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Theoretical Aspects of $\tau \to Kh(h)\nu_\tau$ Decays and Experimental Comparisons

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Predictions for decay rates and distributions for τ decays into final states with kaons are discussed and compared with recent measurements. Special emphasis is put on new constraints for the vector current contribution in the $KK\pi$ decay modes. For the $K\pi\pi$ modes, disagreements with the experimental results can be traced back to the K_1 widths.

1. Introduction

The τ lepton is heavy enough to decay into a variety of hadronic final states. In particular, final states with kaons can provide detailed information about low energy hadron physics in the strange sector. Topics to be studied include:
1) Chiral Perturbation Theory (CHPT) and effective Lagrangians, 2) Resonance parameters $(a_1, \rho, K_1, K^*, \ldots)$ and radial excitations, 3) Tests of SU$(3)_F$ and isospin symmetry, 4) Structure of the charged hadronic current [$(V-A)$ or $(V+A)$], scalar contributions, etc., 5) Determination of the strange quark mass and $\alpha_s(m_\tau)$ measurements for $\Delta S = 1$ transitions, 6) Measurement of the τ neutrino mass, 7) Search for CP violation effects beyond the Standard Model.

Predictions for final states with 2 and 3 meson final states involving one or several kaons based on the "chirally normalized vector meson dominance model" will be discussed and compared with experimental results. The numbers for the experimental new world averages (NWA) are taken from [1]. Special emphasis is put on the vector part in the $KK\pi$ decay modes and on problems in the axial vector part in the $K\pi\pi$ final states, which we believe can be traced back to the K_1 widths. The importance of a detailed analysis of the exclusive final states with the structure function formalism is emphasized [2].

Let us first specify the general structure of the matrix elements for semi-leptonic τ decays. The matrix element $\mathcal{M}$ for the hadronic τ decay into n mesons $h_1, \ldots h_n$

$$\tau(l,s) \to \nu(l',s') + h_1(q_1,m_1) + \ldots h_n(q_n,m_n), \quad (1)$$

can be expressed in terms of a leptonic (M_μ) and a hadronic current (J^μ) as

$$\mathcal{M} = \frac{G}{\sqrt{2}} \binom{\cos\theta_c}{\sin\theta_c} M_\mu J^\mu . \quad (2)$$

In Eq. (2), G denotes the Fermi-coupling constant and θ_c is the Cabibbo angle. The leptonic current is given by

$$M_\mu = \bar{u}(l',s')\gamma_\mu(g_V - g_A\gamma_5)u(l,s) , \quad (3)$$

with $g_V = g_A = 1$ in the Standard Model. The hadronic current J^μ can in general be expressed in terms of a vector and an axial vector current

$$J^\mu(q_1,\ldots,q_n) = \langle h_1(q_1)\ldots h_n(q_n)|V^\mu(0) - A^\mu(0)|0\rangle . \quad (4)$$

In the following, we specify the hadronic matrix elements for hadronic decays into multi meson final states as expected from the Standard Model.

2. $\tau^- \to K^-\nu_\tau$

The decay rate for the simplest decay mode with one kaon is well predicted by the kaon decay

*Talk presented by Erwin Mirkes at the Fourth International Workshop on Tau Lepton Physics (TAU 96), 16–19 September 1996, Estes Park, Colorado, USA constants f_K defined by the matrix element of the axial vector current

$$\langle K(q)|A^\mu(0)|0\rangle \quad = \quad i\sqrt{2}f_K q^\mu . \tag{5}$$

The kaon decay constant can be determined using the precisely measured kaon decay widths $\Gamma(K \to \mu\bar{\nu}_\mu)$. Radiative corrections $\delta R_{\tau/K} = (0.90 \pm 0.22)\%$ to the ratios $\Gamma(\tau \to K\nu)/\Gamma(K \to \mu\nu)$ have been calculated [3]. Using the recent world average $\tau_\tau = (291.6 \pm 1.6)fs$ for the tau lifetime one obtains the following theoretical predictions for the branching ratios

$$\mathcal{B}(K\nu_\tau) \quad = \quad (0.723 \pm 0.006)\% \tag{6}$$

This prediction agrees well with the new world average [1]

$$\mathcal{B}^{exp}(K\nu_\tau) \quad = \quad (0.692 \pm 0.028)\% \tag{7}$$

3. $\tau^- \to [Kh]^-\nu_\tau$

We will now discuss the four decay modes $\overline{K^0}\pi^-, K^-\pi^0, K^-\eta$ and K^-K^0.

