exhibits an invariance under infinitesimal changes of the vector v and that this so called reparametrization invariance [21] results in relations between coefficients of different orders in the $1/m_Q$ expansion.

Full QCD depends only on the momentum of the heavy quark P, and the splitting of this momentum in an on-shell part $m_Q v$ and a residual momentum k corresponding to the covariant derivative acting on the heavy static field h_v is arbitrary. Formally this means that the Lagrangian does not depend on the velocity v, if all orders of the $1/m_Q$ expansion are included.

In order to explore the consequences of reparametrization invariance it is convenient to use the representation (17) and (16), since we may obtain closed expressions for all orders in the $1/m_Q$ expansion. In fact performing an infinitesimal shift of the velocity, combined with the corresponding shift of the covariant derivative and the fields one explicitly checks that the Lagrangian is invariant under the transformation

$$\begin{aligned}
&v \to v + \delta v \qquad v \cdot \delta v = 0 \\
&h_v \to h_v + \frac{\delta\!\!\!/ v}{2}\left(1 + P_- \frac{1}{2m_Q + ivD} i D\!\!\!\!/\,\right) h_v \\
&iD \to iD - m_Q\, \delta v.
\end{aligned} \tag{28}$$

This invariance is the so-called reparametrization invariance, which has non-trivial consequences, since it relates terms of different orders of the $1/m_Q$ expansion.

One of the key points is that reparametrization invariance in fact survives renormalization, which means that the relations between coefficients of the Lagrangian as derived from (28) hold true beyond tree level. As we shall see when we discuss QCD radiative corrections it is reparametrization invariance which implies that the kinetic energy operator $\bar{h}_v (iD)^2 h_v$ is not renormalized to all orders and furthermore one may extract relations between the renormalization constants of the second order terms in the Lagrangian.

Another point related to the way the heavy mass limit is taken is the so called residual mass term [22]. The mass of a quark is not a physical quantity and one has in principle some freedom of how to define the heavy quark mass. In the derivation of the heavy mass limit from QCD we have chosen a phase in such a way that the mass term (with the mass as it appears as a parameter in the Lagrangian) is compensated. However, one could as well choose a different mass $m_Q' = m_Q + \delta m_Q$ and split the momentum of the heavy quark using m_Q'

$$p_Q = m_Q' v + k' = (m_Q - \delta m_Q) v + (k + \delta m_Q v) \tag{29}$$

The heavy quark fields obtained from the two different definitions m_Q and m_Q' differ by a phase factor, such that

$$h_v = e^{i\delta m_Q v x} h_v' \text{ and } iDh_v = e^{i\delta m_Q v x}(iD - \delta m_Q v) h_v' \tag{30}$$

The Lagrangian may be expressed in terms of the field h_v' and reads to leading order

$$\mathcal{L} = \bar{h}_v' v (iD - \delta m_Q v) h_v \tag{31}$$

yielding an equation of motion of the form

$$(iD - \delta m_Q v)h'_v = 0 \tag{32}$$

The quark mass is not a physical quantity, in contrast to the mass of a meson. In the heavy mass limit, the mass of a hadron is given by

$$M_H = m_Q + \bar{\Lambda} \tag{33}$$

and so a redefinition of the heavy quark mass by inserting a residual mass term amounts to a redefinition of $\bar{\Lambda}$. The appearance of a residual mass term is connected to the appearance of renormalon ambiguities in the definition of the heavy quark mass [23], and the question of which mass to chose is a matter of convention. As we shall see below, a particular choice (as e.g. the pole mass or a short distance definition of the mass such as the $\overline{\text{MS}}$-mass) does not change the predictions of the heavy mass expansion as far as physical quantities are concerned, since the nonperturbative matrix elements parameterizing the correction terms to order $1/m_Q$ and higher will depend on the definition of the mass, in such a way that the net result is independent of the definition chosen for the mass.

2.2 Heavy Quark Symmetry

2.2.1 Heavy Flavour and Spin Symmetry

The main benefit of the heavy quark limit is the fact, that additional symmetries beyond the ones present in full QCD arise. In general, the infrared or long distance behaviour is the same in the full and in the effective theory, and hence in its application to strong interactions the nonperturbative long distance contributions are the same in the full and in the effective theory. The benefit of heavy quark effective theory is that it explicitly exhibits the additional symmetries which hold in the limit of infinit heavy quark masses.

The first symmetry which arises is a heavy flavour symmetry. The interacton of the quarks with the gluons is determined by the color quantum numbers and the dependence on flavour enters in full QCD only throught the different quark masses. For the light quarks that fact that the light quark current masses are small compared to the QCD scale Λ_{QCD} yields the well known flavour symmetry for the light quarks; in the heavy mass limit a flavour symmetry arises in a similar manner: once the heavy quark is replaced by a static source of color moving with a definite velocity the flavour does not matter any more. In other words, for two heavy flavours b and c an $SU(2)$ symmetry emerges which relates b and c quarks moving with the same velocity.

For the case of two heavy flavours b and c one has to leading order the Lagrangian

$$\mathcal{L}_{heavy} = \bar{b}_v(v \cdot D)b_v + \bar{c}_v(v \cdot D)c_v, \tag{34}$$

where b_v (c_v) is the field operator h_v for the b (c) quark moving with velocity v and $D = \partial + igA$ is the QCD covariant derivative. This Lagrangian is obviously invariant under the $SU(2)_{HF}$ rotations

$$\begin{pmatrix} b_v \\ c_v \end{pmatrix} \to U_v \begin{pmatrix} b_v \\ c_v \end{pmatrix} \quad U_v \in SU(2)_{HF}. \tag{35}$$

We have put a subscript v for the transformation matrix U, since this symmetry only relates heavy quarks moving with the same velocity. To this end we shall be able to relate e.g. the matrix elements of mesonic transitions as

$$< D(v')|\bar{c}_{v'}\Gamma b_v|B(v) > \quad \text{and} \quad < B(v')|\bar{b}_{v'}\Gamma b_v|B(v) > \quad \text{and} \quad < D(v')|\bar{c}_{v'}\Gamma c_v|D(v) > \tag{36}$$

where Γ is some arbitrary Dirac matrix. We shall discuss below that such matrix elements will be described in terms of a single universal form factor. Note that this symmetry is not a symmetry of the hadron spectrum (we have scaled out the masses) rather it is a symmetry relating form factors.

The second symmetry emerging in the heavy mass limit is the so called spin symmetry. As is well known from electrodynamics, the spin coupling to the gauge field is proportional to the magnetic part of the gauge field and to the Bohr magneton of the corresponding particle. In the strong interaction case this coupling is proportional to the "strong Bohr magneton" $g/2m_Q$ which vanishes as the mass goes to infinity. In the leading order Lagrangian both spin degrees of freedom couple in the same way to gauge field; we may rewrite the leading-order Lagrangian as

$$\mathcal{L} = \bar{h}_v^{+s}(ivD)h_v^{+s} + \bar{h}_v^{-s}(ivD)h_v^{-s}, \tag{37}$$

where $h_v^{\pm s}$ are the projections of the heavy quark field on a definite spin direction s

$$h_v^{\pm s} = \frac{1}{2}(1 \pm \gamma_5 \not{s})h_v, \quad s \cdot v = 0. \tag{38}$$

This Lagrangian has a symmetry under the rotations of the heavy quark spin. In Hilbert space this symmetry is generated by operators $S_v(\epsilon)$ as

$$[h_v, S_v(\epsilon)] = i\not{\epsilon}\not{v}\gamma_5 h_v \tag{39}$$

where ϵ with $\epsilon^2 = -1$ is the rotation axis. Note that the Dirac matrix $\not{\epsilon}\not{v}\gamma_5$ is in fact the generator of a spin rotation around the axis ϵ in the space of Dirac spinors.

Now consider a heavy meson built of the heavy quark and a light antiquark. The mass M of the meson in the spin symmetry limit tends to the quark mass $M = m_Q(1 + \mathcal{O}(\frac{\Lambda_{QCD}}{m_Q}))$. Such a meson is moving with an "infinite" momentum $P^\mu = Mv^\mu$ and obviously the only heavy quark states present in the meson wave-function come from the v^μ sector of the theory. In this sector we have the symmetry generated by the operators $S(v,\epsilon)$ and all hadron states in the sector must occur in multiplets of $\mathrm{SU}(2)_{spin}$. On the other hand, the operators $S(v,\epsilon)$ do not commute with total angular momentum, which means that they in general connect states of different spins.

As an example, consider a heavy pseudoscalar meson in its rest frame. The configuration of the spins may be written as

$$|P(0)>=|\Uparrow\downarrow - \Downarrow\uparrow> \tag{40}$$

where the double lined arrow indicates the heavy quark spin, while the other arrow indicates the spin of the light degrees of freedom. A spin rotation of the heavy quark spin is archived by the matrix Σ_3 defined by

$$\Sigma_3|\Uparrow>=|\Uparrow> \quad \Sigma_3|\Downarrow>=-|\Downarrow> \tag{41}$$

Applying Σ_3 to the pseudoscalar meson state

$$\Sigma_3|P(0)>=\Sigma_3|\Uparrow\downarrow - \Downarrow\uparrow>=|\Uparrow\downarrow + \Downarrow\uparrow>=|V(0,\epsilon=e_z)> \tag{42}$$

one obtains a symmetric combination of the spins which corresponds to a longitudinally polarized vector meson (i.e. with the polarization vector pointing into the z direction) at rest. In a general frame one has the relation

$$S(v,\epsilon)|P(v)>=\eta|V(v,\epsilon)>, \tag{43}$$

System	Δ in GeV^2
(B^*, B)	0.53
(D^*, D)	0.54
(K^*, K)	0.55
(ρ, π)	0.57

Table 1: Value of the splitting Δ or the different systems

where η is a phase factor which may be chosen to be $\eta = 1$.

Thus the spin symmetry rotates a pseudoscalar meson into the corresponding vector meson. Consequently spin symmetry predicts equal masses for the 0^- states and the corresponding 1^- state. In the D system $m_D \approx 1870$ MeV and $m_D^* \approx 2010$ MeV, while in the B system $m_B \approx 5271$ MeV and $m_B^* \approx 5330$ MeV. The coupling of the spin to the gluons is of the order $1/m$; consequently one would expect the relative splittings in the spin symmetry doublets to be of the order of Λ_{QCD}/m. Using $\Lambda_{QCD} = 200$ MeV we have for the above example $\Lambda_{QCD}/m_c \approx 0.1$ and $\Lambda_{QCD}/m_b \approx 0.04$ respectively. Comparing this to the measured splitting between the D and the D^* or the B and the B^* one gets some confidence that for the b and the c quark the heavy quark symmetries may work reasonably well.

Furthermore, since the spin symmetry violating terms are the ones proportional to the "strong Bohr magneton" $g/(2m_Q)$ one would expect that the splitting between the spin symmetry doublets scale as $1/m_Q$. In fact we may consider the quantity

$$\Delta = M^2(1^-) - M^2(0^-) = (M(1^-) + M(0^-))(M(1^-) - M(0^-)) \approx 2m_Q(M(1^-) - M(0^-)) \quad (44)$$

which we should expect to be a constant in the heavy quark systems. As can be seen in table 1 Δ indeed turns out to be constant in the B and D meson systems, but to some surprise one obtains also the same constant looking into light quark systems. The interpretation of this is not clear; while in the heavy quark systems one is led to say that one understands the constant Δ one could as well interpret this in a negative way and say that the $1/m_Q$ dependence of splitting between the 1^- and the 0^- mesons is not an indication for the fact that the heavy mass limit is useful.

2.2.2 Mesons

The two additional symmetries have interesting consequences for matrix elements. We shall study the case of heavy meson transitions first. Starting from full QCD we may consider the bottom current $\bar{b}\gamma_\mu b$, which is conserved in full QCD. Hence the matrix element of this current is given in terms of a single normalized form factor

$$< B^-(p')|\bar{b}\gamma_\mu b|B^-(p) >= (p'_\mu + p_\mu)F(q^2), \quad \text{with } F(q^2 = 0) = 1 \quad (45)$$

where $q = p' - p$. We may now switch to the heavy mass limit and obtain the relation

$$< B^-(v_1)|\bar{b}_{v_1}\gamma^\mu b_{v_2}|B^-(v_2) >= m_B(v_1^\mu + v_2^\mu)\xi(v_1 \cdot v_2) \quad \text{with } \xi(v_1 \cdot v_2 = 1) = 1 \quad (46)$$

In this limit we may employ the heavy flavour symmetry for heavy quark moving with velocity v_1 to rotate the B^- meson into a D^0 and obtain

$$< D^0(v_1)|\bar{c}_{v_1}\gamma^\mu b_{v_2}|B^-(v_2) >= \sqrt{m_B m_D}\xi(v_1 \cdot v_2)(v_1^\mu + v_2^\mu) \quad (47)$$

Due to the symemtry the same form factor appears which is called the Isgur Wise function, and which is normalized due to heavy quark symmetry for equal velocity $\xi(v_1 \cdot v_2 = 1) = 1$.

Similarly we may employ heavy quark spin symmetry for the heavy quark moving with velocity v_1 to rotate the pseudoscalar meson into a vector meson. In this way we may relate the above matrix element of a vector current to a matrix element of left handed current between a heavy pseudoscalar and a heavy vector meson

$$\begin{aligned}
&< D^{*0}(v_1, \epsilon_1)|\bar{c}_{v_1}\gamma^\mu(1-\gamma_5)b_{v_2}|B^-(v_2)> \\
&= - < D^0(v_1)|\bar{c}_{v_1}\gamma_5 \not{v}_1 \not{\epsilon}_1^* \gamma^\mu(1-\gamma_5)b_{v_2}|B^-(v_2)> \\
&= \sqrt{m_B m_D}\xi(v_1 \cdot v_2)(\epsilon_1^{*\mu}(1 + v_1 \cdot v_2) - v_1^\mu v_2 \cdot \epsilon_1^* - i\varepsilon^\mu{}_{abc} v_1^a \epsilon_1^{*b} v_2^c)
\end{aligned} \tag{48}$$

where we have used eqn.(43) and eq.(39) and the gamma matrix relations

$$\begin{aligned}
\gamma_5 \not{v} \not{\epsilon}^* \gamma^\mu(1-\gamma_5) &= (v^\mu \epsilon^{*\nu} - v^\nu \epsilon^{*\mu} - i\varepsilon^{\mu\nu ab} v_a \epsilon_b^*)\gamma_\nu(1-\gamma_5) \\
\gamma^\mu(1-\gamma_5)\gamma_5 \not{v} \not{\epsilon} &= (v^\mu \epsilon^\nu - v^\nu \epsilon^\mu + i\varepsilon^{\mu\nu ab} v_a \epsilon_b)\gamma_\nu(1-\gamma_5)
\end{aligned} \tag{49}$$

These arguments may be further generalized by using the group theory of spin symmetry. In order to implement both spin symmetry and Lorentz covariance it is convenient to represent the mesons by appropriate matrices carrying a heavy quark spinor index A and a light quark index α. In fact, the matrix

$$H(v) = H_{A\alpha}(v) = \frac{1}{2}\sqrt{m_H}\gamma_5(\not{v} - 1) \tag{50}$$

has the correct Lorentz transformation properties for a 0^- particle composed of two spinors: For a connected Lorentz transformation Λ we have

$$H(v) \to S(\Lambda)H(v)S^{-1}(\Lambda) = H(\Lambda v) \tag{51}$$

while a parity transformation P gives

$$H(v) \to PH(v)P^{-1} = -H(\hat{v}) \tag{52}$$

with $\hat{v} = (v_0, -\vec{v})$. Furthermore, the heavy quark is on shell in the limit $m_H \to \infty$ thus we must require

$$(\not{v} - 1)H(v) = 0 \tag{53}$$

Finally, spin rotations of the heavy quark spin rotate the matrix $H(v)$ into the corresponding matrix for the vector meson

$$\Sigma(v, \epsilon)H(v) = \gamma_5 \not{v} \not{\epsilon} H(v) = \frac{1}{2}\sqrt{m_H}\not{\epsilon}(\not{v} - 1) = H^*(v, \epsilon) \tag{54}$$

where the matrix Σ has been defined in (39). It is easy to check that $H^*(v, \epsilon)$ has the correct Lorentz transformation properties for the 1^- state and fulfills the Dirac equation for the heavy quark.

Thus these two matrices have the correct properties under the symmetries involved and may be used to formulate the analogue of the Wigner Eckhard theorem for spin symmetry. If $\mathcal{H}(v)$ denotes either $H(v)$ or $H^*(v, \epsilon)$ and if $|\mathcal{H}(v)>$ denotes the corresponding state , we have

$$< \mathcal{H}(v')|\bar{h}_{v'}\Gamma h_v|\mathcal{H}(v)> = \xi(v \cdot v')\mathrm{Tr}\ \left\{\overline{\mathcal{H}}(v')\Gamma\mathcal{H}(v)\right\} \tag{55}$$

where Γ is some arbitrary combination of Dirac matrices. Eq.(55) is the main result of heavy quark symmetry in the mesonic sector, since it relates *every* matrix element of bilinear heavy to heavy currents between two heavy mesons to a single form factor, the Isgur Wise function $\xi(v \cdot v')$. Furthermore, since the current

$$j_\mu = \bar{h}_v \gamma_\mu h_v \tag{56}$$

generates the heavy flavor symmetry, we have a normalization statement for the Isgur Wise function

$$\xi(v \cdot v' = 1) = 1 \tag{57}$$

Note, finally, that the Isgur Wise function in a group theoretical language is just the reduced matrix element which is universal for the whole spin flavor symmetry multiplet. Furthermore, the trace in (55) is the Clebsch Gordan coefficient which is entirely determined by the current operator and the states of the multiplet.

One may formulate the Wigner Eckart Theorem also for the spin symmetry doublets for excited heavy mesons; the corresponding relations may be found in [24].

2.2.3 Baryons

In a similar way one may derive the consequences of spin symmetry for heavy baryons [25, 26]. The suggested naming scheme for these states in

$$\begin{aligned}
\Lambda_h &= [(qq')_0 h]_{1/2} \\
\Xi'_h &= [(qs)_0 h]_{1/2} \\
\Sigma_h &= [(qq')_1 h]_{1/2} \\
\Xi_h &= [(qs)_1 h]_{1/2} \\
\Omega_h &= [(ss)_1 h]_{1/2} \\
\Sigma^*_h &= [(qq')_1 h]_{3/2} \\
\Xi^*_h &= [(qs)_1 h]_{3/2} \\
\Omega^*_h &= [(ss)_1 h]_{3/2}.
\end{aligned}$$

Here, q, q' refer to u and d quarks, $q \neq q'$ for the Λ_h, but q may be the same as q' for the Σ_h and Σ^*_h. The first subscript (0, 1) is the total spin of the light degrees of freedom, while the second subscript (1/2, 3/2) is the total spin of the baryon. For later convenience, ξ_h may refer to any of the two baryons Λ_h or Ξ'_h. Similarly, we denote any of the three baryons Σ_h, Ξ_h and Ω_h as ω_h, and Σ^*_h, Ξ^*_h and Ω^*_h as ω^*_h.

Let us now exhibit the group theoretical framework of the spin symmetry ($SU(2)_{spin}$) of heavy baryons which arises in the heavy quark limit and identify the representations that correspond to the light baryons. The simplest baryons are the ξ_h's. The nontrivial Lorentz structure of ξ_h is due solely to the two polarization states of the heavy quark. This means that the $SU(2)_{spin}$ rotations constitute a subset of the Lorentz transformations, i.e., that $SU(2)_{spin}$ connects polarization states of the same particle. It follows that the representation which contains ξ_h can be described by an ordinary Dirac spinor $u_{\xi_h}(v)$. The Lorentz transformations are

$$u_{\xi_h}(v) \to D(\Lambda) u_{\xi_h}(\Lambda v), \tag{58}$$

where Λ denotes the fundamental representation matrix of the given Lorentz transformation and $D(\Lambda)$ is its spinorial representation. Note that the spinor indices have been suppressed.

The spin transformations are

$$u_{\xi_h}(v) \to -\gamma_5 \not{v} \not{\epsilon} u_{\xi_h}(v), \tag{59}$$

which transform one polarization state of ξ_h into another.