The hadronic matrix element for the decay $\tau \to h_1 h_2 \nu_\tau$ can be expanded along a set of independent momenta $q_1^\mu - q_2^\mu$ and $Q^\mu = (q_1^\mu + q_2^\mu)$

$$\langle h_1(q_1)h_2(q_2)|V^\mu(0)|0\rangle \quad = \quad [(q_1 - q_2)_\nu \, T^{\mu\nu} \, F_V^{h_1 h_2} + Q^\mu \, F_S^{h_1 h_2}] \tag{8}$$

where $F_V (F_S)$ corresponds to the $J^P = 1^-$ ($J^P = 0^+$) component of the weak charged current and $T^{\mu\nu}$ is the transverse projector, defined by

$$T_{\mu\nu} = g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2} \, . \tag{9}$$

The $\overline{K^0}\pi^-, K^-\pi^0$ and $K^-\eta$ decay modes are expected to be dominated by the K^* resonance, whereas K^-K^0 is dominated by the ρ. We allow for an admixture of radial excitations[2]:

$$T_{K^\star}^{(2m)}(Q^2) = \frac{\mathrm{BW}_{K^\star}(Q^2) + \beta_{K^\star}\,\mathrm{BW}_{K^{\star\prime}}(Q^2)}{1 + \beta_{K^\star}} \tag{10}$$

$$T_\rho^{(2m)}(Q^2) = \frac{\mathrm{BW}_\rho(Q^2) + \beta_\rho\,\mathrm{BW}_{\rho'}(Q^2)}{1 + \beta_\rho} \tag{11}$$

with [4]

$$\begin{aligned} &\beta_{K^\star} = -0.135\,, \\ &m_{K^\star} = 0.892\,\mathrm{GeV}\,, \Gamma_{K^\star} = 0.050\mathrm{GeV}\,, \\ &m_{K^{\star\prime}} = 1.412\,\mathrm{GeV}\,, \Gamma_{K^{\star\prime}} = 0.227\,\mathrm{GeV}\,. \end{aligned} \tag{12}$$

[2]The superscript $(2m)$ stands for “2 meson” resonances

Table 1
Braching ratios for two meson decay modes

	$\beta_{K^*} = -0.135$	$\beta_{K^*} = 0$
$\overline{K^0}\pi^-$	0.906 %	0.65 %
$K^-\pi^0$	0.453 %	0.33 %
$K^-\eta$	$2.0{\cdot}10^{-2}$ %	$0.8{\cdot}10^{-2}$ %
	$\beta_\rho = -0.145$	$\beta_\rho = 0$
K^-K^0	0.11 %	0.056 %

and [5]

$$\begin{aligned} &\beta_\rho = -0.145 \\ &m_\rho = 0.773\,\mathrm{GeV}\,, \Gamma_\rho = 0.145, \mathrm{GeV} \\ &m_{\rho'} = 1.370\,\mathrm{GeV}\,, \Gamma_{\rho'} = 0.510\,\mathrm{GeV}\,. \end{aligned} \tag{13}$$

In Eqs. (10,11), BW denote normalized Breit-Wigner propagators with an energy dependent width

$$\mathrm{BW}_X[Q^2] \equiv \frac{M_X^2}{[M_X^2 - Q^2 - i\sqrt{Q^2}\Gamma_X(Q^2)]} \, . \tag{14}$$

The vector form factors $F_V^{h_1 h_2}$ in Eq. (8) for the various two meson decay modes are given by

$$F_V^{\overline{K^0}\pi^-} = \frac{1}{\sqrt{2}} T_{K^\star}^{(2m)}(Q^2) \qquad F_V^{K^-\pi^0} = T_{K^\star}^{(2m)}(Q^2)$$

$$F_V^{K^-\eta} = \sqrt{\frac{3}{2}} T_{K^\star}^{(2m)}(Q^2) \qquad F_V^{K^0 K^-} = T_\rho^{(2m)}(Q^2)\,.$$