On the other hand, consider the baryon states in which the light degrees of freedom are in a spin one state. These are the ω_h and ω_h^* states which states have nontrivial Lorentz structure coming from both the light system polarizations and the heavy quark polarizations. In this representation there are spin 1/2 states (ω_h), and spin 3/2 states (ω_h^*) [1]. To represent these states we introduce a pseudovector-spinor object R^μ, where $v_\mu R^\mu = 0$ and the spinor indices have been suppressed. Note that in general $\gamma_\mu R^\mu \neq 0$ because R^μ describes a spin 1/2 particle together with a spin 3/2 particle. In other words, R^μ contains a Rarita-Schwinger field as well as a Dirac field. Under the Lorentz transformation

$$R^\mu(v) \to \Lambda^\mu{}_\nu D(\Lambda) R^\nu(\Lambda v), \tag{60}$$

and under the spin transformation

$$R^\mu(v) \to -\gamma_5 \not{v} \not{\epsilon} R^\mu(v). \tag{61}$$

The parity transformation of R^μ is

$$R^\mu(v^\alpha) \to -\gamma^0 R_\mu(v_\alpha). \tag{62}$$

Before we continue it is important to identify the parts of R that correspond to spin 1/2 and those that correspond to spin 3/2 states. Spin 3/2 states can be easily projected by contraction with γ_μ

$$\gamma_\mu R^\mu_{\omega_h^*} = 0, \tag{63}$$

The rest of the independent components of R correspond to ω_h:

$$R^\mu_{\omega_h} = \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma_5 u_{\omega_h}, \tag{64}$$

where u_{ω_h} is the Dirac spinor of the ω_h state.

The power of the spin symmetry formalism lies in the observation that a generic operator $\bar{h}_{v_1}\Gamma h_{v_2}$ transforms as a spinor under $SU(2)_{v_1}$ (and of course independently as a spinor under $SU(2)_{v_2}$). Suppose we look at the matrix element of $\bar{h}_{v_1}\Gamma h_{v_2}$ sandwiched between heavy baryon states. One of the baryon states must correspond to the v_1 velocity sector (and the other one to the v_2 sector). The irreducible representation of which describes the heavy baryon state of velocity v_1 decomposes into a direct sum of the spinors of $SU(2)_{v_1}$. In such a representation the different $SU(2)_{v_1}$ spinors correspond to all the independent polarization states of the light quark system (and for this reason the indices which label the spinors are termed light quark indices). The Wigner-Eckart theorem tells us that for each independent $SU(2)_{v_1}$ spinor in the given representation there is only one $SU(2)_{v_1}$ invariant amplitude that describes the matrix element of $\bar{h}_{v_1}\Gamma h_{v_2}$ [2].

[1] There is a slight ambiguity as to which representation describes the ω_h and ω_h^* states. This is because under the Lorentz group the light quarks' indices can transform either as a pseudovector or as an antisymmetric tensor. The second possibility will not be considered here, details on it may be found in [25, 26, 24].

[2] Likewise, for each spinor in the representation that describes the hadron moving with velocity v_2 there is only one $SU(2)_{v_2}$ invariant amplitude.

On the other hand, Lorentz transformations mix the light quark indices, which means that the set of all the $SU(2)_{spin}$ invariant amplitudes transforms as some tensor representation (not necessarily irreducible) of the Lorentz group. Such a Lorentz tensor can only be built from the velocity vectors v_1^μ, v_2^μ and invariant tensors $g^{\mu\nu}$, $\epsilon^{\mu\nu\alpha\beta}$. In other words, the requirement that the hadron matrix element transforms properly under the Lorentz group means that the light quark indices must be contracted with a Lorentz tensor built out of v_1^μ, v_2^μ, $g^{\mu\nu}$ and $\epsilon^{\mu\nu\alpha\beta}$. The number of irreducible components in such a tensor corresponds to the number of independent form factors describing the heavy hadron matrix element of $\bar{h}_{v_1}\Gamma h_{v_2}$.

As we have discussed above, ξ_h baryons are described by the simplest possible representation, a single spinor. This means that the matrix elements which describe processes involving ξ_h particles will be most constrained by the requirement of the spin symmetry. For example let us look at the matrix element

$$< \xi_h(v)|\bar{h}\Gamma h'|\xi_{h'}(v') >= A(v\cdot v')\bar{u}_{\xi_h}(v)\Gamma u_{\xi_{h'}}(v'), \tag{65}$$

where we have allowed for the possibility of two heavy quark species h and h'. Heavy quark symmetry tells us that there is only one independent amplitude $A(v\cdot v')$. Moreover, if we put $\Gamma = \gamma_\mu$ then the matrix element (65) corresponds to the matrix element of the generator of flavor symmetry and in the forward limit it is uniquely specified

$$A(1) = \sqrt{m_{\xi_h}m_{\xi_{h'}}}\cdot 1, \tag{66}$$

where the factor $\sqrt{m_{\xi_h}m_{\xi_{h'}}}$ means that the hadron states in (65) are normalized relativistically. The case of ξ_h is special because it is the only hadron for which spin symmetry connects different polarization states of the same particle. This means that the number of independent Lorentz amplitudes will be reduced not only for *heavy* $\to$ *heavy* transitions. As we show later in this section, in the case of the semileptonic decays $\xi_h \to$ *light baryon* the *a priori* six independent amplitudes describing such a transition get reduced to two.

Now let us discuss a matrix element related to a $\xi_{h'} \to \omega_h(\omega_h^*)$ transition. Again, the constraint of heavy quark symmetry tells us that only one amplitude consistent with Lorentz invariance *and* spin symmetry can exist:

$$< \omega_h^{(*)}(v)|\bar{h}_v\Gamma h_{v'}|\xi_h >= G(v\cdot v')\,\bar{R}^\mu_{\omega_h^{(*)}}(v)\Gamma u_{\xi_{h'}}(v')\,v'_\mu, \tag{67}$$

In fact equation (67) is consistent only with the subset of the proper Lorentz transformations. If we use the parity properties of R we see that parity forces $G(v\cdot v') = 0$. In ref. [27] spin symmetry arguments in combination with parity and angular momentum conservation explain that (67) in fact had to be equal to zero.

Now consider the matrix element between $\omega_h^{(*)}$ and $\omega_{h'}^{(*)}$

$$\begin{aligned} &< \omega_h^{(*)}(v)|\bar{h}_v\Gamma h_{v'}|\omega_h^{(*)} > \\ &= \bar{R}^\mu_{\omega_h^{(*)}}(v)\Gamma R^\nu_{\omega_h^{(*)}}(v')\,\{B(v\cdot v')g_{\mu\nu} + C(v\cdot v')v'_\mu v_\nu\}. \end{aligned} \tag{68}$$

Heavy quark symmetry allows two independent form factors (without parity there could be a third one).

Finally we derive also a relation for heavy to light transitions [26]. Note that, in general, spin symmetry multiplets contain particles which decay electromagnetically or strongly. This means that even if spin symmetry relates some weak heavy to light amplitudes for the particles in a given representation, many of these relations have little chance of being tested experimentally. One important exception corresponds to the ξ_h particles. Here spin symmetry relates different polarizations of the same particle and imposes interesting constraints. Consider for example the matrix element of an operator $\bar{\ell}\Gamma h_v$ between a heavy ξ_h and a light spin-1/2 baryon B_ℓ. It is described by only two form factors,

$$< B_\ell(p)|\bar{\ell}\Gamma h_v|\xi_h(v) >= \bar{u}_\ell(p)\{F_1(v\cdot p) + \not{p}F_2(v\cdot p)\}\Gamma u_{\xi_h}(v). \tag{69}$$

Thus in this particular case spin symmetry greatly reduces the number of independent Lorentz-invariant amplitudes which may describe the heavy to light transitions.

2.3 Corrections to the Symmetry Limit

Heavy quark symmetries hold in the limit in which the heavy quark mass is considered to be infinitely heavy. However, such a limit is in general only useful if one has some way to at least estimate the corrections to the limiting behaviour.

In the case at hand, there are two sources of corrections, namely QCD radiative corrections, characterized by the small parameter $\alpha_s(m_Q)$ and power corrections governed by the small parameter Λ_{QCD}/m_Q. We shall start our discussion with the perturbatively calculable QCD radiative corrections.

2.3.1 QCD Radiative Corrections

The effective theory approach allows us to separate long and short distance contributions, and the scale where this separation takes place is taken to be the renormalization scale μ. Since we are dealing with QCD, the coupling constant becomes smaller as the scale becomes larger, and thus we may calculate the short distance contributions perturbatively, as long as $\mu \gg \Lambda_{QCD}$.

The usual procedure is to use renormalization group methods to improve perturbation theory. The coupling constant depends on the scale logarithmically, and hence $\alpha_s(m_Q)$ contains nonanalytic dependencies on $1/m_Q$. Renormalization group improvement means that one may resum these logarithms using as an input the perturbatively calculated renormalization group functions.

In order to exhibit this formalism we shall consider a simple example, which we will study in some detail, namely a heavy-to-light current matrix element of the form

$$\langle A|J|M(v)\rangle = \langle A|\bar{q}\Gamma Q|M(v)\rangle = Z\left(\frac{m_Q}{\mu}\right)\langle A|\bar{q}\Gamma h_v|H(v)\rangle|_\mu + \mathcal{O}(1/m_Q) \tag{70}$$

where the left hand side is the matrix element in full QCD (depending on the full heavy quark field Q and on the full pseudoscalar meson state $|M(v)\rangle$) while the right hand side is the static limit containing the static heavy quark field and the state in the static limit $|H(v)\rangle$.

The renormalization constant Z contains the short distance contribution of the matrix element and may be calculated perturbatively. It is a dimensionless quantity and hence it depends (for constant coupling α_s) on the ratio m_Q/μ. Its perturbative expansion takes the form

$$Z\left(\frac{m_Q}{\mu}\right) = a_{00} \tag{71}$$

$$
\begin{aligned}
&+ a_{11}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{10}\alpha_s \\
&+ a_{22}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{21}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{20}\alpha_s^2 \\
&+ a_{33}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^3 + a_{32}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{31}\alpha_s^2\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) \\
&\qquad + a_{30}\alpha_s^3 + \cdots
\end{aligned}
$$

where $\alpha_s = g^2/(4\pi)$ and the coefficients a_{ij} have to be calculated using the Feynman rules of HQET.

Before discussing these Feynman rules and a sample calculation, we shall continue the introduction to the renormalization group methods.

The factorization theorem of the long and short distance contributions implies that the ultraviolet divergencies in the effective theory have to match the logarithmic mass dependences of full QCD. The factorization scale μ is an arbitrary parameter, and the physical quantity $\langle A|\bar{q}\Gamma Q|M(v)\rangle$ does not depend on this parameter. However, calculating the matrix element of this operator in the effective theory and studying its ultraviolet behavior allows us to access the mass dependence of the matrix element $\langle A|\bar{q}\Gamma Q|M(v)\rangle$.

The ultraviolet behavior of the effective theory is investigated by the renormalization group equation (RGE), which is obtained in the usual way from differentiating (70) with respect to the factorization scale μ. The left hand side is independent of the arbitrary renormalization point μ, and one may obtain an equation for the renormalization point dependence of Z

$$
\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_J(g)\right) Z\left(\frac{m_Q}{\mu}, g\right) = 0. \tag{72}
$$

where $\gamma_J(g)$ is the anomalous dimension of the current J which is related to the ultraviolet behavior of the matrix elements of J. The function $\beta(g)$ defines the running of the coupling constant

$$
\frac{d}{d\ln\mu} g(\mu) = \beta(\mu). \tag{73}
$$

Both functions $\gamma_J(g)$ and $\beta(g)$ are calculable in perturbation theory using a loopwise expansion, where the first term of the β function of QCD is well known

$$
\beta(g) = -\frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}n_f\right) g^3 + \cdots, \tag{74}
$$

where n_f is the number of flavors with a mass less than m_Q.

With this input the renormalization group equation may be solved to yield

$$
Z\left(\frac{m_Q}{\mu}\right) = a_{00}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{48\pi^2}{33-2n_f}\gamma_1} \tag{75}
$$

where γ_1 is the first coefficient in the perturbative expansion of the anomalous dimension $\gamma_J = \gamma_1 g^2 + \cdots$ and $\alpha_s(\mu)$ is the one loop expression for the running coupling constant of QCD

$$
\alpha_s(\mu) = \frac{12\pi}{(33-2n_f)\ln(\mu^2/\Lambda_{QCD}^2)} \tag{76}
$$

which is obtained from solving (73) using (74).

This expression corresponds to a summation of the leading logarithms $(\alpha_s \ln m_Q)^n$ which is achieved by a one-loop calculation of the renormalization group functions β and γ_J; in other words, in this way a resummation of the first column of the expansion (71) is achieved.

In a similar way one may also resum the second column of (71), if the renormalization group functions β and γ are calculated to two loops and the non-logarithmic terms of the one loop expression are included.

Eq.(72) describes the renormalization group scaling in the effective theory. It allows to shift logarithms of the large mass scale from the matrix element of J into the coefficient Z: If the matrix element is renormalized at the large scale m_Q the logarithms of the type $\ln m_Q$ will apear in the matrix element of J while the coefficient Z at this scale will simply be

$$Z(1) = a_{00} + a_{10}\alpha_s(m_Q) + a_{20}\alpha_s^2(m_Q) + a_{30}\alpha_s^3(m_Q) + \cdots \tag{77}$$

The renormalization group equation (72) allows to lower the renormalization point from m_Q to μ; the matrix element renormalized at μ will not contain any logarithms of m_Q any more, they will appear in the coefficient Z in the way shown in (71).

In all cases relevant in the present context the matrix elements will be matrix elements involving hadronic states, which are in most cases impossible to calculate from first principles. However, eq.(72) allows to extract the short distance piece, i.e. the logarithms of the large mass m_Q and to separate it into the Wilson coefficients.

Finally, the case we have considered as an example is indeed very simple; in general all operators of a given dimension may mix under renormalization, i.e. instead of a simple anomalous dimension a matrix of anomalous dimensions may occur and the renormalization group equation 72 becomes a system of differential equations.

In order to perform a calculation in HQET one has to derive the Feynman rules of the effective theory. The dimension four piece of the Lagrangian is unique and one may use the leading piece of (17) or (24) to define a renormalizable field theory, and one may read off the Feynman rules for this theory in the usual way. For a static heavy quark moving with velocity v one obtains

	full QCD		HQET	
Propagator of the heavy quark	$\dfrac{i}{\not p - m_Q + i\epsilon}$	$\longrightarrow$	$\dfrac{i}{vk + i\epsilon}$,	$p = mv + k$
Heavy quark gluon vertex	$ig\gamma_\mu T^a$	$\longrightarrow$	$igv_\mu T^a$	

while all other rules remain unchanged.

After having obtained the Feynman rules, one may proceed to calculate the anomalous dimensions by extracting the divergent pieces from loop diagrams. We shall stick to our example of the heavy to light current and give an outline how such a calculation is performed.

We first consider the self energy of a heavy quark which we shall need to obtain the wave function renormalization of the heavy quark. The loop diagram, which is depicted in fig.1, yields a divergent result which we shall regularize in dimensional regularization. As usual these divergencies are compensated by counterterms in the Lagrangian and in the case of the self energy this amounts to add acounterterm of the same form as the leading order Lagrangian. Once the divergent part of the self energy diagram is compensated one has to define the finite

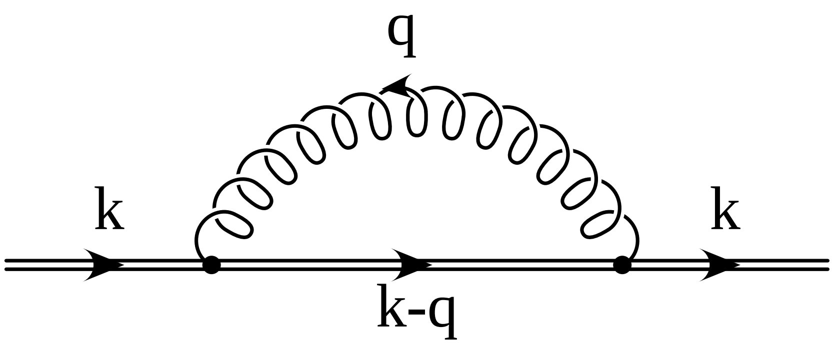

Figure 1: The self energy of a heavy quark. The double line represents the propagator of the static quark with velocity v and residual momentum k or $k-q$ respectively.

part of the diagram, in other words, one may still perform finite renormalizations corresponding to a change of the renormalization point μ. Hence a counter term is added as

$$\mathcal{L} = \bar{h}_v^{bare}(ivD)h_v^{bare} = \bar{h}_v(ivD)h_v - (1-Z_h)\bar{h}_v(ivD)h_v = Z_h\bar{h}_v(ivD)h_v \tag{78}$$

which corresponds to a replacement of the bare field by the renormalized one

$$h_v^{bare} = \sqrt{Z_h(\mu)}h_v|_\mu \tag{79}$$

where we have explicitly displayed the μ dependence and $h_v|_\mu$ is the heavy quark field renormalized at the point μ.

In dimensional regularization, the dependence on the renormalization scale μ enters through the fact that in this regularization scheme one has to introduce some scale parameter which keeps the coupling dimensionless; this scale parameter is to be identified with the renormalization scale μ.

The one-loop integral one has to perform in order to calculate Z_h is

$$\begin{aligned} i\Sigma_H(k) &= (-ig)^2(\mu)^\epsilon \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} v_\lambda T^a \frac{i}{v(k+q)} v^\lambda \frac{-i}{q^2} T^a \\ &= -\frac{4}{3}g^2(\mu)^\epsilon \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \frac{1}{q^2\, v(k+q)} \end{aligned} \tag{80}$$

Since we only want to extract the wave function renormalization we only need to evaluate the divergent part of the integral, i.e. the contribution proportional to the pole in ϵ. One obtains

$$\Sigma_H(k) = \frac{16}{3}(vk)\frac{g^2}{16\pi^2}\left(\frac{1}{\epsilon} + \ln(\mu)\right) + \mathcal{O}(1) = (vk)(Z_h - 1) \tag{81}$$

where we have neglected finite terms as $\epsilon \to 0$ and $\mu \to 0$. Obviously the divergence may be compensated by defining a the wave function renormalization in such a way that the pole in ϵ is removed. However, one may still add a finite renormalization on top of the infinite one corresponding to a choice of the renormalization point μ. One could choose the scale m_Q as a reference point and thus the logarithm appearing here corresponds to the logarithms in (71) and the renormalization constant Z_h becomes

$$\sqrt{Z_h} = 1 - \frac{8}{3}\frac{g^2}{16\pi^2}\left(\frac{1}{\epsilon} + \ln(\mu)\right) \tag{82}$$

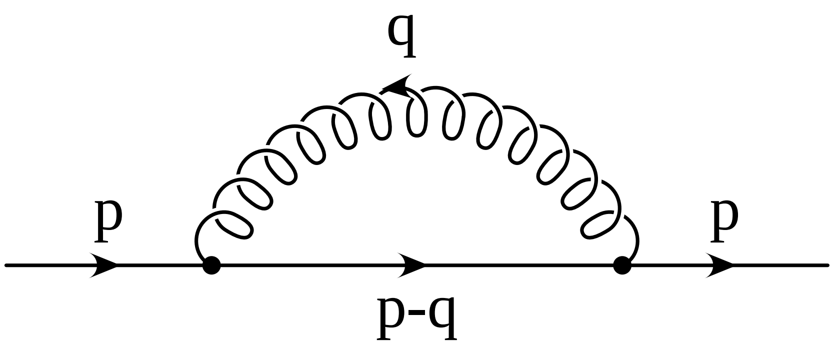

Figure 2: The self energy of a light quark.

Similarly one may as well calculate the self energy of a quark in full QCD as depicted in fig.2. The wave function renormalization of a dynamic quark comes out to be

$$\sqrt{Z_q} = 1 - \frac{4}{3}\frac{g^2}{16\pi^2}\left(\frac{1}{\epsilon} + \ln(\mu)\right) \tag{83}$$

As a side remark we note that the result for the wave function renormalization of the heavy quark may be related to finite terms contained in the renormalized self energy of a dynamic quark. Keeping the mass of the quark one finds in the finite renormalized self energy close to mass shell terms of the form

$$\Sigma^{ren}_{full}(p) = -\frac{16}{3}\frac{g^2}{16\pi^2}(\not{p} - m_Q)\frac{p^2 - m_Q^2}{m_Q^2}\ln\left(\frac{m_Q^2}{m_Q^2 - p^2}\right) + \cdots \tag{84}$$

where the dots denote nonlogarithmic terms. Splitting the momentum of the heavy quark as $p = m_Q v + k$ and projecting on the large components one finds

$$\Sigma^{ren}_{full}(p) = -\frac{16}{3}\frac{g^2}{16\pi^2}P_+(vk)\ln\left(\frac{m_Q^2}{vk}\right) + \cdots \tag{85}$$

which contains a term which diverges logarithmically as $m_Q \to \infty$. The coefficient of this logarithmic divergence is equal to the divergent contribution as calculated in the effective theory, showing that the ultraviolett behaviour of the effective theory is indeed related to the mass dependences in the full theory.

Our goal is to perform a calculation of the one-loop anomalous dimension of heavy quark currents [28]. It is well known that in full QCD the vector and the axial vector currents have vanishing anomalous dimension, since both are conserved in the massless limit. On the other hand, the rsult for these currents will exhibit logarithmic dependences on the quark masses, which become logarithmic divergencies in the heavy mass limit. Thus we expect nontrivial anomalous dimension once one of the heavy quarks becomes static. In other words, we shall now evaluate the anomalous dimension of the heavy-light current

$$J = \bar{q}\Gamma h_v \tag{86}$$

The one loop diagram that needs to be evaluated is depicted in fig.3, where the circle-cross vertex represents the Dirac matrix Γ. The result is again divergent and this divergence needs

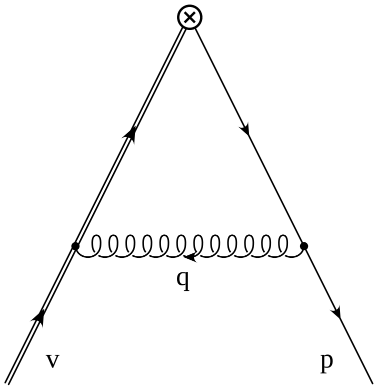

Figure 3: The one loop correction to the heavy to light current operators. The double line represents a static heavy quark moving with velocity v.

to be compensated by an appropriate counterterm. This counterterm takes the form

$$\text{counterterm for } J = \left[\frac{\sqrt{Z_h Z_q}}{Z_J} - 1\right] J = [(\sqrt{Z_h} - 1) + (\sqrt{Z_q} - 1) - (Z_J - 1)]J + \mathcal{O}(g^4) \quad (87)$$

where the appropriate renormalization factors for the external lines have been taken into account.

Inorder to evaluate the divergent part one may neglect all external momenta and the necessary integral I becomes

$$\begin{aligned} I &= (-ig)^2(\mu)^\epsilon \int \frac{d^{4-\epsilon}q}{(2\pi)^{4-\epsilon}} \gamma_\lambda T^a \frac{i\not{q}}{q^2} \Gamma \frac{i}{vq} v^\lambda T^a \frac{-i}{q^2} \\ &= -\frac{8}{3}\frac{g^2}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\mu\right) \end{aligned} \quad (88)$$

where we have again dropped all finite terms as $\epsilon \to 0$ and $\mu \to 0$. From this result we obtain for the renormalization constant Z_J

$$Z_J = 1 + \frac{12}{3}\frac{g^2}{16\pi^2}\left(\frac{1}{\epsilon} + \ln\mu\right) \quad (89)$$

From the renormalization constant Z_J we evaluate the anomalous dimension as

$$\gamma_J = \mu\frac{\partial}{\partial\mu}\ln Z_J = \frac{12}{3}\frac{g^2}{16\pi^2} + \mathcal{O}(g^4) \quad (90)$$

which yields for the renormalization group improved result for Z_J

$$Z_J = \left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{6}{33-2n_f}} \quad (91)$$

In a similar fashion one may evaluate the counter term for the heavy-heavy current

$$\mathcal{J}(v, v') = \bar{h}_{v'}\Gamma h_v \quad (92)$$

and derive from this the anomalous dimension. Here a specific feature of HQET enters the game, namely the anomlous dimension depends on the velocities of the heavy quarks. At the

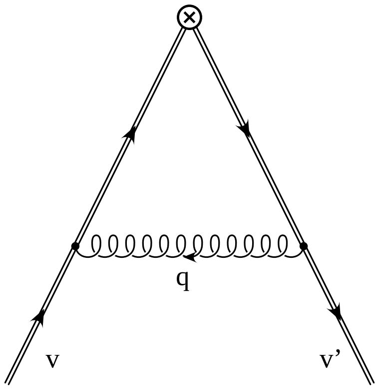

Figure 4: The one loop correction to the heavy to heavy current operators. The double lines represent static heavy quarks moving with velocity v and v' respectively.

first sight this seems to be strange, since in general the anomalous dimensions do not depend on the kinematic variables of the matrix element. However, in HQET the velocity of the heavy quark is not a kinematic quantity rather it is an external parameter, while the real kinematic quantities are the residual momenta of which the anomalous dimension is independent.

For the one loop correction to the heavy-heavy current one has to evaluate the diagram shown in fig.4. The counter term reads

$$\text{counterterm for } \mathcal{J} = \left[\frac{Z_h}{Z_J} - 1\right] J = [(Z_h - 1) - (Z_{\mathcal{J}} - 1)]\mathcal{J} + \mathcal{O}(g^4) \tag{93}$$

and the renormalization constant is found to be

$$Z_{\mathcal{J}} = 1 - \frac{16}{3}\frac{g^2}{16\pi^2}[(vv')r(vv') - 1]\left(\frac{1}{\epsilon} + \ln\mu\right) \tag{94}$$

where we have defined a function

$$r(x) = \frac{1}{\sqrt{x^2 - 1}}\ln[\sqrt{x^2 - 1} + x] \tag{95}$$

From the renormalization constant we derive the anomlous dimension to be

$$\gamma_{\mathcal{J}} = -\frac{16}{3}\frac{g^2}{16\pi^2}[(vv')r(vv') - 1] + \mathcal{O}(g^4) \tag{96}$$

which vanishes as $vv' \to 1$. This is in fact a result which holds to all orders in perturbation theory since the heavy-heavy current for equal velocities is the generator of heavy flavour symmetry and hence a conserved current, which has to have vanishing anomalous dimension in all order of perturbation theory.

Inserting this result into the formula for the resummation of the leading logarithms one has

$$Z_{\mathcal{J}} = \left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{8}{33 - 2n_f}[vv'r(vv') - 1]} \tag{97}$$

As an application we consider the left handed current

$$L_\nu = \bar{c}\gamma_\nu(1 - \gamma_5)b \tag{98}$$

and start at some large scale as e.g. the scale of the weak bosons M_W. Since in full QCD this current does not have an anomalous dimension, there will not be large logarithms of the form $\ln(M_W^2/m_b^2)$. Scaling down using the renormalization group one arrives at the scale of the b quark mass. At that point we match to an effective theory in which the b quark becomes a static quark, where tree level matching is sufficient as long as we stick to the one-loop result for the renormalization group functions. Hence we find for some scale μ below m_b

$$\bar{c}\gamma_\nu(1-\gamma_5)b = \left(\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right)^{-\frac{6}{25}} (\bar{c}\gamma_\nu(1-\gamma_5)b_v)|_\mu \tag{99}$$

The charm quark mass is still a perturbative scale and hence we may scale down further using the perturbatively calculated renormalization group functions. At the charm quark mass scale we replace also the charm quark by a static quark and scale down further to some scale μ below m_c which is assumed to be still large enough to use perturbation theory. Thus one has

$$\begin{aligned} &\langle D(p' = M_D v')|\bar{c}\gamma_\nu(1-\gamma_5)b|B(p = M_B v)\rangle \\ &= \left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)^{-\frac{6}{25}} \left(\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right)^{-\frac{8}{33-2n_f}[vv'r(vv')-1]} \\ &\qquad \times\langle D(p' = M_D v')|\bar{c}_{v'}\gamma_\nu(1-\gamma_5)b_v|B(p = M_B v)\rangle_\mu \end{aligned} \tag{100}$$

where we have sandwiched the current now between states to indicate that the velocities entering the heavy quark operators are the velocities of the initial and final state particles.

Using the renomalization group we have thus achieved a resummation of logarithms. The scaling between m_b and m_c has moved logarithms of the form $\ln(m_b/m_c)$ from the matrix element into the renormalization (or Wilson coefficient) Z_j, while the scaling between m_c and μ has moved the logarithms of the form $\ln(m_c/\mu)$ into the renormalization constant $Z_{\mathcal{J}}$. In fact, for the example discussed above the result is known including the two loop anomlaous dimension [29]

The procedure described here has the disadvantage that numerically it is useful only in the limit $m_b \gg m_c$ such that $\ln(m_b/m_c) \gg 1$ is a large logarithm and terms of order $(m_c/m_b)^n\alpha_s(m_b)\ln(m_b/m_c))$ may be neglected. In real life we have $m_b/m_c \sim 3$ and one may think of simply performing the one loop calculation including the masses in full QCD and use this to match directly QCD to a theory with two static quarks, where heavy quark symmetries hold. In this way one obtains all terms of order $(m_c/m_b)^n\alpha_s\ln(m_b/m_c))$. However, the price to pay is a scale ambiguity in the scale of α_s which has to be taken at some scale $\bar{m}$ between m_b and m_c. Numerically this is not a problem since α_s does not run much between m_c and m_b.

Recently the latter procedure has been applied at the two loop level [30] such that the terms of order $(m_b/m_c)^n\alpha_s^2(\bar{m})\ln(m_b/m_c))$ are now known. The usual way to parametrize the corrections is by introducing corrections factors

$$V_\mu \to \eta_V V_\mu, \qquad A_\mu \to \eta_A A_\mu \tag{101}$$

for the $b \to c$ vector and axial vector current respectively. The coefficients η_V and η_A are known up to terms of order $\alpha_s^3(\bar{m})$. Taking the numbers of [30] one has

$$\eta_V = 1 + 0.018 + 0.004 + \mathcal{O}(\alpha_s^3(\bar{m})) = 1.022 \pm 0.004 \tag{102}$$
$$\eta_A = 1 - 0.033 - 0.007 + \mathcal{O}(\alpha_s^3(\bar{m})) = 0.960 \pm 0.007 \tag{103}$$

The uncertainty given here is the uncertainty due to the terms of order $\alpha_s^3(\bar{m})$ and is conservatively estimated by the size of the calculated $\alpha_s^2(\bar{m})$ corrections.

Given the fact that the calculated QCD corrections are already below the level of 1% one needs to worry also about QED corrections. In fact, the QED corrections to processes of this type have been calculated already some time ago in the context of μ decay[31]. Similar to the QCD corrections they may be factorized and in principle one could apply also renormalization group methods. However, the running of the QED coupling is negligible at this level and the QED corrections factor is given by

$$\eta_{A/V} \to \eta_{A/V} \left[1 + \frac{\alpha_{QED}}{\pi} \ln \frac{M_Z}{m_b}\right] \sim 1.013\ \eta_{A/V} \tag{104}$$

Hence short distance QED corrections enhance the matrix elements by 1.3%.

2.3.2 Power Corrections

While the QCD radiative corrections are related to logarithmic dependences on the heavy mass, one also expects power corrections which are connected with inverse powers of the heavy mass. As we shall see, the numerator is some scale related to the light degrees of freedom which is thus of order Λ_{QCD}. This type of corrections involves long distance contributions and hence is generically nonperturbative. Consequently, at higher orders in $1/m_Q$ new, nonperturbative parameters or even form factors enter the game, which cannot be calculated in the framework of HQET and which are in general not related to the nonperturbative input used in the static limit.

As an example we consider again the matrix element of a heavy-light current $\bar{q}\Gamma Q$ mediating a transition between a heavy meson and some arbitrary state $|A\rangle$ involving only light degrees of freedom. The full QCD Lagrangian $\mathcal{L}$ and the fields Q are expanded in terms of a power series in $1/m_Q$ in the way described in section 2, and the matrix element under consideration up to order $1/m_Q$ takes the form:

$$\begin{aligned}\langle A|\bar{q}\Gamma Q|M(v)\rangle &= \langle A|\bar{q}\Gamma h_v|H(v)\rangle \\ &+\frac{1}{2m_Q}\langle A|\bar{q}\Gamma P_- i\not{D} h_v|H(v)\rangle - i\int d^4x \langle A|T\{L_1(x)\bar{q}\Gamma h_v\}|H(v)\rangle + \mathcal{O}(1/m^2)\end{aligned} \tag{105}$$

where L_1 are the first-order corrections to the Lagrangian as given in (17) or (24). Furthermore, $|M(v)\rangle$ is the state of the heavy meson in full QCD, including all its mass dependence, while $|H(v)\rangle$ is the corresponding state in the infinite mass limit.

Expression (105) displays the generic structure of the power corrections as they appear in any HQET calculation. There will be local contributions coming from the expansion of the full QCD field; these may be interpreted as the corrections to the currents. The nonlocal contributions, i.e. the time-ordered products, are the corresponding corrections to the states and thus in the r.h.s. of (105) only the states of the infinite-mass limit appear.

However, it has already been discussed in section 2 that local terms may be shifted form the field and vice versa by appropriate field redefinitions. Hence the distinction between corrections to the states (as being the time-ordered products with the corrections to the Lagrangian) is not unique.

Another example for higher order terms is the mass of the heavy hadron, for which also a $1/m_Q$ expansion exists. In the infinite mass limit this mass is given in terms of the quark mass plus some "binding energy" $\bar{\Lambda}$. The corrections are of order $1/m_Q$ and are given by the matrix elements of the leading correction term of the Lagrangian. One obtains

$$m_H = m_Q \left(1 + \frac{\bar{\Lambda}}{m_Q} + \frac{1}{2m_Q^2} (\lambda_1 + d_H \lambda_2) + \mathcal{O}(1/m_Q^3) \right) \tag{106}$$

where $d_H = 3$ for the 0^- and $d_H = -1$ for the 1^- meson. Up to this order no nonlocal terms appear; such terms show up the first time at order $1/m_Q^3$. The parameters $\bar{\Lambda}$, λ_1 and λ_2 correspond to matrix elements involving higher order terms that appear in the effective theory Lagrangian

$$\bar{\Lambda} = \frac{\langle 0 | q \, iv\overleftarrow{D} \, \gamma_5 h_v | H(v) \rangle}{\langle 0 | q \gamma_5 h_v | H(v) \rangle} \tag{107}$$

$$\lambda_1 = \frac{\langle H(v) | \bar{h}_v (iD)^2 h_v | H(v) \rangle}{2M_H} \tag{108}$$

$$\lambda_2 = \frac{\langle H(v) | \bar{h}_v \sigma_{\mu\nu} iD^\mu iD^\nu h_v | H(v) \rangle}{2M_H} \tag{109}$$

where the normalization of the states is chosen to be $\langle H(v) | \bar{h}_v h_v | H(v) \rangle = 2M_H = 2(m_Q + \bar{\Lambda})$. These parameters may be interpreted as the binding energy of the heavy meson in the infinite mass limit ($\bar{\Lambda}$), the expectation value of the kinetic energy of the heavy quark (λ_1) and its energy due to the chromomagnetic moment of the heavy quark (λ_2) inside the heavy meson. The latter two parameters play an important role since they parametrize the non-perturbative input needed in the subleading order of the $1/m_Q$ expansion.

As has been discussed already in connections with the residual mass term the matrix elements $\bar{\Lambda}$, λ_1 and λ_2 depend on the renormalization point and/or on the definition of the mass which is used as an expansion parameter. Another issue are the renormalon ambiguities which again are related to the definition of the mass. We shall not discuss this issue here, rather refer the reader to the literature [23].

The only parameter which is easy to access is λ_2, since it is related to the mass splitting between $H(v)$ and $H^*(v, \epsilon)$. From the B-meson system we obtain

$$\lambda_2(m_b) = \frac{1}{4}(M_{H^*} - M_H) = 0.12 \text{ GeV}^2; \tag{110}$$

from the charm system the same value is obtained. This shows that indeed the spin-symmetry partners are degenerate in the infinite mass limit and the splitting between them scales as $1/m_Q$.

The other parameters appearing in (106) are not simply related to the hadron spectrum. Using the pole mass for m_Q in (106), QCD sum rules yield for a value of $\bar{\Lambda} = 570 \pm 70$ MeV [7]. More problematic is the parameter λ_1; from its definition one is led to assume $\lambda_1 < 0$; a more restrictive inequality

$$-\lambda_1 > 3\lambda_2 \tag{111}$$

has been derived in a quantum mechanical framework in [32] and using heavy-flavour sum rules [33]. Furthermore, there exists also a sum rule estimate [34] for this parameter:

$$\lambda_1 = -0.52 \pm 0.12 \text{ GeV}^2. \tag{112}$$

This value is compatible with the bounds; however, it is unexpectedly large, since it corresponds to a rms-momentum of the heavy quark inside the meson of

$$\sqrt{\langle \vec{p}^2 \rangle} \sim 720 \text{ MeV} \tag{113}$$

which is large compared to the naive guess of $-\lambda_1 \sim (\Lambda_{QCD})^2$ This is the reason why also smaller values of λ_1 have been used in the literature.

Recently there has been an attempt [35] to extract $\bar{\Lambda}$ and λ_1 from the shape of the lepton energy spectrum in inclusive semileptonic B decays (see sections 4.3 and 4.4). The values obtained from this analysis are $\bar{\Lambda} = 0.39 \pm 0.11$ GeV and $-\lambda_1 = 0.19 \pm 0.10$ GeV^2, where the $\overline{\text{MS}}$ definition of the mass has been used. The uncertainties quoted are only the 1σ statistical ones; the systematical uncertainties of this approach are difficult to estimate.

Finally we shall review an important results concerning $1/m_Q$ corrections, one of which is called Luke's theorem [36]. It is a generalization of the Ademollo Gatto theorem [37], which states that in the presence of explicit symmetry breaking the matrix elements of the currents that generate the symmetry are still normalized up to terms which are second order in the symmetry breaking interaction.

For the case at hand the relevant symmetry is the heavy flavor symmetry. This symmetry is an $SU(2)$ symmetry and is generated by three operators $Q_\pm$ and Q_3 with

$$\begin{aligned}
&Q_+ = \int d^3x\, \bar{b}_v(x) c_v(x) \quad Q_- = \int d^3x\, \bar{c}_v(x) b_v(x) \\
&Q_3 = \int d^3x\, (\bar{b}_v(x) b_v(x) - \bar{c}_v(x) c_v(x)) \\
&[Q_+, Q_-] = Q_3 \qquad [Q_+, Q_3] = -2Q_+ \qquad (Q_+)^\dagger = Q_-
\end{aligned} \tag{114}$$

Let us denote the ground state flavour symmetry multiplet as $|B\rangle$ and $|D\rangle$. Then the operators act in the following way

$$\begin{aligned}
&Q_3|B\rangle = |B\rangle \qquad Q_3|D\rangle = -|D\rangle \\
&Q_+|D\rangle = |B\rangle \qquad Q_-|B\rangle = |D\rangle \\
&Q_+|B\rangle = Q_-|D\rangle = 0.
\end{aligned} \tag{115}$$

The Hamiltonian of this system has a $1/m_Q$ expansion of the form

$$\begin{aligned}
H &= H_0^{(b)} + H_0^{(c)} + \frac{1}{2m_b} H_1^{(b)} + \frac{1}{2m_c} H_1^{(c)} + \cdots \\
&= H_0^{(b)} + H_0^{(c)} + \frac{1}{2}\left(\frac{1}{2m_b} + \frac{1}{2m_c}\right)(H_1^{(b)} + H_1^{(c)}) \\
&\qquad\qquad + \frac{1}{2}\left(\frac{1}{2m_b} - \frac{1}{2m_c}\right)(H_1^{(b)} - H_1^{(c)}) + \cdots \\
&= H_{symm} + H_{break}.
\end{aligned} \tag{116}$$

In the second equation, the first line is still symmetric under heavy flavour $SU(2)$ while the term in the second line does not commute any more with $Q_\pm$, but it still commutes with Q_3. In other words, to order $1/m_Q$ we still have common eigenstates of H and Q_3, which we shall denote as $|\tilde{B}\rangle$ and $|\tilde{D}\rangle$. Sandwiching the commutation relation we get

$$\begin{aligned}
1 &= \langle \tilde{B}|Q_3|\tilde{B}\rangle = \langle \tilde{B}|[Q_+, Q_-]|\tilde{B}\rangle \\
&= \sum_n \left[\langle \tilde{B}|Q_+|\tilde{n}\rangle\langle \tilde{n}|Q_-|\tilde{B}\rangle - \langle \tilde{B}|Q_-|\tilde{n}\rangle\langle \tilde{n}|Q_+|\tilde{B}\rangle\right] \\
&= \sum_n \left[|\langle \tilde{B}|Q_+|\tilde{n}\rangle|^2 - |\langle \tilde{B}|Q_-|\tilde{n}\rangle|^2\right]
\end{aligned} \tag{117}$$

where $|\tilde{n}\rangle$ form a complete set of states of the Hamiltonian $H_{symm} + H_{break}$. The matrix elements may be written as

$$\langle \tilde{B}|Q_\pm|\tilde{n}\rangle = \frac{1}{E_B - E_n}\langle \tilde{B}|[H_{break}, Q_\pm]|\tilde{n}\rangle \tag{118}$$

where E_B and E_n are the energies of the states $|\tilde{B}\rangle$ and $|\tilde{n}\rangle$ repectively. In the case $|\tilde{n}\rangle = |\tilde{D}\rangle$ the matrix element will be of order unity, since both the numerator and the energy difference in the denominator are of the order of the symmetry breaking. For all other states the energy difference in the denomiantor is nonvanishing in the symmetry limit, and hence this difference is of order unity; thus the matrix element for these states will be of the order of the symmetry breaking. From this we conclude

$$\langle \tilde{B}|Q_+|\tilde{D}\rangle = 1 + \mathcal{O}\left[\left(\frac{1}{2m_b} - \frac{1}{2m_c}\right)^2\right]. \tag{119}$$

In particular, the weak transition currents at the nonrecoil point $v = v'$ are proportional to these symmetry generators and hence we may conclude that for some of these matrix elements we only have corrections of the order $1/m_Q^2$.