For the $\Delta S = 1$ transition $\tau \to K\pi\nu_\tau$, the form factor F_S is expected to receive a sizable contribution ($\sim 5\%$ to the decay rate) from the $K_0^*(1430)$ with $J^P = 1^+$ [6]. However, we will neglect this contribution in the following discussion. Predictions for the $K\pi$ and KK decay modes are compared with experimental results in Fig. 1. The sensitivity of our theoretical predictions to the parameter β_K^* and β_ρ is indicated in table 1, which also includes a prediction for the $K\eta$ final state. This decay mode is also completely fixed by the parameters of the $T_{K^\star}^{(2m)}$ resonance. Note that our prediction for $\mathcal{B}(K\eta)$ and $\mathcal{B}(KK)$ are both very sensitive to the choice of β_{K^*} and β_ρ. The results based on our favorite values are consitent with the experimental numbers. The branching ratio $\mathcal{B}(K\eta)$ was recently measured by CLEO [1] $\mathcal{B}(K\eta) = (2.6 \pm 0.5 \pm 0.5) \cdot 10^{-2}\%$ and ALEPH [1] $\mathcal{B}(K\eta) = (2.9 \pm 1.3 \pm 0.7) \cdot 10^{-2}\%$. Other theoretical predictions for the $K\eta$ decay mode are $\mathcal{B}^{th}(K\eta) = (1.2 - 1.4) \cdot 10^{-2}\%$ [7] and $\mathcal{B}^{th}(K\eta) = 2.22 \cdot 10^{-2}\%$ [8]. Predictions for the

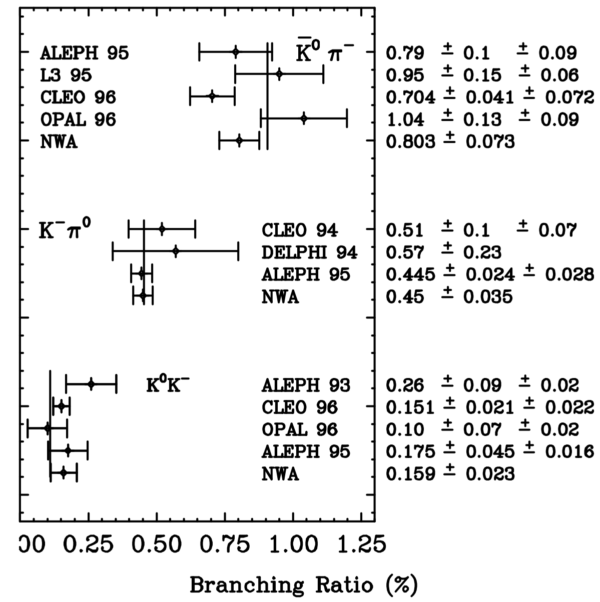

Figure 1. $\tau \to [Kh]^-\nu_\tau$ branching ratio measurements. The vertical lines are the theoretical predictions from the second column in table 1.

K^-K^0 decay mode can also be obtained via CVC from $e^+e^- \to \pi^+\pi^-$ data after applying $SU(3)$-breaking effects. The result is $\mathcal{B}^{CVC}(K^-K^0) = (0.111 \pm 0.3)\%$ [9].

4. $\tau^- \to [KK\pi]^-\nu_\tau$

The hadronic matrix elements for three meson final states have a much richer structure. The decay modes involving kaons allow for axial and vector current contributions at the same time [10,11,4]. The most general ansatz for the matrix element of the quark current J^μ in Eq. (4) is characterized by four form factors F_i [2], which are in general functions of $s_1 = (q_2+q_3)^2, s_2 = (q_1+q_3)^2$, $s_3 = (q_1+q_2)^2$ and Q^2 (which is conveniently chosen as an additional variable)

$$J^\mu(q_1,q_2,q_3) \;=\; V_1^\mu\,F_1 + V_2^\mu\,F_2 + i\,V_3^\mu\,F_3 + V_4^\mu\,F_4\;, \qquad (15)$$

with

$$\begin{aligned} V_1^\mu &= (q_1-q_3)_\nu\,T^{\mu\nu}\;,\\ V_2^\mu &= (q_2-q_3)_\nu\,T^{\mu\nu}\;,\\ V_3^\mu &= \epsilon^{\mu\alpha\beta\gamma}q_{1\,\alpha}q_{2\,\beta}q_{3\,\gamma}\;,\\ V_4^\mu &= q_1^\mu+q_2^\mu+q_3^\mu = Q^\mu\;. \end{aligned} \qquad (16)$$

$T^{\mu\nu}$ denotes again the transverse projector as defined in Eq. (9). The form factors F_1 and $F_2(F_3)$ originate from the $J^P = 1^+$ axial vector hadronic current ($J^P = 1^-$ vector current) and correspond to a hadronic system in a spin one state, whereas F_4 is due to the $J^P = 0^+$ spin zero part of the axial current matrix element. The contribution to F_4 is expected to be small [12] and we will neglect this contribution in the subsequent discussion.

The form factors F_1 and F_2 can be predicted by chiral Lagrangians at small momentum transfer whereas the vector form factor is related to the Wess-Zumino anomaly [13,14]. To saturate the form factors in the region of large Q^2 by resonances with momentum independent couplings is a natural choice in the context of the vector dominance model (VDM) but perhaps the most problematic assumption, which has to be tested by measurements of differential distributions. A particular powerful tool is provided by the analyses of angular distributions. The relevant information is conveniently encoded in structure functions [2] which in turn allow to reconstruct the form factors. τ decays are therefore a unique tool to study hadron physics in the low momentum region in order to test a variety of theoretical approaches and to derive resonance parameters which are not accessible elsewhere.

The resulting choice for the form factors F_i in this chirally normalized vector meson dominance model is summarized by [4]

$$F_1^{(abc)}(Q^2,s_2,s_3) \;=\; \frac{2\sqrt{2}A^{(abc)}}{3f_\pi}G_1^{(abc)}(Q^2,s_2,s_3)\;, \qquad (17)$$

$$F_2^{(abc)}(Q^2,s_1,s_3) \;=\; \frac{2\sqrt{2}A^{(abc)}}{3f_\pi}G_2^{(abc)}(Q^2,s_1,s_3)\;, \qquad (18)$$

$$F_3^{(abc)}(Q^2,s_1,s_2,s_3) \;=\; \frac{A^{(abc)}}{2\sqrt{2}\pi^2 f_\pi^3}G_3^{(abc)}(Q^2,s_1,s_2,s_3)\;. \qquad (19)$$

where the Breit-Wigner functions $G_{1,2,3}$ and the normalizations $A^{(abc)}$ are listed for $abc \equiv K\pi K$ in Tab. 3.

Let us briefly discuss the resonance structure in Tab. 3 (for details see [4]).

Table 2
Predictions for the branching ratios $\mathcal{B}(abc)$ in % for the $KK\pi$ decay modes. The contribution from the vector current is given in parentheses.

channel (abc)	$T_\rho^{(3m)}$ [4]	$T_\rho^{(3m)}$ Eq.(20).
$K^-\pi^-K^+$	0.20 (0.08)	0.18 (0.063)
$K^0\pi^-\overline{K^0}$	0.20 (0.08)	0.18 (0.063)
$K_S\pi^-K_S$	0.048 (0.014)	0.045 (0.011)
$K_S\pi^-K_L$	0.10 (0.052)	0.092 (0.039)
$K^-\pi^0K^0$	0.16 (0.057)	0.15 (0.045)