Reparametrization invariance also puts restrictions on the higher order corrections, since it relates different orders of the $1/m_Q$ expansion. In particular for the correction terms to the Lagrangian reparametrization invariance yields restrictions, i.e. it relates different oreders.

3 Exclusive Decays

The most straightforward application of heavy quark symmetry are exclusive decays. In particular the exclusive heavy-to-heavy transitions of the form $b \to c$ are strongly restricted by heavy quark symmetries and the phenomenological implications are very interesting.

In this section we shall consider exclusive decays of heavy hadrons, focussing mainly on semileptonic decays. Heavy quark symmetries and HQET has not (yet?) brought any significant progress in the description of exclusive nonleptonic decays and hence only a brief comment is added on this kind of decays.

3.1 Semileptonic Heavy to Heavy Transitions

3.1.1 Meson Decays: $B \to D^{(*)}\ell\nu$

The the exclusive semileptonic decays $B \to D^{(*)}\ell\nu$ are the "master example" for the heavy-to-heavy decays which are strongly constrained by heavy quark symmetries. The hadronic matrix elements acquire in the heavy mass limit exactly the form of the Wigner-Eckart theorem (55) and are thus described interms of a single normalized form factor, the Isgur Wise function.

We shall first list the relevant matrix elements and count the form factors without assuming heavy quark symmetry. One has

$$\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle = \sqrt{m_B m_D}\left[\xi_+(y)(v_\mu + v'_\mu) + \xi_-(y)(v_\mu - v'_\mu)\right] \tag{120}$$

$$\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu b|B(v)\rangle = i\sqrt{m_B m_{D^*}}\xi_V(y)\varepsilon_{\mu\alpha\beta\rho}\epsilon^{*\alpha}v'^\beta v^\rho \tag{121}$$

$$\begin{aligned}\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B(v)\rangle = \sqrt{m_B m_{D^*}}\Big[&\xi_{A1}(y)(vv'+1)\epsilon^*_\mu - \xi_{A2}(y)(\epsilon^* v)v_\mu \\ &-\xi_{A2}(y)(\epsilon^* v)v'_\mu\Big],\end{aligned} \tag{122}$$

where we have defined $y = vv'$. Thus in general there are six form factors, which in the heavy mass limit for both the b and the c quark may be related to the Isgur Wise function as introduced in (55)

$$\xi_i(y) = \xi(y) \text{ for } i = +, V, A1, A3, \qquad \xi_i(y) = 0 \text{ for } i = -, A2. \tag{123}$$

In particular, at the non-recoil point $v = v'$ we have due to heavy quark symmetry and Luke's theorem

$$\xi_i(1) = 1 + \mathcal{O}(1/m_Q^2) \text{ for } i = +, V, A1, A3, \quad \xi_i(1) = \mathcal{O}(1/m_Q) \text{ for } i = -, A2. \tag{124}$$

Note that the form factors for which there is no normalization at $v = v'$ there is also no protection against corrections of lienar order in $1/m_Q$.

The differential rates for the exclusive semiletonic $b \to c$ transitions may be expressed in terms of the six form factors as

$$\frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \tag{125}$$
$$= \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 \left(m_D\sqrt{y^2-1}\right)^3 \left|\xi_+(y) - \frac{m_B - m_D}{m_B + m_D}\xi_-(y)\right|^2$$
$$\frac{d\Gamma}{dy}(B \to D^*\ell\nu_\ell) \tag{126}$$
$$= \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B - m_{D^*})^2 m_{D^*}^2 \left(m_{D^*}\sqrt{y^2-1}\right)(y+1)^2|\xi_{A1}(y)|^2 \sum_{i=0,\pm}|H_i(y)|^2$$

with

$$|H_\pm(y)|^2 = \frac{m_B^2 - m_{D^*}^2 - 2ym_Bm_{D^*}}{(m_B - m_{D^*})^2}\left[1 \mp \sqrt{\frac{y-1}{y+1}}R_1(y)\right]^2 \tag{127}$$
$$|H_0(y)|^2 = \left(1 + \frac{m_B(y-1)}{m_B - m_{D^*}}[1 - R_2(y)]\right)^2 \tag{128}$$

where we have defined the form factor ratios

$$R_1(y) = \frac{\xi_V(y)}{\xi_{A1}(y)}, \quad R_2(y) = \frac{\xi_{A3}(y) + \frac{m_B}{m_{D^*}}\xi_{A2}(y)}{\xi_{A1}(y)} \tag{129}$$

In the heavy mass limit these differential rates depend only on the Isgur-Wise function

$$\frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \stackrel{m_b,m_c\to\infty}{=} \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2\left(m_D\sqrt{y^2-1}\right)^3|\xi(y)|^2 \tag{130}$$
$$\frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \stackrel{m_b,m_c\to\infty}{=} \tag{131}$$
$$\frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B - m_{D^*})^2 m_{D^*}^2\left(m_{D^*}\sqrt{y^2-1}\right)(y+1)^2 \tag{132}$$
$$\left[1 + \frac{4y}{y+1}\frac{m_B^2 - m_{D^*}^2 - 2ym_Bm_{D^*}}{(m_B - m_{D^*})^2}\right]|\xi(y)|^2$$

These relations allow for a test of heavy quark symmetry, since the ratios of the differential rates do not depend on any unknown form factor any more. In particular the ratios R_1 (R_2) measures the ratio of the differential transverse (longitudinal) rate and the total differential rate. In the heavy mass limit both R_1 and R_2 are unity; this has to be compared to the measurements by CLEO [38]

$$R_1 = 1.24 \pm 0.26 \pm 0.12 \tag{133}$$
$$R_2 = 0.72 \pm 0.18 \pm 0.07 \tag{134}$$

3.1.2 The determination of V_{cb} using HQET

From the measured lepton invariant mass spectrum one may determine V_{cb} in a model independent way by extrapolating to the kinematical endpoint of maximal momentum transfer to the leptons, corresponding to the point $v = v'$. At this point heavy quark symmetries determine the absolute normalization of some of the form factors and the corrections to this normalization have been discussed in section 2.

The mode $B \to D^*\ell\nu_\ell$ has the advantage of a higher branching fraction and hence we shall start the discussion with this decay. The relevant formula may be derived from (126) and reads

$$\lim_{y\to 1} \frac{1}{\sqrt{y^2-1}} \frac{d\Gamma}{dy}(B \to D^*\ell\nu_\ell) = \frac{G_F^2}{4\pi^3}(m_B - m_{D^*})^2 m_{D^*}^3 |V_{cb}|^2 |\xi_{A1}(1)|^2 \tag{135}$$

The form factor ξ_{A1} is normalized due to heavy quark symmetries and is hence protected against $1/m_Q$ corrections at $v = v'$ by Lukes theorem [36]. Hence we have

$$\xi_{A1}(1) = \eta_A(1 + \delta_{1/m^2}) \tag{136}$$

Including QED corrections and the estimate of the $1/m_Q^2$ corrections in the way discussed in section 2 one obtains

$$\xi_{A1}(1) = 0.92 \pm 0.03 \tag{137}$$

From this value one may extract from the extrapolations shown in figs.5 and 6 a value for V_{cb}

$$|V_{cb}| = 0.0362 \pm 0.0019 \pm 0.0020 \pm 0.0014 \text{ (CLEO)[39]} \tag{138}$$
$$|V_{cb}| = 0.0345 \pm 0.0025 \pm 0.0027 \pm 0.0015 \text{ (ALEPH)[41]} \tag{139}$$

where the last error reflects the theoretical uncertainty of the $1/m_Q$ corrections.

Recently CLEO has also measured the leptonic invariant mass spectrum for the decay $B \to D\ell\nu_\ell$. In a similar way one may perform an extrapolation to obtain V_{cb}. Here one gets from (125)

$$\begin{aligned} &\lim_{y\to 1} \left(\frac{1}{\sqrt{y^2-1}}\right) \frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \\ &\quad = \frac{G_F^2}{48\pi^3}(m_B + m_D)^2 m_D^3 |V_{cb}|^2 \left|\xi_+(1) - \frac{m_B - m_D}{m_B + m_D}\xi_-(1)\right|^2 \end{aligned} \tag{140}$$

In this case a form factor enters which is not protected by Lukes theorem, $\xi_-(1) = \mathcal{O}(1/m_Q)$. However, this does not spoil the possibility to determine $|V_{cb}|$ from this mode, since the $1/m_Q$ corrections are kinematically suppressed by the factor $(m_B - m_D)/(m_B + m_D)$. Here we have

$$\left|\xi_+(1) - \frac{m_B - m_D}{m_B + m_D}\xi_-(1)\right| = \eta_V(1 + \Delta_{1/m_Q}) \tag{141}$$

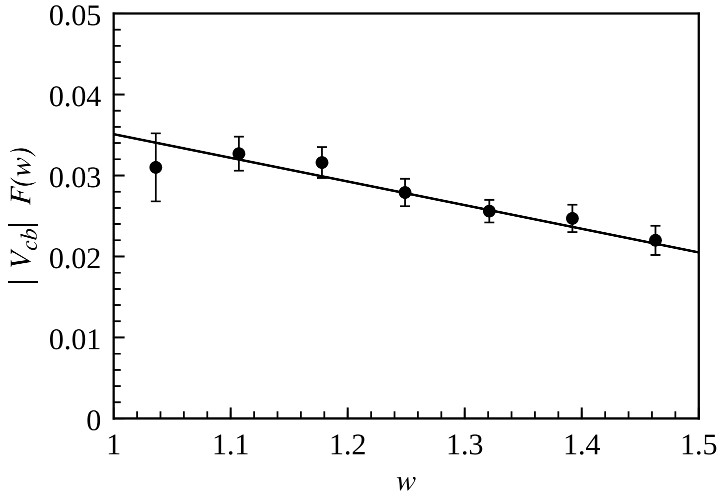

Figure 5: CLEO data[39] and the extrapolation used to obtain V_{cb}. The figure istaken from the recent review [40].

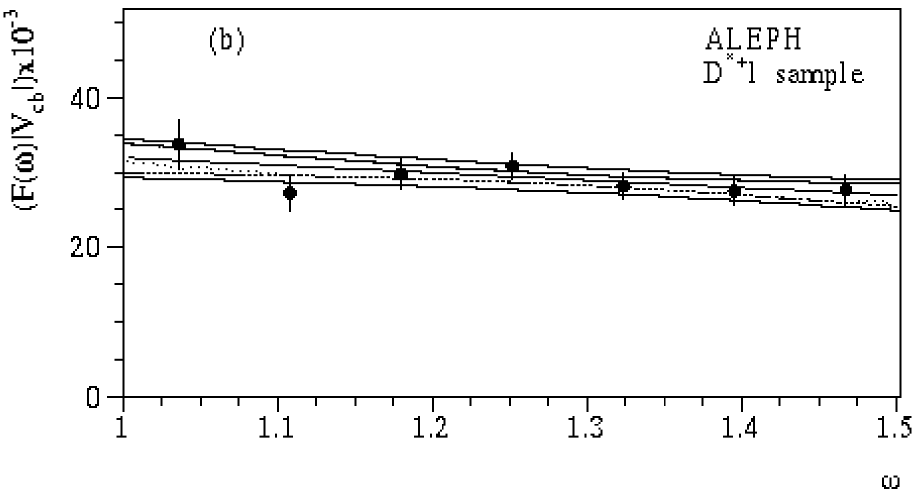

Figure 6: ALEPH data[41] and the extrapolation used to obtain V_{cb}.

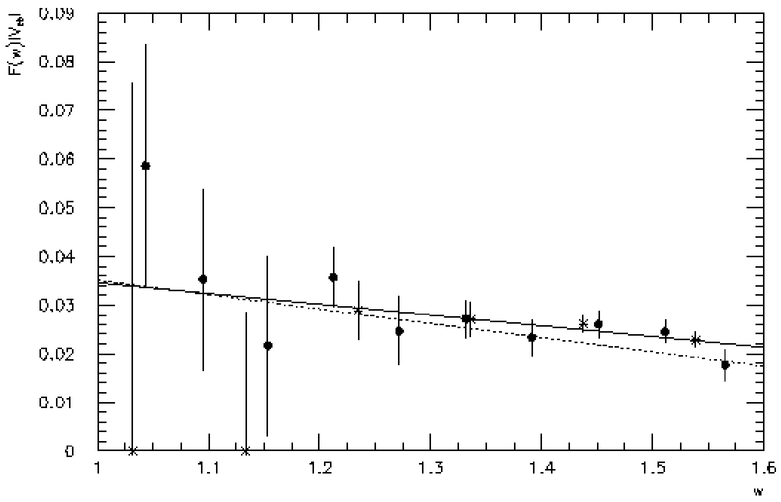

Figure 7: Data and the extrapolation used to obtain V_{cb} from $B \to D\ell\nu_\ell$. The figure is taken from [44].

where Δ_{1/m_Q} are the $1/m_Q$ corrections induced by $\xi_-(1)$. These corrections have been estimated recently [42]

$$\left|\xi_+(1) - \frac{m_B - m_D}{m_B + m_D}\xi_-(1)\right| = 0.98 \pm 0.07 \tag{142}$$

The data from CLEO [44] are shown in fig.7. From the extrapolation shown in fig.7 one obtains the value

$$|V_{cb}| = 0.0353 \pm 0.0046 \pm 0.0044 \pm 0.0025 \tag{143}$$

where the last error again reflects the theoretical uncertainty. Both values of V_{cb} are in very good agreement and are also consistent with values obtained form other methods, in particular with the results from inclusive semileptonic decays [43]

The data on the leptonic invariant mass spectrum extend over the whole rage of y and the extrapolation to $y = 1$ has to rely on some ansatz for the Isgur Wise function. This may be used in turn to extract a value also for the slope of the Isgur Wise function.

Close to the point $y = 1$ one thus parametrizes the Isgur Wise function as

$$\xi(y) = 1 - \rho^2(y-1) + \cdots \tag{144}$$

The theoretical predictions[45] for the slope parameter ρ^2 depend on matrix elements involving excited D mesons which are hard to estimate; thus the theoretical value is quite uncertain and ranges between

$$0.5 \leq \rho^2 \leq 1.1 \tag{145}$$

Data on the slope parameter come from the LEP experiments as well as from ARGUS and CLEO, the results obtained from experiment are

$$\rho^2 = \begin{cases} 0.29 \pm 0.21 \pm 0.12 & \text{ALEPH[46]} \\ 0.84 \pm 0.12 \pm 0.08 & \text{CLEO[47]} \\ 1.17 \pm 0.22 \pm 0.06 & \text{ARGUS[48]} \\ 0.81 \pm 0.16 \pm 0.10 & \text{DELPHI[49]} \end{cases} \tag{146}$$

which are compatible with the theoretical expectation.

3.1.3 Baryon Decays: $\Lambda_b \to \Lambda_c \ell \nu$

Exclusive decays of the heavy-to-heavy type of baryons may also be considered using the heavy mass limit. The easiest example of this type is the decay $\Lambda_b \to \Lambda_c \ell \nu$, since in both baryons the light degrees of freedom are spinless.

In general the vector current between two spin-1/2/ baryons is bgiven in terms of three form factors

$$\begin{aligned} < B', p', s' | \bar{q}\gamma_\mu q | B, p, s > &= \bar{u}(p', s')[f_1(q^2)\gamma_\mu - i f_2(q^2)\sigma_{\mu\nu}q^\nu \\ &+ f_3(q^2)q_\mu] u(p, s), \end{aligned} \tag{147}$$

where f_1, f_2 and f_3 are real.

Employing heavy quark symmetry we find for the Λ's, (c.f. eq. 65)

$$f_1 = A, \quad f_2 = f_3 = 0. \tag{148}$$

Finally, for the form factors for the transition between the ω_h's, we find (c.f. eq. 68)

$$\begin{aligned} f_1 &= -\frac{1}{3}\left[B(v\cdot v' - 4) + C(v\cdot v' - 1)(v\cdot v' - 3)\right], \\ f_2 &= -\frac{4}{3(M + M')}\left[B + C(v\cdot v' - 1)\right], \\ f_3 &= 0. \end{aligned} \tag{149}$$

where M and M' are the masses of the initial and final baryon.

Let us now discuss the semileptonic decays of a heavy Λ, say Λ_b. Its semileptonic weak decay into Λ_c involves the matrix element of the charged current which, due to spin symmetry, may be parametrized in terms of a single form factor:

$$< \Lambda_c(v') | \bar{c}\gamma_\mu(1-\gamma_5) b | \Lambda_b(v) > = A(v\cdot v')\bar{u}_{\Lambda_c}(v')\gamma_\mu(1-\gamma_5)u_{\Lambda_b}(v). \tag{150}$$

As has been pointed out previously, the form factor A is normalized at $v \cdot v' = 1$. Consequently, as in the case of the semileptonic decays of heavy mesons, this fact may be used to extract V_{cb} of the Cabibbo-Kobayashi-Maskawa (CKM) matrix using these semileptonic decays.

However, data on this type of decay is still sparse compared to the corresponding meson decays. Semileptonic decays of the Λ_b have up to now only been observed at LEP from b quark hadronization into a Λ_b and only one out of about ten b quarks fragment into a Λ_b [50]

From the theoretical side the Λ_Q type heavy baryons are the simplest objects, since here the light degrees of freedom are in a spinless state. Due to this one has also a strong restriction on the possible $1/m_Q$ corrections for semileptonic $\Lambda_b \to \Lambda_c$ decays.

Let us study a transition mediated by a heavy quark bilinear current between Λ_b and Λ_c. Our representation follows the paper by Georgi, Grinstein and Wise [51]; following this reference we shall discuss the matrix element

$$< \Lambda_c | \bar{c}\Gamma b | \Lambda_b > . \tag{151}$$

To leading order in heavy quark effective theory this matrix element is given in terms of a single form factor A

$$< \Lambda_c(v') | \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} | \Lambda_b(v) > = A(v\cdot v')\bar{u}_c(v')\Gamma u_b(v) \tag{152}$$

To order $1/m_c$ we shall start with the non-local corrections originating from the Lagrangian. The operators are given in (17) or (24). To be specific, we use the Lagrangian (17) and write the corrections as

$$\mathcal{L}_1 = \frac{1}{2m_c}\bar{h}_v (iD)^2 h_v \tag{153}$$
$$\mathcal{L}_2 = \frac{1}{2m_c}\bar{h}_v (ivD)^2 h_v \tag{154}$$
$$\mathcal{L}_3 = \frac{1}{2m_c}\bar{h}_v (-i\sigma_{\mu\nu})(iD^\mu)(iD^\nu) h_v \tag{155}$$

The first two pieces $\mathcal{L}_1$ and $\mathcal{L}_2$ transform trivially under the spin symmetry and thus their contributions look exactly like the one from the lowest order (152), i.e. they just renormalize the form factor A

$$\begin{aligned}&(-i)\int d^4x \sum_{k=1}^{2} C_k(\mu) < \Lambda_c(v')|T\left[\mathcal{L}_k(x)\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}(0)\right]|\Lambda_b(v) > \\ &= \Delta A(v\cdot v')\bar{u}_c(v')\Gamma u_b(v)\end{aligned} \tag{156}$$

Finally, we shall study the contribution of the chromomagnetic moment operator $\mathcal{L}_3$. From spin symmetry this matrix element is given in terms of an antisymmetric tensor $\xi_{\mu\nu}$

$$\begin{aligned}&(-i)\int d^4x\, C_3(\mu) < \Lambda_c(v')|T\left[\mathcal{L}_3(x)\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}(0)\right]|\Lambda_b(v) > \\ &= \xi^{\mu\nu}(v'\cdot v')\bar{u}_c(v')\sigma_{\mu\nu}\left(\frac{\not{v}'+1}{2}\right)\Gamma u_b(v)\end{aligned} \tag{157}$$

There are only two vectors available and thus $\xi^{\mu\nu}$ has to be of the form

$$\xi_{\mu\nu}(v'\cdot v') = \left(v_\mu v'_\nu - v'_\mu v_\nu\right)\eta(v\cdot v') \tag{158}$$

This shows that the chromomagnetic operator does not contribute, since one has

$$\bar{u}_c(v')\sigma_{\mu\nu}\left(\frac{\not{v}'+1}{2}\right)v^{\mu\prime} = 0. \tag{159}$$

and thus from spin symmetry

$$(-i)\int d^4x\, C_3(\mu) < \Lambda_c(v')|T\left[\mathcal{L}_3(x)\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}(0)\right]|\Lambda_b(v) >= 0 \tag{160}$$

The second class of contributions are the matching contributions. In the following we shall neglect QCD scaling and discuss only the operators appearing at the matching scale m_c. From spin symmetry one obtains for the following matrix element

$$< \Lambda_c(v')|\bar{h}_{v'}^{(c)} i \overleftarrow{D}_\mu\, \Gamma h_v^{(b)}|\Lambda_b(v) >= \bar{u}_c(v')\Gamma u_b(v)\left[E v_\mu + F v'_\mu\right] \tag{161}$$

where the arrow over the covariant derivative indicates that it acts to the left on the c quark. Here two form factors E and F appear. These are not independent, since one has from the equations of motion for the c quark

$$0 = < \Lambda_c(v')|\bar{h}_{v'}^{(c)} i(v'\cdot \overleftarrow{D})\Gamma h_v^{(b)}|\Lambda_b(v) > \tag{162}$$
$$= \bar{u}_c(v')\Gamma u_b(v)\left[E(v\cdot v') + F\right] \tag{163}$$

and thus $E(v \cdot v') = -F$. Furthermore, momentum conservation relates the the form factor E (or F) to the lowest order form factor A, since we have

$$
\begin{aligned}
& i\partial_\mu < \Lambda_c(v')|\bar{h}^{(c)}_{v'}\Gamma h^{(b)}_v|\Lambda_b(v) > \\
& = \left[(m_{\Lambda_b} - m_b)v_\mu - (m_{\Lambda_c} - m_c)v'_\mu\right] A(v \cdot v')\bar{u}_c(v')\Gamma u_b(v) \\
& = \bar{\Lambda}(v_\mu - v'_\mu)A(v \cdot v')\bar{u}_c(v')\Gamma u_b(v)
\end{aligned} \tag{164}
$$

where we may equate $m_{\Lambda_b} - m_b$ with $m_{\Lambda_c} - m_c$ to the order in the $1/m$ expansion we are working. We denote the mass difference between the baryon and the quark by $\bar{\Lambda}$ which is typical scale from the binding of the light degrees of freedom and thus is of the order of Λ_{QCD}.