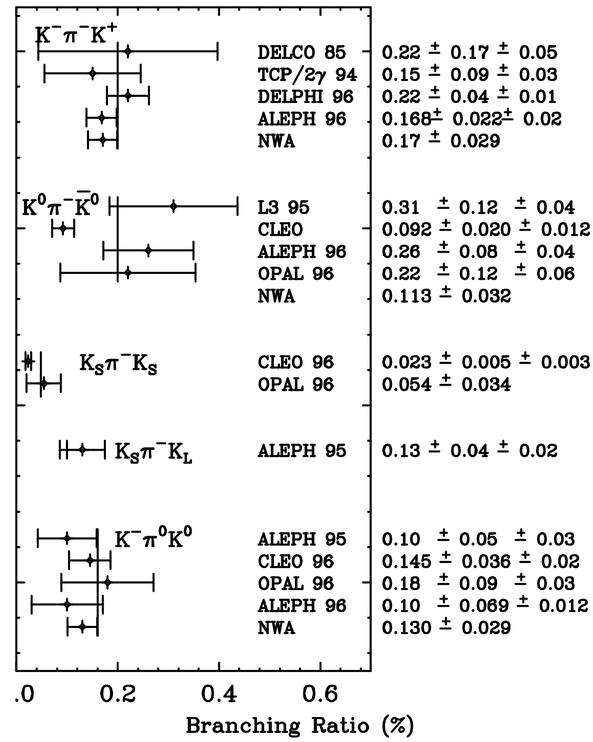

Figure 2. $\tau \to [K\pi K]^-\nu_\tau$ branching ratio measurements. The vertical lines are the theoretical predictions from the second column in table 2.

The form factors F_1 and F_2 are governed by the a_1 resonance with energy dependent width

$$\mathrm{BW}_{a_1}(Q^2) = \frac{m_{a_1}^2}{m_{a_1}^2 - Q^2 - i m_{a_1}\Gamma_{a_1} g(Q^2)/g(m_{a_1})}$$

with $m_{a_1} = 1.251$ GeV , $\Gamma_{a_1} = 0.475$ GeV . The function $g(Q^2)$ has been calculated in [5].

The two meson resonances $T_\rho^{(2m)}$ and $T_{K^\star}^{(2m)}$ are defined in Eqs. (10) and (11). The ω resonance $T_\omega(Q^2) = \frac{1}{1+\epsilon}[\mathrm{BW}_\omega(Q^2) + \epsilon \mathrm{BW}_\Phi(Q^2)]$ in the vector form factor F_3 in Tab. 3 allows for a contribution of the ϕ with a relative strength $\epsilon = 0.05$.

The admixture of the radial excitations in the three meson non-strange vector resonance in F_3, denoted by $T_\rho^{(3m)}$, is expected to differ from the corresponding two meson vector resonance $T_\rho^{(2m)}$ in Eq. (11). In [4,11,15], a form for $T_\rho^{(3m)}$ including ρ, ρ' and ρ'' was used, which was obtained from a fit to (fairly poor) $e^+e^- \to \eta\pi\pi$ data [16,17].

Predictions based on these parametrizations and the sub-resonance structure as given in table 3 are compared with experimental results in Fig. 2. The branching ratios are also listed in the second column of table 2.

Using $SU(3)$ symmetry, $T_\rho^{(3m)}$ can also be directly obtained from $\tau \to \eta\pi^-\pi^0\nu_\tau$. The matrix element for this decay mode is directly proportional to the product $T_\rho^{(3m)} \cdot T_\rho^{(2m)}$ [11]. New measurements of the $\eta\pi^-\pi^0$ mass spectrum in $\tau \to \eta\pi^-\pi^0\nu_\tau$ have become available [18,19] allowing now for such a direct determination of $T_\rho^{(3m)}$. We found, that the $\eta\pi^-\pi^0$ mass spectrum in τ decays is poorly described by $T_\rho^{(3m)}$ parametrization based on the $e^+e^- \to \eta\pi\pi$ data [20]. A direct fit for the three body vector resonance (using the model for $\tau \to \eta\pi^-\pi^0\nu_\tau$ in [11]) to the new $\tau \to \eta\pi^-\pi^0\nu_\tau$ data yields [20]

$$T_\rho^{(3m)} = \frac{\left[\mathrm{BW}_\rho + \lambda \mathrm{BW}_{\rho'} + \nu \mathrm{BW}_{\rho''}\right]}{1+\lambda+\nu} \tag{20}$$

with

$$\begin{aligned} \lambda &= -0.22 \pm 0.03 \\ \nu &= -0.10 \pm 0.01 \\ \chi^2/\mathrm{d.o.f.} &= 11.0/14 \end{aligned} \tag{21}$$

where we fix the parameters to the PDG [21] values,

$$\begin{aligned} m_\rho &= 0.773 \text{ GeV}, & \Gamma_\rho &= 0.145 \text{ GeV}, \\ m_{\rho'} &= 1.465 \text{ GeV}, & \Gamma_{\rho'} &= 0.310 \text{ GeV}, \\ m_{\rho''} &= 1.70 \text{ GeV}, & \Gamma_{\rho''} &= 0.235 \text{ GeV}. \end{aligned} \tag{22}$$

The invariant mass distribution and the decay rate for the $\tau \to \eta\pi^-\pi^0\nu_\tau$ are well described by these parameters [20] and we will use this parametrization for $T_\rho^{(3m)}$ also in the following for the $KK\pi$ decay modes. Predictions for the $KK\pi$ branching ratios based on these parameters for $T_\rho^{(3m)}$ are shown in the third column of table 2. Note that the differences to the results in the second column are entirely due to the different vector current contribution. Whereas the differences in the branching ratios are fairly small ($\sim 10\%$) the differences in the $KK\pi$ mass spectra are much larger. Fig. 3 shows the Q^2 distribution for the s_1, s_2 integrated structure functions $w_A(Q^2)$ and $w_B(Q^2)$ (for a definition of the structure functions see [2]). Sizable differences are seen in the vector structure function $w_B(Q^2)$ depending on the choice of the $T_\rho^{(3m)}$ parametrization.

The resonance structure in the $K^-\pi^-K^+$ decay mode is shown in Fig. 4. The large peak in the $K^+\pi^-$ invariant mass around the $K^*(892)$ resonance shows that these decay modes are dominated by the $T_{K^*}^{(2m)}$-two meson sub-resonance compared to the $\rho(\to K^+K^-)$ channel. The results are in good agreement with the measurements in [22].

τ decay modes with an axial *and* a vector current contribution offer a unique possibility to measure the relative sign between V and A in the hadronic current of Eq. (4). This is possible by just measuring the sign of one axial vector-vector interference structure function $W_{F,G,H,I}$. Predictions for the Q^2 distribution for the structure functions $w_{F,G} = \int ds_1 ds_2 W_{F,G}$ are shown in Fig. 5 for the for the $\tau \to K^-\pi^-K^+\nu_\tau$ decay mode. The size of these structure functions is comparable to the structure functions w_A and w_B in Fig. 3 which determine the decay rate. Further predictions for V and A interference structure functions in the $KK\pi$ and $K\pi\pi$ decay modes based on the model in [11] are given in [23].

Structure function measurements allow also for a (model independent or model dependent) separation of vector, axial-vector and scalar contributions in semi-leptonic τ decays [24]. Furthermore, CHPT predicts interesting effects in the structure functions w_D and w_E for the two three pion decay modes [25].

Our result for $\mathcal{B}(K_S\pi^-K_S)$ appears to be considerably higher than the experimental result, whereas the other predictions agree fairly well. The rates for $K_L\pi^-K_L$ and $K_S\pi^-K_S$ are identical, the rate for $K_S\pi^-K_L$ is about a factor 2.1-2.4 higher. Note that the first relation is a strict consequence of CP symmetry, the second one depends on the dynamics of the decay amplitude (in particular on the a_1 parameters [4]). Our results for the $K^-\pi^-K^+$ final state differ from those in [17], where the contribution of the axial-vector channel amounts to less than 10% to the decay rate in this channel. In fact, our predictions for the axial-vector contribution is about 60-75% (see table 2). This result is fairly insensitive towards the details of the $T_{K^*}^{(2m)}$ parametrization. It is, however, sensitive towards the a_1 parameters. Use of $\Gamma_{a_1} = 0.599$ GeV reduces the axial-vector contribution by about 15%.