Pulling the derivative inside one obtains from (164)

$$
\begin{aligned}
& i\partial_\mu < \Lambda_c(v')|\bar{h}^{(c)}_{v'}\Gamma h^{(b)}_v|\Lambda_b(v) >= \\
& < \Lambda_c(v')|\bar{h}^{(c)}_{v'} i \overleftarrow{D}_\mu \Gamma h^{(b)}_v|\Lambda_b(v) > + < \Lambda_c(v')|\bar{h}^{(c)}_{v'}\mu\Gamma iD_\mu h^{(b)}_v|\Lambda_b(v) >
\end{aligned} \tag{165}
$$

Combining this with (164), contracting with v_μ and using the equation of motion for the b quark the second term on the right side of (165) does not contribute and one finds a relation between E and A

$$
E(1 + v \cdot v') = \bar{\Lambda}A \tag{166}
$$

Combining the two contributions from the Lagrangian and from the matching one finds that at the Λ_{QCD}/m_c level there is still only one form factor. This is entirely due to the fact that the Λ baryons are in fact the simplest objects from the point of view of heavy quark effective theory. For mesons and for more complicated baryons there are additional independent form factors at order Λ_{QCD}/m_c [36, 52].

Finally may check Luke's theorem for the present case. in wich it states that the shift ΔA of the form factor A due to the two operators $\mathcal{L}_1$ and $\mathcal{L}_2$ vanishes at the normalization point; i.e. the single form factor to order Λ_{QCD}/m_c is still normalized. This may be seen very similar to the discussion in section 2.3.2 by splitting the two operators $\mathcal{L}_1$ and $\mathcal{L}_2$ into their singlet and triplet piece with respect to heavy flavor symmetry:

$$
\mathcal{L}_1 + \mathcal{L}_2 = \frac{1}{2m_c}\bar{h}^{(c)}_v[(iD)^2 - (iv \cdot D)^2]h^{(c)}_v \tag{167}
$$

$$
\begin{aligned}
& = \frac{1}{4m_c}\left\{\bar{h}^{(c)}_v[(iD)^2 - (iv \cdot D)^2]h^{(c)}_v + \bar{h}^{(b)}_v[(iD)^2 - (iv \cdot D)^2]h^{(b)}_v\right\} \\
& + \frac{1}{4m_c}\left\{\bar{h}^{(c)}_v[(iD)^2 - (iv \cdot D)^2]h^{(c)}_v - \bar{h}^{(b)}_v[(iD)^2 - (iv \cdot D)^2]h^{(b)}_v\right\}
\end{aligned} \tag{168}
$$

where the first combination is singlet under heavy flavor symmetry, while the second one behaves like a triplet.

Inserting the triplet piece into the the matrix element (156) one easily sees that its matix element vanishes due to heavy flavor symmtry and thus only the singlet piece contributes to the matrix element under consideration. However, since only the singlet piece gives a contribution the vector current for equal velocities of the heavy quark $\bar{h}^{(c)}_v\gamma_\mu h^{(b)}_v$ still generates a symmetry, even including the $1/m_c$ terms. Thus the form factor – including the correction ΔA – is still normalized:

$$
(A + \Delta A)|_{v \cdot v' = 1} = 1 \tag{169}
$$

which is Luke's theorem for the present case.

3.2 (Semi)leptonic Heavy to Light transitions

Heavy quark symmetries may also be used to restrict the independent form factors appearing in heavy to light decays [53]. For the decays of heavy mesons into light 0^- and 1^- particles heavy quark symmetries restrict the number of independent form factors to six, which is just the number needed to parametrize the semileptonic decays of this type. Furthermore, no absolute normalization of form factors may be obtained from heavy quark symmetries in the heavy to light case; only the relative normalization of B meson decays heavy to light transitions may be obtained from the corresponding D decays.

In general we shall discuss matrix elements of a heavy to light current which have the following structure

$$J = \langle A|\bar{\ell}\Gamma h_v|H(v)\rangle, \tag{170}$$

where Γ is an arbitrary Dirac matrix, ℓ is a light quark (u, d or s) and A is a state involving only light degrees of freedom.

Spin symmetry implies that the heavy quark index hooks directly to the heavy quark index of the Dirac matrix of the current. Thus one may write for the transition matrix element (176)

$$\langle A|\bar{\ell}\Gamma h_v|H(v)\rangle = \text{ Tr } (\mathcal{M}_A \Gamma H(v)) \tag{171}$$

where the matrix $H(v)$ representing the heavy meson has been given in (50). The matrix $\mathcal{M}_A$ describes the light degrees of freedom and is the most general matrix which may be formed from the kinematical variables involved. Furthermore, if the energies of the particles in the state A are small, i.e. of the order of Λ_{QCD}, the matrix $\mathcal{M}_A$ does not depend on the heavy quark; in particular it does not depend on the heavy mass m_H. In the following we shall discuss some examples.

3.2.1 Heavy Meson Decay Constant

The first example is the heavy meson decay constant, where the state A is simply the vacuum state. The heavy meson decay constant is defined by

$$\langle 0|\bar{\ell}\gamma_\mu\gamma_5 h_v|H(v)\rangle = f_H m_H v_\mu, \tag{172}$$

and since $|A\rangle = |0\rangle$ the matrix $\mathcal{M}_0$ is simply the unit matrix times a dimensionful constant[3] and one has, using (171)

$$\langle 0|\bar{\ell}\gamma_\mu\gamma_5 h_v|H(v)\rangle = \kappa \text{ Tr } (\gamma\gamma_5 H(v)) = 2\kappa\sqrt{m_H} v_\mu. \tag{173}$$

As discussed above the constant κ does not depend on the heavy mass and thus one infers the well-known scaling law for the heavy meson decay constant from the last two equation

$$f_H \propto \frac{1}{\sqrt{m_H}} \tag{174}$$

Including the leading and subleading QCD radiative corrections one obtains a relation between f_B and f_D

$$f_B = \sqrt{\frac{m_c}{m_b}} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25} \left[1 + 0.894\frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right] f_D \sim 0.69 f_D. \tag{175}$$

[3]Note that contributions proportional to $\not{v}$ may be eliminated using

$$H(v)\not{v} = -H(v).$$

3.2.2 Semileptonic Decays: $B \to (\pi/\rho)\ell\nu$

The second example are transitions of a heavy meson into a light pseudoscalar meson, which we shall denote as π. The matrix element corresponding to (170) is

$$J_P = \langle \pi(p)|\bar{\ell}\Gamma h_v|H(v)\rangle, \tag{176}$$

where p is the momentum of the light quark,

The Dirac marix $\mathcal{M}_P$ for the light degrees of freedom appearing now in (171) depends on p and v. It may be expanded in terms of the sixteen independent Dirac matrices 1, γ_5, γ_μ, $\gamma_5\gamma_\mu$, and $\sigma_{\mu\nu}$ taking into account that it has to behave like a pseudoscalar. The form factors appearing in the decomposition of $\mathcal{M}_P$ depend on the variable $v \cdot p$, the energy of the light meson in the rest frame of the heavy one. In order to compare different heavy to light transition by employing heavy flavor symmetry this energy must be sufficiently small, since the typical scale for the light degrees of freedom has to be of the order of Λ_{QCD} to apply heavy quark symmetry[4]. For the case of a light pseudoscalar meson the most general decomposition of $\mathcal{M}_P$ is

$$\mathcal{M}_P = \sqrt{v \cdot p}A(\eta)\gamma_5 + \frac{1}{\sqrt{v \cdot p}}B(\eta)\gamma_5 \not{p}, \tag{177}$$

where we have defined the dimensionless variable

$$\eta = \frac{v \cdot p}{\Lambda_{QCD}}. \tag{178}$$

The form factors A and B are universal in the kinematic range of small energy of the light meson, i.e. where the momentum transfer to the light degrees of freedom is of the order Λ_{QCD}; in this region η is of order unity. This universality of the form factors may be used to relate various kinds of heavy to light transitions, e.g. the semileptonic decays like $D \to \pi e\nu$, $D \to K e\nu$ or $B \to \pi e\nu$ and also the rare decays such as $B \to K\ell^+\ell^-$ or $B \to \pi\ell^+\ell^-$, where ℓ denotes an electron or a muon.

The relvant hadronic current for the case of a $0^- \to 0^-$ transition may be expressed in terms of two form factors

$$\begin{aligned} \langle \pi(p)|\bar{\ell}\gamma(1-\gamma_5)h_v|H(v)\rangle &= F_1(v \cdot p)m_H v_\mu + F_2(v \cdot p)p_\mu \\ &= F_+(v \cdot p)(m_H v_\mu + p_\mu) + F_-(v \cdot p)q_\mu \end{aligned} \tag{179}$$

where

$$F_\pm(v \cdot p) = \frac{1}{2}\left(F_1(v \cdot p) \pm F_2(v \cdot p)\right) \tag{180}$$

Inserting this into (176) one may express $F_\pm$ in terms of the universal form factors A and B

$$F_1(v \cdot p) = F_+(v \cdot p) + F_-(v \cdot p) = -2\sqrt{\frac{v \cdot p}{m_H}}A(\eta) \tag{181}$$

$$F_2(v \cdot p) = F_+(v \cdot p) - F_-(v \cdot p) = -2\sqrt{\frac{m_H}{v \cdot p}}B(\eta) \tag{182}$$

[4]Note that in this case the variable $v \cdot p$ ranges between 0 and $m_H/2$ where we have neglected the pion mass. Thus at the upper end of phase space the variable $v \cdot p$ scales with the heavy mass and heavy quark symmetries are not applicable anymore.

From these relations one may read off the scaling of the form factors with the heavy mass which was already derived in [53]. In particular it follows that for small $v \cdot p$ F_1 is suppressed by one power of the heavy mass compared to F_2. In other words, in the infinite mass limit the heavy to light transitions of the form $B \to \pi$ are described in terms of a single form factor, since $F_+ = -F_-$

The case of a heavy meson decaying into a light vector meson may be treated similarly. The matrix element for the transition of a heavy meson into a light vector meson (denoted generically as ρ in the following) is given again by (170) and is in this case

$$J_V = \langle \rho(p,\epsilon)|\bar{\ell}\Gamma h_v|H(v)\rangle. \tag{183}$$

Using (171) one has

$$\langle \rho(p,\epsilon)|\bar{\ell}\Gamma h_v|H(v)\rangle = \text{ Tr } \left(\mathcal{M}_V \Gamma H(v)\right), \tag{184}$$

where now the Dirac matrix $\mathcal{M}_V$ has to be a linear function of the polarization of the light vector meson.

The most general decomposition is given in terms of four dimensionless form factors

$$\mathcal{M}_V = \sqrt{v\cdot p}C(\eta)(v\cdot\epsilon) + \frac{1}{\sqrt{v\cdot p}}D(\eta)(v\cdot\epsilon)\not{p} + \sqrt{v\cdot p}E(\eta)\not{\epsilon} + \frac{1}{\sqrt{v\cdot p}}F(\eta)\not{p}\not{\epsilon} \tag{185}$$

where the variable η has been defined in (178).

Similar to the case of the decays into a light pseudoscalar meson (184) may be used to relate various exclusive heavy to light processes in the kinematic range where the energy of the outgoing vector meson is small. For example, the semileptonic decays $D \to \rho e\nu$, $D \to K^* e\nu$ and $B \to \rho e\nu$ are related among themselves and all of them may be related to the rare heavy to light decays $B \to K^*\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ with $\ell = e, \mu$.

3.2.3 Heavy to light decays of baryons

Finally we comment on the heavy to light transitions of baryons. For the Λ-type heavy baryons spin symmetry relates different polarizations of the same particle and thus imposes interesting constraints. Consider for example the matrix element of an operator $\bar{\ell}\Gamma h_v$ between a heavy Λ_Q and a light spin-1/2 baryon B_ℓ. It is described by only two form factors,

$$\langle B_\ell(p)|\bar{\ell}\Gamma h_v|\Lambda_Q(v)\rangle = \bar{u}_\ell(p)\{F_1(v\cdot p) + \not{p}F_2(v\cdot p)\}\Gamma u_{\Lambda_Q}(v). \tag{186}$$

Thus in this particular case spin symmetry greatly reduces the number of independent Lorentz-invariant amplitudes which describe the heavy to light transitions.

This has some interesting implications for exclusive semileptonic Λ_c decays. For the case of a left handed current $\Gamma = \gamma_\mu(1-\gamma_5)$, the semileptonic decay $\Lambda_c \to \Lambda\ell\bar{\nu}_\ell$ is in general parametrized in terms of six form factors

$$\begin{aligned}\langle \Lambda(p)|\bar{q}\gamma_\mu(1-\gamma_5)c|\Lambda_c(v)\rangle &= \bar{u}(p)\left[f_1\gamma_\mu + if_2\sigma_{\mu\nu}q^\nu + f_3 q^\mu\right]u(p') \\ &+ \bar{u}(p)\left[g_1\gamma_\mu + ig_2\sigma_{\mu\nu}q^\nu + g_3 q^\mu\right]\gamma_5 u(p'),\end{aligned} \tag{187}$$

where $p' = m_{\Lambda_c}v$ is the momentum of the Λ_c whereas $q = m_{\Lambda_c}v - p$ is the momentum transfer. From this one defines the ratio G_A/G_V by

$$\frac{G_A}{G_V} = \frac{g_1(q^2=0)}{f_1(q^2=0)}. \tag{188}$$

In the heavy c quark limit one may relate the six form factors f_i and g_i $(i = 1, 2, 3)$ to the two form factors F_j $(j = 1, 2)$

$$f_1 = -g_1 = F_1 + \frac{m_\Lambda}{m_{\Lambda_c}} F_2 \tag{189}$$

$$f_2 = f_3 = -g_2 = -g_3 = \frac{1}{m_{\Lambda_c}} F_2 \tag{190}$$

from which one reads off $G_A/G_V = -1$. This ratio is accessible by measuring in semileptonic decays $\Lambda_c \to \lambda \ell \bar{\nu}_\ell$ the polarization variable α

$$\alpha = \frac{2G_A G_V}{G_A^2 + G_V^2} \tag{191}$$

which is predicted to be $\alpha = -1$ in the heavy c quark limit. The subleading corrections to the heavy c quark limit have been estimated and found to be small [54]

$$\alpha < -0.95, \tag{192}$$

and recent measurements yield

$$\alpha = -0.91 \pm 0.49 \quad \text{ARGUS [55]} \tag{193}$$

$$\alpha = -0.89^{+0.17+0.09}_{-0.11-0.05} \quad \text{CLEO[56]} \tag{194}$$

and are in satisfactory agreement with the theoretical predictions.

Recently the CLEO collaboration also measured the ratio of the form factors F_1 and F_2, averaged over phase space. Heavy quark symmetries do not fix this form factor ratio, at least not for a heavy to light decay, while for a heavy to heavy decay the form factor F_2 vanishes in the heavy mass limit for the final state quark. CLEO measures [57]

$$\left\langle \frac{F_2}{F_2} \right\rangle_{\text{phase space}} = -0.25 \pm 0.14 \pm 0.08 \tag{195}$$

which is in good agreement with model estimates [58].

3.3 Comment on Exclusive Non-Leptonic Decays

The effective weak Hamiltonian mediating nonleptonic decays consists of four quark operators, the matrix elements of which need to be calculated to obtain the amplitude for an exclusive nonleptonic decay. In constructing the effective hamiltonian from the standard model of weak interaction one integrates out the heavy weak bosons and by the usual machinery of effective theory one may calculate rediative corrections to resum logarithms of the mass M_W of the weak boson. These corrections are known to next-to-leading logarithmic precision [59].

At the scale of the bottom quark one may apply heavy quark effective theory to access also the logarithms of the bottom quark mass m_b and even of the charm mass m_c, both treated as perturbative scales. These corrections are known to leading logarithmic accuracy [60].

At present no theoretically clean method exists to evaluate these matrix elements. Furthermore, the restrictions due to symmetries (heavy quark symmetries as well as the symmetries of the light degrees of freedom) do not restrict these decays as it is the case for exclusive semileptonic decays [61] and hence the heavy mass limit has not added anything to our knowledge concerning exclusive nonleptonic decays.

In order to obtain some numbers to compare to data one commonly used approach is the so called factorization assumption. It has been discussed in [62] that for specific decays this assumption may hold, but in general factorization is completely ad hoc and may even be shown not to hold in the general case.

This assumption consists of rewriting the matrix element of the four quark operator as a product of two matrix elements of quark currents. Once this assumption is made, one may relate the the exclusive nonleptonic decays to the corresponding semileptonic ones and the meson decay constants [63].

Under this assumption one may thus make use of heavy quark symmetry as it restricts the form factors of semileptonic decays. We shall not discuss any details here since anything that can be said under the assumption of factorization is already covered in the sections on semileptonic decays.

4 Inclusive Decays

In order to describe inclusive decays in the framework of the $1/m_Q$ expansion one combines the method of the operator product expansion with heavy quark effective theory [8]-[15]. The approach applied here resembles very much the description of deep inelastic scattering.

The first step consists of an operator product expansion which yields an infinite sum of operators with increasing dimension. The dimensions of the operators are compensated by inverse powers of a large scale, which is in general of the order of the heavy mass scale. The decay probability is then given as forward matrix elements of these operators between the state of the decaying heavy hadron; these matrix elements still have a mass dependence, which then may be extracted in terms of a $1/m_Q$ expansion using HQET as for exclusive decays.

In the following sections we shall first develop the theoretical framework and then consider some applications to various processes. The method described below also allows us to deal with inclusive non-leptonic processes and hence in principle opens the possibility for a calculation of lifetimes and branching fractions in the framework of the $1/m_Q$ expansion. Applying this idea to the energy spectra of the charged lepton in inclusive semileptonic decays of heavy mesons, the relevant expansion parameter is not $1/m_Q$, but rather $1/(m_Q - 2E_\ell)$; the denominator is thus the energy release of the decay. In almost all phase space the energy release is of the order of the heavy mass; it is only in the endpoint region that it becomes small and hence the expansion breaks down. This problem may be fixed by a resummation of terms in the operator product expansion, which strongly resembles the summation corresponding to leading twist in deep inelastic scattering. Analogously to the parton-distribution function, a universal function appears, which determines all inclusive heavy-to-light decays.

4.1 Operator Product Expansion

This section is devoted to a short introduction into the meothod employed to set up a $1/m_Q$ expansion for inclusive processes. The effective Hamiltonian for a decay of a heavy (down-type) quark is in general linear in the decaying heavy flavoured quark

$$\mathcal{H}_{eff} = \bar{Q} R \tag{196}$$

where the operator R describes the decay products. In the following we shall consider semileptonic decays, for which

$$R_{sl} = \frac{G_F}{\sqrt{2}} V_{Qq}\ \gamma_\mu(1-\gamma_5)q\ (\bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\ell), \tag{197}$$

where q is an up-type quark (c or u, since we shall consider b decays). Similarly, for nonleptonic decays the Cabbibo allowed contribution corresponds to

$$\begin{aligned} R_{nl} = \frac{G_F}{2\sqrt{2}} V_{Qq} V^*_{q'q''} & [(C_+(m_b) + C_-(m_b))\gamma_\mu(1-\gamma_5)q\ (\bar{q}'\gamma^\mu(1-\gamma_5)q'') \\ & +(C_+(m_b) - C_-(m_b))\gamma_\mu(1-\gamma_5)q''\ (\bar{q}'\gamma^\mu(1-\gamma_5)q)], \end{aligned} \tag{198}$$

where q' (q'') is a down-type (up-type) quark and V_{Qq} the corresponding CKM matrix element. The coefficients $C_\pm(m_b)$ are the QCD corrections obtained from the renormalization group running between M_W and m_b; in leading logarithmic approximation these coefficients are [59]

$$C_\pm(m_b) = [\frac{\alpha_S(M_W^2)}{\alpha_S(m_b^2)}]^{\gamma_\pm},\ \text{with}\ \gamma_+ = \frac{6}{33-2N_f} = -\frac{1}{2}\gamma_- \tag{199}$$

where $\alpha_s(\mu)$ is the one-loop expression (76) for the running coupling coupling constant of QCD.

Finally, for radiative rare decays we have

$$R_{rare} = \frac{G_F}{\sqrt{2}} V_{tb} V^*_{ts} C_7(m_b) \frac{e}{16\pi^2} m_b \sigma_{\mu\nu}(1+\gamma_5) s F^{\mu\nu} \tag{200}$$

where $C_7(m_b)$ is again a coefficient obtained from running between M_W and m_b. Its value is $C_7(m_b) \sim 0.3$, the corresponding analytical expression may be found in [64].

The inclusive decay rate for a heavy hadron H containing the quark Q may be related to a forward matrix element by

$$\begin{aligned} \Gamma \propto & \sum_X (2\pi)^4 \delta^4(P_B - P_X) |\langle X | \mathcal{H}_{eff} | H(v)\rangle|^2 = \int d^4x\, \langle H(v)|\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)|H(v)\rangle \\ = & \ 2\ \mathrm{Im} \int d^4x\, \langle H(v)|T\{\mathcal{H}_{eff}(x)\mathcal{H}^\dagger_{eff}(0)\}|H(v)\rangle. \end{aligned} \tag{201}$$

where $|X\rangle$ is the final state, which is summed over to obtain the inclusive rate.

The matrix element appearing in (201) contains a large scale, namely the mass of the heavy quark. The first step towards a $1/m_Q$ expansion is to make this large scale explicit. This may be done by a phase redefinition. This leads to

$$\Gamma \propto 2\ \mathrm{Im} \int d^4x\, e^{-im_Q vx}\, \langle H(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}|H(v)\rangle \tag{202}$$

where

$$\widetilde{\mathcal{H}}_{eff} = \bar{Q}_v R \qquad Q_v = e^{-im_Q vx} Q \tag{203}$$

This relation exhibits the similarity between the cross-section calculation in deep inelastic scattering and the present approach to total rates. In deep inelastic scattering there appears a large scale, which is the momentum transfer to the leptons, while here the mass of the heavy quark appears as a large scale.

The next step is to perform an operator product expansion of the product of the two Hamiltonians. After the phase redefinition the remaining matrix element does not involve large momenta of the order of the heavy quark mass any more and hence a short-distance expansion becomes useful if the mass m_Q is large compared to the scale $\bar{\Lambda}$ determining the matrix element. The next step is thus to perform an operator-product expansion, which has the general form

$$\int d^4x\, e^{im_Q vx} \langle H(v)|T\{\widetilde{\mathcal{H}}_{eff}(x)\widetilde{\mathcal{H}}^\dagger_{eff}(0)\}|H(v)\rangle = \sum_{n=0}^{\infty} \left(\frac{1}{2m_Q}\right)^n \hat{C}_{n+3}(\mu)\langle H(v)|\mathcal{O}_{n+3}|H(v)\rangle_\mu,$$

where $\mathcal{O}_n$ are operators of dimension n, with their matrix elements renormalized at scale μ, and $\hat{C}_n$ are the corresponding Wilson coefficients. These coefficients encode the short distance physics related to the heavy quark mass scale and may be calculated in perturbation theory. All long distance contributions connected to the hadronic scale $\bar{\Lambda}$ are contained in the matrix elements of the operators $\mathcal{O}_{n+3}$.

Still the matrix elements of $\mathcal{O}_{n+3}$ are not independent of the heavy quark mass scale, but this mass dependence may be expanded in powers of $1/m_Q$ by means of heavy quark effective theory. This is achived by expanding the heavy quark fields appearing in the operators $\mathcal{O}_n$ using (16) (or, equivalently, (23)) as well as the states by including the corrections to the Lagrangian given in (17) (or (24)) as time-ordered products. In this way the mass dependence of the total decay rate may be accessed completely within an expansion in $1/m_Q$.

The lowest-order term of the operator product expansion are the dimension-3 operators. Due to Lorentz invariance and parity there are only two combinations which may appear, namely $\bar{Q}_v \not{v} Q_v$ or $\bar{Q}_v Q_v$. Note that the Q_v operators differ from the full QCD operators only by a phase redefinition, and hence $\bar{Q}_v \not{v} Q_v = \bar{Q} \not{v} Q$ and $\bar{Q}_v Q_v = \bar{Q} Q$. The first combination is proportional to the Q-number current $\bar{Q}\gamma_\mu Q$, which is normalized even in full QCD, while the second differs from the first one only by terms of order $1/m_Q^2$

$$\bar{Q}_v Q_v = v_\mu \bar{Q}_v \gamma_\mu Q_v + \frac{1}{2m_Q^2}\bar{h}_v \left[(iD)^2 - (ivD)^2 + \frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}\right] h_v + \mathcal{O}(1/m_Q^3). \tag{204}$$

where $G_{\mu\nu}$ is the gluon field strength.

Thus the matrix elements of the dimension-3 contribution is known to be normalized; in the standard normalization of the states this implies

$$\langle H(v)|\mathcal{O}_3|H(v)\rangle = \langle H(v)|\bar{Q}_v \not{v} H_v|B(v)\rangle = 2m_H \tag{205}$$

where m_H is the mass of the heavy hadron. To lowest order in the heavy mass expansion we may furthermore replace $m_B = m_Q$ and hence we may evaluate the leading term in the $1/m_q$ expansion without any hadronic uncertainty. Generically the dimension-3 contribution yields the free quark decay rate. This has been previously used as a model for inclusive decays, but now it turns out to be the first term in a systematic $1/m_Q$ expansion of total rates.

A dimension-four operators contains an additional covariant derivative, and thus one has matrix elements of the type

$$\langle H(v)|\mathcal{O}_4|H(v)\rangle \propto \langle H(v)|\bar{Q}_v \Gamma D_\mu Q_v|H(v)\rangle = \mathcal{A}_\Gamma v_\mu \tag{206}$$

Since the equations of motion apply for this tree level matrix element, one finds that the constant $\mathcal{A}_\Gamma$ has to vanish, and thus there are no dimension-four contributions. This statement is completely equivalent to Luke's theorem [36], since we are considering a forward matrix element, i.e. a matrix element at zero recoil [65].

The first non trivial non-perturbative contribution comes from dimension-5 operators and are of order $1/m_Q^2$. For mesonic decays there are only the two parameters λ_1 and λ_2 given in

(108) and (109), which correspond to matrix elements of the subleading terms of the Lagrangian. They parametrize the non-perturbative input in the order $1/m_Q^2$. For Λ_Q-type baryons the parameter λ_2 vanishes due to heavy quark spin symmetry, while the kinetic energy parameter λ_1 is nonzero as well. In the framework of the $1/m_Q$ expansion this leads to a difference in lifetimes between mesons and baryons.

4.2 Semileptonic Decays

In the following subsections we shall apply the method outlined above to inclusive semileptonic processes. Before quoting the results some remarks concerning the reliability of the method are in order.

When considering semileptonic processes the effective hamiltonian may be split into a hadronic current and a leptonic one. Hence the operator product expansion is in fact an expansion for the two hadronic currents which takes the form

$$H^{\mu\nu} = \int d^4x \, e^{i(m_Q v - q)x} \, \langle H(v)|T\{\tilde{J}^{\mu}_{had}(x)\tilde{J}^{\dagger\nu}_{had}(0)\}|H(v)\rangle \tag{207}$$

where the coefficients A depend on q, the momentum transferred to the leptons. The hadronic tensor $H_{\mu\nu}$ may be expressed in terms of scalar functions which will depend on vq and q^2. As a function of these two variables the analytic structure of $H_{\mu\nu}$ may be studied to gain some insight into the reliability of the operator product expansion. For fixed q^2 we consider $H_{\mu\nu}$ as a function of vq: $H_{\mu\nu}$ exhibits a cut along the real axis for values $\sqrt{q^2} < vq < (m_Q^2 + q^2 - m_*^2)/(2m_b)$ corresponding (for $vq > 0$) to the decay process we are considering. Here m_* is the mass of the lowest lying hadronic state that may appear as a decay product. There are also other processes yielding the analytical structure depicted in fig.8. Calculating the energy spectrum or the total rate involves an integration over the momentum of the neutrino which corresponds to an integration over the variable q with a weight given by the leptonic tensor. This integration samples the cut of $H_{\mu\nu}$ and may be represented by a contour integration along the curve C as shown in fig.8.

Close to the upper edge of the left hand cut $H_{\mu\nu}$ is dominated by resonances and hence is not well represented by the partonic expression. On the other hand, the integration (at least in almost all phase space) over q^2 induces sufficient "smearing" such that the intergals of the "true" (i.e. hadronic) $H_{\mu\nu}$ may be identified with the corresponding integral of the partonic expression for $H_{\mu\nu}$. This assumption, namely that weighted averages of the hadronic and partonic expressions are equal, is usually called global duality.

The notion of global duality has to be contrasted with the notion of local duality, where the equality between partonic and hadronic quantities is assumed not only in the average, but even for the function itself. As we shall see below, local dualtity is needed for the description of inclusive nonleptonic processes, while the milder assumption of global dualitity is needed for semileptonic processes, for both the total rate as well as the energy spectrum.

4.2.1 Total Rates

Inserting R_{sl} as given in (197) one obtains for the total inclusive semileptonic decay rate $B \to X_c \ell \nu$

$$\Gamma(B \to X_c \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[\left(1 + \frac{\lambda_1}{2m_c^2}\right) f_1\left(\frac{m_c}{m_b}\right) - \frac{9\lambda_2}{2m_c^2} f_2\left(\frac{m_c}{m_b}\right) \right], \tag{208}$$

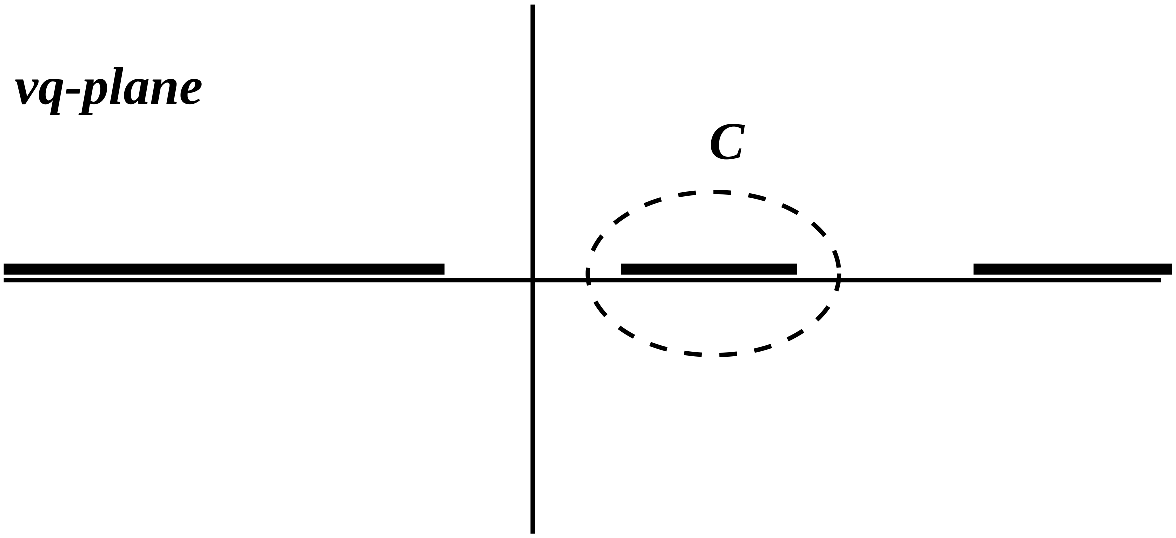

Figure 8: Sketch of the complex vq plane for fixed $q^2 > 0$ for inclusive semileptonic decays. The thick lines along the real axis indicate the branch cuts of the hadronic tensor. They correspond to $vq < -\sqrt{q^2}$, $\sqrt{q^2} < vq < (m_Q^2 + q^2 - m_*2)/(2m_Q)$ and $vq > ...$, where m_* is the mass of the lightest hadronic final state which may appear in the inclusive decay. The countour C is the integration contour corresponding to the integration over the neutrino momentum.

where the two f_j are phase-space functions

$$\begin{aligned} f_1(x) &= 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log x, \\ f_2(x) &= 1 - \frac{8}{3}x^2 - 8x^4 + 8x^6 + \frac{5}{3}x^8 + 8x^4 \log x. \end{aligned} \tag{209}$$

The result for $B \to X_u \ell \nu_\ell$ is obtained from (208) as the limit $m_c \to 0$ and the replacement $V_{cb} \to V_{ub}$

$$\Gamma(B \to X_u \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left[1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} \right]. \tag{210}$$

As discussed above, the leading non-perturbative corrections in (208) and (210) are parametrized by λ_1 and λ_2. Estimates for these parameters have been discussed in section 2; in order to estimate the total effect of the non-perturbative effects we shall insert a range of values $-0.3 > \lambda_1 > -0.6$ GeV^2; from this we obtain

$$\frac{\lambda_1 - 9\lambda_2}{2m_b^2} \sim -(3 \cdots 4)\% \tag{211}$$

This means that the non-perturbative contributions are small, in particular compared to the perturbative ones, which have been calculated some time ago [66, 67]. For the decay $B \to X_u \ell \bar{\nu}_\ell$ the lowest order QCD corrections are given by

$$\Gamma(B \to X_u \ell \bar{\nu}_\ell) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{ub}|^2 \left[1 + \frac{2\alpha}{3\pi} \left(\frac{25}{4} - \pi^2 \right) \right] = 0.85 |V_{ub}|^2 \Gamma_b, \tag{212}$$

and thus the typical size of QCD radiative corrections is of the order of ten to twenty percent.

4.3 Energy Spectrum

The method of the operator-product expansion may also be used to obtain the non-perturbative corrections to the charged lepton energy spectrum [10]. In this case the operator product expansion is applied not to the full effective Hamiltonian, but rather only to the hadronic currents. The rate is written as a product of the hadronic and leptonic tensor

$$d\Gamma = \frac{G_F^2}{4m_B}|V_{Qq}|^2 W_{\mu\nu}\Lambda^{\mu\nu} d(PS), \tag{213}$$

where $d(PS)$ is the phase-space differential. The short-distance expansion is then performed for the two currents appearing in the hadronic tensor. Redefining the phase of the heavy-quark fields as in (9) one finds that the momentum transfer variable relevant for the short-distance expansion is $m_Q v - q$, where q is the momentum transfer to the leptons. After integration over the momentum of the neutrino the expansion variable is the energy release $(m_Q/2) - E_\ell$, where E_ℓ is the energy of the charged lepton.

The structure of the expansion for the spectrum is identical to the one of the total rate. The contribution of the dimension-3 operators yields the free-quark decay spectrum, there are no contributions from dimension-4 operators, and the $1/m_b^2$ corrections are parametrized in terms of λ_1 and λ_2. Calculating the spectrum for $B \to X_c \ell\nu$ yields [11]-[14]

$$\begin{aligned} \frac{d\Gamma}{dy} = & \frac{G_F^2 \mid V_{cb}|^2 m_b^5}{192\pi^3}\Theta(1-y-\rho)y^2\left[\left\{3(1-\rho)(1-R^2)-2y(1-R^3)\right\}\right. \\ & +\frac{\lambda_1}{[m_b(1-y)]^2}(3R^2-4R^3)-\frac{\lambda_1}{m_b^2(1-y)}(R^2-2R^3) \\ & -\frac{3\lambda_2}{m_b^2(1-y)}(2R+3R^2-5R^3)+\frac{\lambda_1}{3m_b^2}[5y-2(3-\rho)R^2+4R^3] \\ & \left.+\frac{\lambda_2}{m_b^2}[(6+5y)-12R-(9-5\rho)R^2+10R^3]\right]+\mathcal{O}\left[(\Lambda/[m_b(1-y)])^3\right] \end{aligned} \tag{214}$$

where we have defined

$$\rho = \left(\frac{m_c}{m_b}\right)^2 \qquad R = \frac{\rho}{1-y} \tag{215}$$

and

$$y = 2E_\ell/m_b \tag{216}$$

is the rescaled energy of the charged lepton.

This expression is somewhat complicated, but it simplifies for the decay $B \to X_u \ell\nu$ since then the mass of the quark in the final state may be neglected. One finds

$$\begin{aligned} \frac{d\Gamma}{dy} = & \frac{G_F^2 \mid V_{ub}|^2 m_b^5}{192\pi^3}\left[\left(2y^2(3-2y)+\frac{10y^2}{3}\frac{\lambda_1}{m_b^2}+2y(6+5y)\frac{\lambda_2}{m_b^2}\right)\Theta(1-y)\right. \\ & \left.-\frac{\lambda_1+33\lambda_2}{3m_b^2}\delta(1-y)-\frac{\lambda_1}{3m_b^2}\delta'(1-y)\right] \end{aligned} \tag{217}$$

Figure 9 shows the distributions for inclusive semileptonic decays of B mesons. The spectrum close to the endpoint, where the lepton energy becomes maximal, exhibits a sharp spike as $y \to y_{max}$. In this region we have we have

$$\frac{d\Gamma}{dy} \propto \Theta(1-y-\rho)\left[2+\frac{\lambda_1}{(m_Q(1-y))^2}\left(\frac{\rho}{1-\rho}\right)^2\left\{3-4\left(\frac{\rho}{1-\rho}\right)\right\}\right], \tag{218}$$

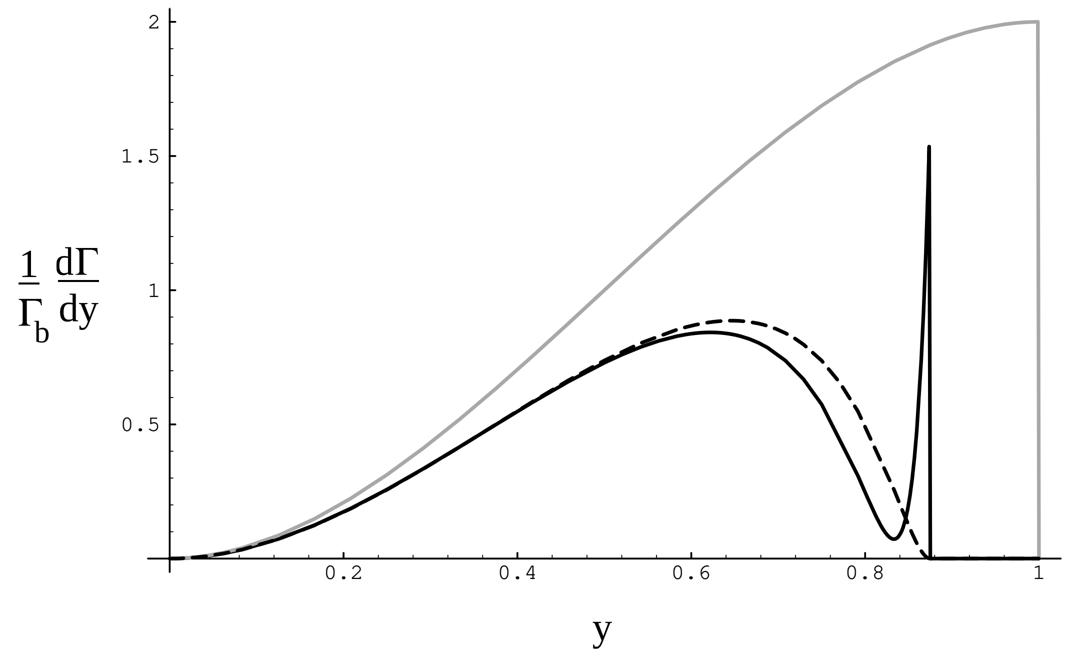

Figure 9: The electron spectrum for free quark $b \to c$ decay (dashed line), free quark $b \to u$ decay (grey line), and $B \to X_c e\bar{\nu}_e$ decay including $1/m_b^2$ corrections (solid line) with $\lambda_1 = -0.5$ GeV^2 and $\lambda_2 = 0.12$ GeV^2. The figure is from [13].

which behaves like δ-functions and its derivatives as $\rho \to 0$, which can be seen in (217). This behaviour indicates a breakdown of the operator product expansion close to the endpoint, since for the spectra the expansion parameter is not $1/m_Q$, but rather $1/(m_Q - qv)$, which becomes $1/(m_Q[1-y])$ after the integration over the neutrino momentum. In order to obtain a description of the endpoint region, one has to perform some resummation of the operator product expansion.

4.3.1 The Endpoint Region

Very close to the endpoint of the inclusive semileptonic decay spectra only a few resonances contribute. In this resonance region one may not expect to have a good description of the spectrum using an approach based on parton-hadron duality; here a sum over a few resonances will be appropriate.

In the variable y the size of this resonance region is however of the order of $(\bar{\Lambda}/m_Q)^2$ and thus small. In a larger region of the order $\bar{\Lambda}/m_Q$, which we shall call the endpoint region, many resonances contribute and one may hope to describe the spectrum in this region using parton-hadron duality.

In order to obtain an adequate description of this region we consider again the correlator of two currents and contract the light quark field

$$T(q^2, v\cdot q) = \int \mathrm{d}^4x\, e^{iQ\cdot x} \langle B(v)|\, T\left\{\bar{h}_v(x)\,\Gamma_1\, S_q(x,0)\,\Gamma_2\, h_v(0)\right\} |B(v)\rangle\,. \qquad (219)$$

with $Q = q - m_b v$ and Γ_1 and Γ_2 two arbitrary Dirac matrices. Here $S_q(x,0)$ is the propagator of the q-quark in the background field of the light constituents of the B-meson. The Fourier transform of $S_q(x,0)$ has the form

$$\begin{aligned} S_q(Q) &= \frac{1}{\not{Q} + i\not{D} + i\epsilon} \\ &= (\not{Q} + i\not{D}) \frac{1}{(Q^2 + 2iQ\cdot D - -\not{D}\not{D} + i\epsilon)} . \end{aligned} \tag{220}$$

Note that a derivative acting on the rescaled heavy quark field corresponds to the residual momentum $k = p_b - m_b v$. Since the residual momentum results from the interactions of the heavy quark with light degrees of freedom, it is of order Λ_{QCD}.

We will be interested in the region where the components of Q are large (of order m_b), since otherwise the operator product expansion breaks down. To leading order in the heavy mass expansion, we can then neglect the covariant derivative in the numerator, as well as the term containing two derivatives in the denominator. This gives

$$S_q(Q) = \frac{\not{Q}}{Q^2 + 2iQ\cdot D + i\epsilon} + \mathcal{O}(\Lambda_{QCD}/m_b) . \tag{221}$$

The term in the denominator containing the covariant derivative is of order Λm_b. To see whether it is important, we have to distinguish different kinematic regions. When $Q^2 \sim m_b^2$, the derivative term is suppressed by a factor Λ/m_b. To leading order, the propagator corresponds to the propagator of a free quark with momentum Q and we recover the standard operator product expansion as considered above.

However, close to the endpoint, Q^2 is small although the components of this vector are large. In the kinematic region where $Q^2 \sim \Lambda_{QCD} m_b$ (which we shall call endpoint region in the following), it is not possible to neglect the derivative term, since it is of the same order as the Q^2 term. For even smaller values $Q^2 - m_q^2 \sim \Lambda m_q$, the derivative term becomes the dominant term in the denominator, and the operator product expansion breaks down. This is the resonance region, where the invariant mass Q^2 of the hadronic final state is close to the mass-shell of the lowest-lying resonances containing a q-quark

It is possible to simplify the result further. To this end, we write $Q = (v\cdot Q)(n + \delta n)$ and choose the vectors n and δn such that n is a null-vector on the forward light cone satisfying $n^2 = 0$ and $n \cdot v = 1$, and thus $v \cdot \delta n = 0$. In the region where the derivative term in (221) is important, i.e. for $v\cdot Q$ of order m_b but Q^2 of order Λm_b, we have $n\cdot \delta n \sim \Lambda/m_b$. As long as we restrict ourselves to the leading term in the large-m_b limit, we can thus neglect δn. This leads to the final expression for the effective background-field propagator:

$$S_q(Q) \simeq \int \mathrm{d}k_+ \, \delta(k_+ - iD_+) \frac{\not{Q}}{Q^2 + 2k_+ \, v\cdot Q + i\epsilon} , \tag{222}$$

where we define $n\cdot D \equiv D_+$. In light-cone gauge, $n\cdot A \equiv A_+ = 0$, the operator iD_+ reduces to $i\partial_+$ and corresponds to the light-cone residual momentum k_+ of the heavy quark inside the b-meson.

The δ-function appearing in (222) is still operator valued, since it contains a covariant derivative. To calculate the rate one has to take a matrix element which defines a distribution function of the residual light cone momentum of the heavy quark

$$f(k_+) = \langle B(v)| \, \bar{h}_v \, \delta(k_+ - iD_+) \, h_v \, |B(v)\rangle \tag{223}$$

which corresponds in the language of deep inelastic scattering to the leading-twist structure function that determines the probability to find a b-quark with light-cone residual momentum k_+ inside the B-meson. We use a mass-independent normalization of states such that $\langle B(v)|\,\bar{h}_v\,h_v\,|B(v)\rangle = 1$. It then follows that the structure function is normalized to unity:

$$\int \mathrm{d}k_+\, f(k_+) = 1\,. \tag{224}$$

It will later be convenient to introduce the Fourier transform of the structure function, which is given by the forward matrix element of a bilocal operator:[5]

$$\begin{aligned}\tilde{f}(t) &= \int \mathrm{d}k_+\, e^{-ik_+t}\, f(k_+) \\ &= \langle B(v)|\,\bar{h}_v(0)\,P\exp\Big[-i\int\limits_0^t \mathrm{d}u\, A_+(un)\Big]\, h_v(tn)\,|B(v)\rangle\,.\end{aligned} \tag{225}$$

This function obeys the normalization condition $\tilde{f}(0) = 1$. Note that the path-ordered exponential is absent in light-cone gauge.

From the definition of the distribution function f we may infer some interesting properties. The moments of $f(k_+)$ are given by forward matrix elements of leading-twist, higher-dimension operators in the heavy quark effective theory. They form a set of low-energy parameters A_n defined by

$$A_n = \int \mathrm{d}k_+\, k_+^n\, f(k_+) = i^n\, \tilde{f}^{(n)}(0) = \langle B(v)|\,\bar{h}_v\,(iD_+)^n\,h_v\,|B(v)\rangle\,, \tag{226}$$

where $\tilde{f}^{(n)}(0)$ is a shorthand notation for the n-th derivative of $\tilde{f}(t)$ evaluated at $t = 0$. Using the equation of motion of the heavy quark effective theory, one obtains for the first three moments

$$\begin{aligned} A_1 &= 0\,, \qquad A_2 = -\frac{\lambda_1}{3}\,, \\ A_3 &= -\frac{v_\nu}{6}\,\langle B(v)|\,\bar{h}_v\,[D_\mu, g_s G^{\mu\nu}]\,h_v\,|B(v)\rangle\,, \end{aligned} \tag{227}$$

where $g_s G^{\mu\nu} = i[D^\mu, D^\nu]$ is the gluon field-strength tensor, and λ_1 has been introduced in (108).

The function f is a universal distribution function, which appears in all heavy-to-light inclusive decays; another example is the decay $B \to X_s\gamma$ [69, 32], where this function determines the photon-energy spectrum in a region of order $1/m_Q$ around the K^* peak.

In principle f has to be determined by other methods than the $1/m_Q$ expansion, e.g. from lattice calculations or from a model, or it has to be determined from experiment by measuring the photon spectrum in $B \to X_s\gamma$ or the lepton spectrum in $B \to X_u \ell\bar{\nu}$. In the context of the model ACCMM model [67] f has been calculated in [70].

In addition to the moments also the support of f is known; it is nonvanishing only for $-\infty < k_+ < \bar{\Lambda}$. A simple one parameter model incorporating these properties has been suggested in [71]

$$f(k_+) = \frac{32}{\pi^2\bar{\Lambda}}\,(1-x)^2 \exp\Big\{-\frac{4}{\pi}\,(1-x)^2\Big\}\,\Theta(1-x), \tag{228}$$

where $x = k_+/\bar{\Lambda}$, and the choice $\bar{\Lambda} = 570$ MeV yields reasonable values for the moments.

[5]The definition of a heavy quark structure function as the Fourier transform of the bilocal operator in (225) was used, in a different context, in Ref. [68].

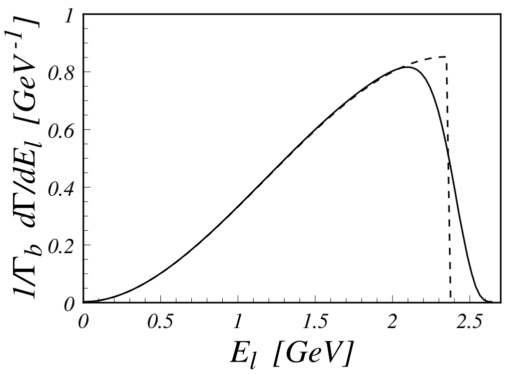

Figure 10: Charged-lepton spectrum in $B \to X_u \ell \bar{\nu}$ decays. The solid line is (229) with the ansatz (228), the dashed line shows the prediction of the free-quark decay model. The figure is from [71].

The spectrum may be calculated in terms of the distribution function f and one finds (assuming the final state quark to be massless)

$$\begin{aligned} \frac{d\Gamma}{dE_\ell} &= \frac{G_F^2 \mid V_{qb} \mid^2}{12\pi^3} E_\ell^2 \int dk_+ \, f(k_+) \, \Theta\Big[m_b^* - 2\,E_\ell\Big] \\ &\times \Big\{ 3m_b^{*2} - 4m_b^* \, E_\ell \Big\} . \end{aligned} \tag{229}$$

Note that the heavy quark mass m_b does no longer appear explicitly. For this reason, and in particular when the focus is on the endpoint region, it would be unnatural to introduce the rescaled lepton energy $y = 2E_\ell/m_b$. Hence, we will hereafter present our results as function of the lepton energy E_ℓ, which is the quantity that is actually measured in experiments. Note, in particular, that (to the order we are working) the maximum value of the lepton energy is correctly reproduced.

The result (229) represents a resummation of the most singular contributions in the endpoint region, which correspond to a resummation of the highest derivatives of δ-functions as they appear in (217), where the explixit caslulation to order $1/m_Q^2$ has been performed. In order to illustrate the effect of the convolution (229) we show in fig. 10 the spectrum for $B \to X_u \ell \nu_\ell$ using the ansatz (228).

Including the non-perturbative effects yields a reasonably behaved spectrum in the endpoint region and the δ-function-like singularities have disappeared. Furthermore, the spectrum now extends beyond the parton model endpoint; it is shifted from $E_\ell^{max} = m_Q/2$ to the physical endpoint $E_\ell^{max} = M_H/2$, since f is non-vanishing for positive values of $k_+ < \bar{\Lambda} = M_H - m_Q$.

4.3.2 Perturbative Corrections

On top of the large nonperturbative corrections in the endpoint region there are also large perturbative corrections which have been known for a long time [66]. While the corrections to the total rate are of he expected size (see 212), the endpoint region contains large logarithms, of which the leading ones are the Sudakov-like double logarithms. Up to terms vanishing at the endpoint we have to order α_2

$$\frac{d\Gamma}{dy} = \frac{d\Gamma^{(0)}}{dy} \left[1 - \frac{2\alpha_s}{3\pi} \left(\ln^2(1-y) + \frac{31}{6} \ln(1-y) + \frac{5}{4} + \pi^2 \right) \right], \tag{230}$$

where $y = 2E_\ell/m_b$ is again the rescaled energy variable and $\Gamma^{(0)}$ is the parton model rate which becomes clsoe to the endpoint

$$\frac{1}{\Gamma_b}\frac{d\Gamma^{(0)}}{dy} = 2y^2(3-2y)\Theta(1-y) \to 2\Theta(1-y) \tag{231}$$

The doubly logarithmic terms are believed to exponentiate, yielding

$$\frac{d\Gamma}{dy} = \frac{d\Gamma^{(0)}}{dy}\exp\left[-\frac{2\alpha_s}{3\pi}\ln^2(1-y)\right]. \tag{232}$$

up to singly logarithmic terms which have been neglected in (232). Note that the exponentiation strongly reduces the rate close to the endpoint resulting in a strong modification of the parton model result.

Since singly logarithmic terms are ommitted, the choice of the scale μ in α_s is not obvious and various suggsetions have been made. In particular the choice $\mu^2 = m_b^2(1-y)$ yields a drastic modification of the spectrum in the endpoint region; using the one loop result for α_s we have

$$\frac{d\Gamma}{dy} = \frac{d\Gamma^{(0)}}{dy}\exp\left[-\frac{8}{25}\frac{\ln^2(1-y)}{\ln[(m_b^2/\Lambda_{QCD}^2)(1-y)]}\right]. \tag{233}$$

yielding a strong damping in the endpoint region.

Unlike the nonperturbative corrections the perturbative ones cannot shift the endpoint of the spectrom away from the one given by the parton model. Still both types, the perturbative and the nonperturbative corrections are strongly entangled in the endpoint region and a simultaneous treatment of both is difficult [72].

4.4 Non-Leptonic Decays

In contrast to exclusive nonleptonic processes the $1/m_Q$ expansion allows a systematic, QCD-based description of fully inclusive nonleptonic decays. In this way one may indeed calculate lifetimes and branching fractions.

We shall first point out some points concerning the operator product expansion for inclusive nonleptonic decays and then consider a few applications.

4.4.1 Total Rate

The method of combining the operator product expansion with the $1/m_Q$ expansion for the matrix elements may also be applied to total rates for inclusive nonleptonic processes. In order to discuss the assumption which have to be used we write

$$\Phi(q^2, vq)\int d^4x\, e^{-iqx}\langle H(v)|T\{\mathcal{H}_{eff}(x)\mathcal{H}_{eff}^\dagger(0)\}|H(v)\rangle. \tag{234}$$

which is related to the total rate by $\Gamma \propto \mathrm{Im}\Phi(0,0)$. The resason why the variable q^2 is introduced is that one may study the analytical structure of the correlator of two effective hamiltonians. Similarly to the semileptonic case we consider Φ as a function of vq for fixed q^2. The analytical structure looke very similar and is depcited in fig.11.

However, there is an important difference between semileptonic decays and nonleptonic processes. While in the semileptonic case the integration over the neutrino momentum corresponded to a contour integration in the complex vq plane, in the nonleptonic case one has

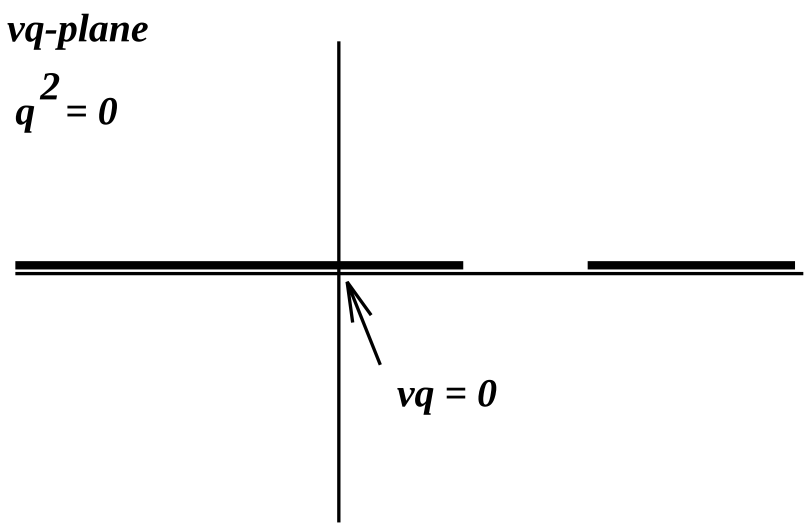

Figure 11: Sketch of the complex vq plane for $q^2 = 0$ for inclusive nonleptonic decays. The thick lines along the real axis indicate the branch cuts of the function Φ. They correspond to $vq < (m_Q^2 - m_* 2)/(2m_Q)$ and $vq > ...$, where m_* is the mass of the lightest hadronic final state which may appear in the inclusive decay. The arrow points at $vq = 0$ which is the point to be evaluated for the rate. The resonance region is at the right edge of the left-hand cut.

to evaluate Φ at a specific point $q^2 = 0$ and $vq = 0$ (equivalent to $q = 0$). In other words, there is no "smearing" in the nonleptonic case and hence one has to use local duality. Although this is in general a dangerous assumption, in the present case it seems to be justified, since Φ is evaluated at a point far away from the resonance region, where (for $q^2 = 0$) $vq = (m_Q^2 - m_*^2)/(2m_Q) = \mathcal{O}(m_Q)$ [73]. Still it is generally believed that the application of the operator product expansion is not as well justified as in the semileptonic case.

4.4.2 Lifetimes of $B^\pm$, B^0 and Λ_b

The subject of heavy hadron lifetimes is strongly related to nonleptonic processes, which have been considered already some time ago [74, 75]; however, the systematic application of the $1/m_Q$ expansion has turned many assumptions into quantitative arguments. As outlined in the last section the $1/m_Q$ expansion allows to calculate total rates, even for non-leptonic processes, and hence a QCD-based calculation of lifetimes becomes possible.

Mainly due to the LEP experiments the data on b hadron lifetimes is quite precise. In particular the lifetime ratios are known with precisions well below ten percent [76]

$$
\begin{aligned}
\frac{\tau(B^-)}{\tau(B_d)} &= 1.02 \pm 0.04\,, \\
\frac{\tau(B_s)}{\tau(B_d)} &= 1.01 \pm 0.07\,, \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} &= 0.78 \pm 0.05\,. \qquad (235)
\end{aligned}
$$

where $\tau(B_s)$ is the averaged B_s-meson lifetime.

To leading order in the $1/m_Q$ expansion all lifetime ratios are unity, since the lifetimes are then determined by the free quark decay. If we assume light flavour symmetry for the matrix elements λ_1 and λ_2, the corrections to the ratio between two meson lifetimes has to be of order $1/m_Q^3$ and is hence given by dimension six operators, which we have not yet discussed in the previous sections. Since the value of λ_1 is different in a meson and a baryon and $\lambda_2 = 0$ for a Λ_Q baryon, we expect

$$\begin{aligned}
\frac{\tau(B^-)}{\tau(B_d)} &= 1 + O(1/m_b^3)\,, \\
\frac{\tau(B_s)}{\tau(B_d)} &= 1 + O(1/m_b^3)\,, \\
\frac{\tau(\Lambda_b)}{\tau(B_d)} &= 1 + O(1/m_b^2)
\end{aligned} \tag{236}$$

and one already recognizes a potential problem with the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$.

In order to discuss the lifetime ratios for mesons one needs to consider higher orders in the $1/m_Q$ expansion. Unfortunately, in general the number of dimension six operators number of dimension six operators that appear at order $1/m_Q^3$ is quite large. However, not all of these operators contribute to lifetime ratios, since they have to be sensitive to the light quark inside the heavy hadron.

These types of spectator effects have been reconsidered recently in [77] and we shall follow this line of argument. Although intuitively these dimension six operators seem to yield only small contributions, their effect is enhanced by a phase space factor. The non-spectator effects typically yield at the partonic level at least a three particle phase space, while the spectator effects have to leading order only a two particle final state. This yields an enhancement of these contributions by a phase space factor $16\pi^2$. Qualitatively, this means that the $1/m_Q$ expansion for the lifetime ratios takes the form

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + \frac{1}{m_b^3}\left[a_0 + \frac{1}{m_b}a_1 + \cdots\right] + \frac{16\pi^2}{m_b^3}\left[b_0 + \frac{1}{m_b}b_1 + \cdots\right] \tag{237}$$

and hence the coefficient b_0 is the leading contribution to the lifetime ratio.

We start from the effective hamiltonian for nonleptonic weak decays (see also (198)

$$\begin{aligned}
H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{cb} \Big\{ &(C_+(m_b) + C_-(m_b)) \left[\bar{d}'_L\gamma_\mu u_L\, \bar{c}_L\gamma^\mu b_L + \bar{s}'_L\gamma_\mu c_L\, \bar{c}_L\gamma^\mu b_L\right] \\
&+ (C_+(m_b) - C_-(m_b)) \left[\bar{c}_L\gamma_\mu u_L\, \bar{d}'_L\gamma^\mu b_L + \bar{c}_L\gamma_\mu c_L\, \bar{s}'_L\gamma^\mu b_L\right] + \text{h.c.}\,,
\end{aligned}$$

where $d' = d\cos\theta_c + s\sin\theta_c$ and $s' = s\cos\theta_c - d\sin\theta_c$ are the Cabibbo-rotated down type quarks and $C_+(\mu)$ and $C_-(\mu)$ are the Wilson coefficients from the running from M_W down to the scale μ which have been given to leading logarithmic sccuracy in (199).

The correction to the decay rate due to the spectator effects may be expressed in terms of four quark operators, which we choose to be

$$\begin{aligned}
O^q_{V-A} &= \bar{b}_L\gamma_\mu q_L\, \bar{q}_L\gamma^\mu b_L\,, \\
O^q_{S-P} &= \bar{b}_R\, q_L\, \bar{q}_L\, b_R\,, \\
T^q_{V-A} &= \bar{b}_L\gamma_\mu t_a q_L\, \bar{q}_L\gamma^\mu t_a b_L\,, \\
T^q_{S-P} &= \bar{b}_R\, t_a q_L\, \bar{q}_L\, t_a b_R\,,
\end{aligned} \tag{238}$$

where $t_a = \lambda_a/2$ are the generators of colour SU(3). In terms of these operators the total decay rate becomes [77]

$$\begin{aligned}
\Gamma = \Gamma_0 &+ \frac{2G_F^2 m_b^2}{\pi}\,|V_{cb}|^2\,(1-z)^2 \left\{ \left(2c_1c_2 + \frac{1}{N_c}\,(c_1^2+c_2^2)\right) \;< O_{V-A}^u > + 2(c_1^2+c_2^2) \;< T_{V-A}^u > \right\} \\
&- \frac{2G_F^2 m_b^2}{3\pi}\,|V_{cb}|^2\,(1-z)^2 \left\{ \left(2c_1c_2 + \frac{1}{N_c}\,c_1^2 + N_c\,c_2^2\right) \left[\left(1+\frac{z}{2}\right) \;< O_{V-A}^{d'} > - (1+2z) \;< O_{S-P}^{d'} > \right] \right. \\
&\qquad \left. + 2c_1^2 \left[\left(1+\frac{z}{2}\right) \;< T_{V-A}^{d'} > - (1+2z) \;< T_{S-P}^{d'} > \right] \right\} \\
&- \frac{2G_F^2 m_b^2}{3\pi}\,|V_{cb}|^2\,\sqrt{1-4z} \left\{ \left(2c_1c_2 + \frac{1}{N_c}\,c_1^2 + N_c\,c_2^2\right) \left[(1-z) \;< O_{V-A}^{s'} > - (1+2z) \;< O_{S-P}^{s'} > \right] \right. \\
&\qquad \left. + 2c_1^2 \left[(1-z) \;< T_{V-A}^{s'} > - (1+2z) \;< T_{S-P}^{s'} > \right] \right\},
\end{aligned} \tag{239}$$

where $z = m_c^2/m_b^2$, and N_c=3 is the number of colours.

We shall focus here on the mesonic case, the baryonic case is discussed in [77]. The matrix elements of the four quark operators are parametrized as

$$\begin{aligned}
\frac{1}{2m_{B_q}}\,\langle B_q|\,O_{V-A}^q\,|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}\,B_1\,, \\
\frac{1}{2m_{B_q}}\,\langle B_q|\,O_{S-P}^q\,|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}\,B_2\,, \\
\frac{1}{2m_{B_q}}\,\langle B_q|\,T_{V-A}^q\,|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}\,\varepsilon_1\,, \\
\frac{1}{2m_{B_q}}\,\langle B_q|\,T_{S-P}^q\,|B_q\rangle &\equiv \frac{f_{B_q}^2 m_{B_q}}{8}\,\varepsilon_2\,.
\end{aligned} \tag{240}$$

where this parametrization is motivated by factorization, which has been briefly discussed in connection with exclusive nonleptonic decays. Assuming factorization yields $B_i = 1$ and $\varepsilon_i = 0$. Factorization holds exactly in the limit of large N_c and hence one expects

$$B_i = O(1)\,, \qquad \varepsilon_i = O(1/N_c)\,. \tag{241}$$

Following [77] one may now evaluate the lifetime ratio between charged and neutral B mesons in terms of B_i and ε_i

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + k_1 B_1 + k_2 B_2 + k_3\varepsilon_1 + k_4\varepsilon_2\,, \tag{242}$$

where the coefficients k_i are determined by the mass ratio z and the Wilson coefficients c_1 and c_2.

In earlier work [78] the assumption of factorization has been made and hence the matrix elements of the octet-octet operators ε_i vanish. Using this assumption one obtains for the lifetime ratio

$$\frac{\tau(B^-)}{\tau(B_d)} = 1 + 0.05 \cdot \frac{f_B^2}{(200\ \mathrm{MeV})^2} \tag{243}$$

which is certainly compatible with data.

μ	k_1	k_2	k_3	k_4
$m_b/2$	$+0.044$	0.003	-0.735	0.201
m_b	$+0.020$	0.004	-0.697	0.195
$2m_b$	-0.008	0.007	-0.665	0.189

Table 2: Values of the coefficients k_i for different choices of the renormalization scale μ. The value of z is fixed at $z = 0.085$. The table is taken from [77].

On the other hand, the coefficients k_3 and k_4 of the octett-octett operators are larger by a factor five to ten, and if one expects these matrix elements to be suppressed by a factor $1/N_C$ it is a possible scenario that the nonfactorizable contributions actually dominate the corrections to the lifetime ratio.

The lifetime of the Λ_b baryon has also been considered within the framework of the $1/m_Q$ expansion. The general conclusion of the discussion is that the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ should be slightly smaller than unity, but cannot be as low as the measured value for "reasonable" values of the parameters. This fact constitutes a potential problem for the $1/m_Q$ expansion for inclusive nonleptonic decays, which is still subject of topical research.

4.4.3 Semileptonic Branching Fraction

Once one is able to calculate lifetimes one may also directly calculate the semileptonic branching fraction of b hadrons within the $1/m_Q$ expansion.

The total decay rate is composed of three contributions

$$\Gamma_{tot} = \Gamma(b \to c\ell\nu) + \Gamma(b \to c\bar{u}d') + \Gamma(b \to c\bar{c}s') \tag{244}$$

where the prime indicates the CKM rotated fields

$$\begin{aligned} d' &= V_{ud}d + V_{us}s + V_{ub}b && (245)\\ s' &= V_{cd}d + V_{cs}s + V_{cb}b && (246) \end{aligned}$$

For further use it is useful to define the ratios

$$R_{ud} = \frac{\Gamma(b \to c\bar{u}d')}{3\Gamma(b \to ce\nu)} \quad R_{cs} = \frac{\Gamma(b \to c\bar{c}s')}{3\Gamma(b \to ce\nu)} \tag{247}$$

where the semileptonic rate is given in terms of the semielectronic one by

$$\Gamma(b \to c\ell\nu) = 3f(m_\tau)\Gamma(b \to ce\nu) \tag{248}$$

where $f(m_\tau)$ is a known function that takes into account phase space effects due to the non-negligible τ mass, $f(m_\tau = 1777 \text{ MeV}) = 0.74$. These relations allow us to express the sum of the ratios $R_{ud} + R_{cs}$ in terms of the semileptonic branching fraction

$$R_{ud} + R_{cs} = f(m_\tau)\frac{1 - Br(b \to c\ell\nu)}{Br(b \to c\ell\nu)} \tag{249}$$

Using the 1996 Particle Data Group data [79]

$$\begin{aligned} Br(B \to Xe\nu) &= (10.4 \pm 0.4)\% && (250)\\ Br(B \to X\mu\nu) &= (10.3 \pm 0.5)\% && (251)\\ Br(B \to X\tau\nu) &= (2.7 \pm 0.4)\% && (252) \end{aligned}$$

one obtains the result

$$R_{ud} + R_{cs} = 2.37 \pm 0.12 \tag{253}$$

Up to here we have only used data, and now we discuss the theoretical predictions for the sum $R_{ud} + R_{cs}$. Due to the mass of the c quark one intuitively expects that the calculation of R_{ud} might be more reliable than the evaluation of R_{cs}, since the latter channel has two heavy quarks in the final state and one is thus closer to the resonance region.

Furthermore, as has been mentioned already several times, the non-perturbative corrections are small compared to the perturbative ones, and hence the latter will contribute the bulk part of the effect, while the non-perturbative corrections are almost negligible. The leading and subleading perturbative corrections have been available for some time, neglecting the masses of the final state quarks [80]. Recently the complete calculation of the subleading terms including the masses of the final state quarks has been performed [81, 82]. However, the mass effects are not drastic and do not change the conclusions drawn from the massless case, as far as he semileptonic branching fraction is concerned.

The theoretical expressions for R_{ud} and R_{cs} depend on both quark masses m_b and m_c. In the following analysis we shall exploit the fact that the mass difference $m_b - m_c$ is quite well known; in the heavy mass limit it simply becomes $< M_B > - < M_D >$, where $< M_H >$ denotes the spin averaged mass on the heavy ground state meson

$$< M_H >= \frac{1}{4}(m_H + 3m_{H^*}) \tag{254}$$

such that the mass difference is of the order $1/m_Q^2$ with

$$m_b - m_c = \lambda_1 \left(\frac{1}{2m_b} - \frac{1}{2m_c} \right) \tag{255}$$

Hence a good choice of input parameters is the b quark mass m_b, the renormalization scale μ entering the expressions for the QCD radiative corrections and the QCD scale parameter $\Lambda^{(5)}_{\overline{\mathrm{MS}}}$.

First we shall put in some "reasonable" values $\mu = m_b = 4.8$ GeV and $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 180$ MeV and evaluate from this the sum $R_{ud} + R_{cs}$; we obtain

$$R_{ud} + R_{cs} = 1.73, \tag{256}$$

while for each individual contribution we have $R_{ud} = 1.27$ and $R_{cs} = 0.46$. As we have argued above we expect that R_{ud} may be calculated more reliably in the $1/m_Q$ expansion, so we may take this calculation and vary the renormalization scale. Since R_{ud} becomes larger as the scale is lowered, we may think of $\mu = m_b/3$ as a reasonable choice of the renormalization scale, since the energy release is shared among the three quarks in the final state. For this choice one obtains $R_{ud} = 1.52$; using the data we would conclude $R_{cs} = 0.85$ which is from the theoretical estimates on the large side.

Large values like that may be obtained for instance by lowering the value of the b quark mass m_b; for a value $m_b = 4.4$ GeV one would obtain a R_{cs} in the region as discussed above; however, this would imply a value of the c quark mass as low as $m_c = 1.1$ GeV, which is at the lower edge of the commonly accepted values of this parameter.

Another possibility are potentially large QCD radiative corrections appearing in the $c\bar{c}s$ channel. These corrections have been studied previously by Ball et al.[81, 82], who find larger QCD radiative corrections in this channel due to the long range Coulomb-like interaction between the two heavy c quarks in the final state. These corrections shift the result into the right direction, but they do not seem sufficiently large to explain the data.

Finally, all the calculations of the non-leptonic rate are based on the concept of parton-hadron duality, which is expected to hold on intuitive grounds; however, it may fail for yet unknown reasons, rendering the calculation of the non-leptonic inclusive rates useless. On the other hand, it has been pointed out [78] that such a failure should show up somehow in the $1/m_Q$ expansion, but the $1/m_Q^3$ terms considered so far do not give any hint that local duality may fail.

A large value of R_{cs} has phenomenological implications of its own, since it will enhance the number of charm and anticharm quarks produced in b decays. Defining the number n_c of charm quarks produced per b decay as (note that it is the sum of c and $\bar{c}$)

$$n_c = Br(b \to c\ell\nu) + Br(b \to c\bar{u}d') + 2Br(b \to c\bar{c}s') \tag{257}$$

we may express this quantity in terms of the ratio R_{cs} as

$$n_c = 1 + R_{cs} \frac{Br(b \to c\ell\nu)}{f(m_\tau)}. \tag{258}$$

Thus we conclude that for a value of R_{cs} large enough to explain the semileptonic branching ratio we have to have $n_c \sim 1.3$.

On the other hand, we may obtain n_c experimentally by summing over the various semiinclusive modes. There are experimental data available at high and at low energies. At high energies the value of n_c is [83] $n_c = 1.23 \pm 0.07$ while the average at low energies is [84] $n_c = 1.12 \pm 0.05$. The weighted average of these values is $n_c = 1.16 \pm 0.04$, which is somewhat lower than what one has from the semileptonic branching fraction, but this discrepancy is not yet significant.

Thus the potential problem is not the semileptonic branching fraction alone, but rather the semileptonic branching fraction in combination with the charm counting in b decays. Possible resolutions of this problem are that the data on charm counting improve and indeed indicate a larger value such as $n_c \sim 1.3$. However, if future data confirm a value of n_c as low as $n_c = 1.1$ with significantly smaller errors, this would either imply a failure of parton-hadron duality even in the "safe" channel $c\bar{u}d$, or, if duality were shown to hold in this channel, could indicate a large branching fraction of B mesons into non-charmed final states, as large as

$$Br(b \to charmless\ final\ states) \sim 20\%.$$

This is very unlikely in the SM, but possible in non-SM scenarios, which enhance the channel $b \to sg$.

5 Concluding Remarks

Heavy quark physics, in particular the physics of the bottom quark, will provide a deeper insight into the question of quark mixing and the origin of CP violation. The possibility of testing this sector of the standard model with unprecendented precision has motivated a very strong effort of a large part of the particle physics community.

In particular from the experimental side impressive progress has been made, mainly due to technological progress, among which the development of silicon vertex detectors was the most prominent. It is mainly this piece of equipment which allowed the LEP experiments to compete with the present dedicated B physics experiments such as CLEO and ARGUS. This fact is very astonishing in retrospect, since by the time LEP started no one would have guessed that e.g. the B hadron lifetime measurements would be dominated by the LEP experiments.

The future efforts from the experimental side spans practically all possibilities of how to study b hadrons. On one hand there are three dedicated e^+e^- machines running at energies about the $\Upsilon(4s)$: The CLEO experiment located at the symmetric machine CESR at Cornell, and two asymmetric B factories, one at SLAC (the BaBar experiment) and one at KEK (the BELLE experiment). The latter two are designed in an asymmetric way in order to make a time dependent measurement of the CP asymmetries in the B_d system possible.

On the other hand there are experiments based on hadronic collisions. There is one fixed target experiment planned at DESY in Hamburg (the HERA-B experiment) which has as its main target the CP asymmetries in the decay $B_d \to J/\Psi K_s$ and the CDF experiment at FERMILAB is now upgraded to allow for measurements of bottom hadron decays. In the longer run a dedicated b physics experiment is planned at the LHC, the LHC-B experiment, which will due to the very large statistics allow additional measurements.

Parallel to these experimental efforts there has been a breakthrough in the theoretical description. Many of the calculations of weak processes of heavy quarks are meanwhile based on the $1/m_Q$ expansion of QCD and the model uncertainties have been substantially reduced. The purpose of the present review was to summarize the theoretical development based on the heavy mass expansion using the methods of effective field theory.

Ultimately all the unknown nonperturbative quantities will be evaluated on the lattice, still the heavy mass expansion has turned out to be a useful tool to obtain a better understanding of the lattice data. At present the computer power is still too limited to calculate e.g. the form factors of a weak transition over the full kinematic range, but first steps in this direction exist.

5.1 Conclusions and Open Questions: Exclusive Decays

As far as exclusive decays are concerned the theoretical description seems to be in quite good shape. The master example is the determination of V_{cb} from the q^2 spectrum of the leptons in $B \to D^*\ell\bar{\nu}_\ell$, where heavy quark symmetries allow access to the absolute value of the relevant form factor at maximum q^2. Uncertainties enter here through new unknown matrix elements at the level of $1/m_c^2$, which have been estimated to be about five percent and hence quite small. On the other hand, with present theoretical technology this seems to be a serious limitation which will be hard to overcome. The expectation from the future B factories on the uncertainties V_{cb} is that by the end of the millenium V_{cb} will be known with an uncertainty of five percent. This has to be compared to our present knowledge of the Cabibbo angle, i.e. V_{us} which is known at a precision of about one percent.

Exclusive semileptonic heavy to light transitions are not as strongly retricted by heavy quark symmetries. Still relative normalizations are possble which are, however, not protected against $1/m_Q$ corrections. In spite of this, exclusive heavy to light decays will play some role in the determination of V_{ub}, but this requires measurement of the lepton spectrum in $b \to u$ decays. The determination of V_{ub} – at least judging from present theoretical technology – has to rely on mix of different methods, including also inclusive decays. A realistic estimate on the theoretical uncertainty on V_{ub}, which can be achieved with present methods, is ten percent.

Exclusive nonleptonic decays remain the most difficult kind of processes despite of the heavy quark limit. In exclusive semileptonic decays heavy quark symmetries allow us in some cases to ged rid of the details of QCD dynamics and hence get access to weak interaction quantities without "solving" QCD. However, in nonleptonic decays QCD plays a crucial role and there is no way to employ heavy quark symmetries to "hide" the strong interaction problems.

5.2 Conclusions and Open Questions: Inclusive Decays

In order to set up a $1/m_Q$ expansion for inclusive decays one makes use of the fact that the mass of the heavy quark set a large scale and performs an operator product expansion. For semileptonic decays one need to assume global duality (i.e. that partonic and hadronic quantities coincide in the average) which is generally believed to be safe assumption. For nonleptonic processes one need the stronger assumption of local duality (i.e. that hadronic and partonic quantities concide at some kinematic point) and a part of the community believes that this may be the reason for the problems in describing the lifetimes, although local duality is assumed at a point far away from the resonance region.

One may also calculate the lepton energy spectrum in inclusive semileptonic decays using the operator product expansion. In this case the expansion parameter is the energy release $m_Q - 2E_\ell$ which is in almost all phase space of the order of the heavy quark mass. However, close to the maximal energy this expansion breaks down, yielding unphysical spikes in the spectrum. However, in the endpoint region, which is assumed to be wide enough to still employ duality, one may perform a resummation of the most singular terms, yielding a distribution function for the light cone residual momentum of the heavy quark inside the heavy meson, in close analogy to what happens in deep inelastic scattering. Even closer to the endpoint one enters the resonance region, where the spectrum is dominated by the lowest resonances which can appear as final states in the inclusive process.

Both CKM parameters V_{ub} and V_{cb} may be determined from inclusive decays. As far as V_{cb} is concerned, the inclusive method can compete with the exclusive one, since the theoretical uncertainties are comparable. It is already a nontrivial check that the values of V_{cb} determined from inclusive and exclusive processes indeed coincide very nicely. V_{ub} has been determined from the endpoint region of the inclusive decay $B \to X_u \ell \bar{\nu}_\ell$ looking at events beyond the endpoint of charmed final states. With improved technology it may be possible to determine the hadronic invariant mass of the final state and hence allow us to obtain a better determination of V_{ub} at future B factories.

5.3 Outlook

In this review considered the implications of heavy quark symmetry and the $1/m_Q$ expansion of QCD without using additional input It is obvious that one may add on top of heavy quark symmetries other methods which have been developed in QCD. One additional input which has been discussed is to implement also the chiral symmetry of the light degrees of freedom [85] which has provided some insight e.g. into the form factors of heavy light transitions.

Another input to supplement the nonperturbative information could be QCD sum rules or even models. In particular the latter may be checked, since a simple requirement for models is compatibility with heavy quark symmetries. The ultimate way to determine nonperturbative parameters are lattic calculations which eventually will not use the heavy mass expansion but rather deal with full QCD.

Recently heavy quark symmetries, lattice data and constraints from unitarity and analyticity have been combined to contrain form factors [86]. In particular for heavy to light transitions this input basically fixes the possible shape of the form factors and will eventually yield a reliable way to determine V_{ub} from exclusive decays.

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