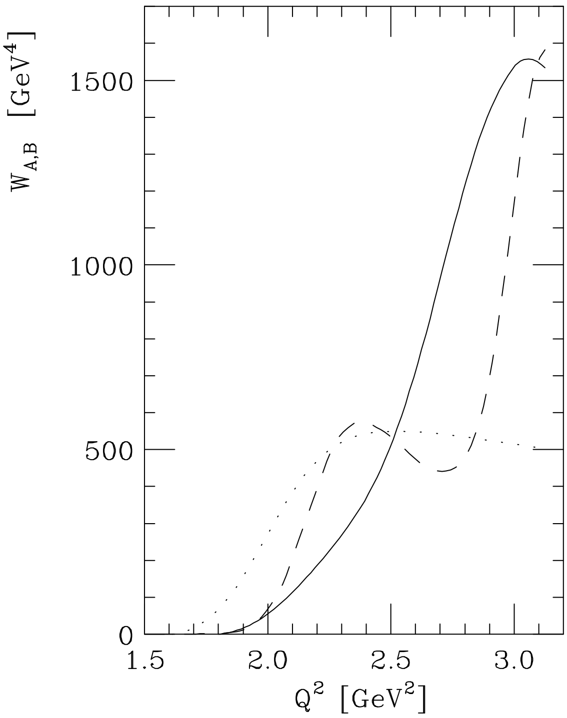

Figure 3. Axial vector structure function $w_A(Q^2)$ (dotted) and vector structure function $w_B(Q^2)$ for the $\tau \to K^-\pi^-K^+\nu_\tau$ decay mode. The solid line shows w_B based on the $T_\rho^{(3m)}$ parametrization in Eqs. (20,21), whereas the dashed curve shows w_B for $T_\rho^{(3m)}$ as defined in [4] (denoted as $T_\rho^{(2)}$ there).

5. $\tau^- \to [K\pi\pi]^-\nu_\tau$

The parametrization for the form factors F_1, F_2, F_3 in Eqs. (17-19) for the $K\pi\pi$ decay

Table 3
Parametrization of the form factors F_1 F_2 and F_3 in Eqs. (17,18,19) for $KK\pi$ decay modes.

channel (abc)	$A^{(abc)}$	$G_1^{(abc)}(Q^2,s_2,s_3)$	$G_2^{(abc)}(Q^2,s_1,s_3)$
$K^-\pi^-K^+$	$\frac{-\cos\theta_c}{2}$	$\mathrm{BW}_{a_1}(Q^2)T_\rho^{(2m)}(s_2)$	$\mathrm{BW}_{a_1}(Q^2)T_{K^\star}^{(2m)}(s_1)$
$K^0\pi^-\overline{K^0}$	$\frac{-\cos\theta_c}{2}$	$\mathrm{BW}_{a_1}(Q^2)T_\rho^{(2m)}(s_2)$	$\mathrm{BW}_{a_1}(Q^2)T_{K^\star}^{(2m)}(s_1)$
$K_S\pi^-K_S$	$\frac{-\cos\theta_c}{4}$	$\mathrm{BW}_{a_1}(Q^2)T_{K^\star}^{(2m)}(s_3)$	$\mathrm{BW}_{a_1}(Q^2)\times [T_{K^\star}^{(2m)}(s_1)+T_{K^\star}^{(2m)}(s_3)]$
$K_S\pi^-K_L$	$\frac{-\cos\theta_c}{4}$	$\mathrm{BW}_{a_1}(Q^2)\times [2T_\rho^{(2m)}(s_2)+T_{K^\star}^{(2m)}(s_3)]$	$\mathrm{BW}_{a_1}(Q^2)\times [T_{K^\star}^{(2m)}(s_1)-T_{K^\star}^{(2m)}(s_3)]$
$K^-\pi^0K^0$	$\frac{3\cos\theta_c}{2\sqrt{2}}$	$\mathrm{BW}_{a_1}(Q^2)\times \left[\frac{2}{3}T_\rho^{(2m)}(s_2)+\frac{1}{3}T_{K^\star}^{(2m)}(s_3)\right]$	$\frac{1}{3}\mathrm{BW}_{a_1}(Q^2)\times \left[T_{K^\star}^{(2m)}(s_1)-T_{K^\star}^{(2m)}(s_3)\right]$
		$G_3^{(abc)}(Q^2,s_1,s_2,s_3)$	
$K^-\pi^-K^+$	$-\cos\theta_c$	$T_\rho^{(3m)}(Q^2)(\sqrt{2}-1)\left[\sqrt{2}T_\omega(s_2)+T_{K^\star}^{(2m)}(s_1)\right]$	
$K^0\pi^-\overline{K^0}$	$\cos\theta_c$	$T_\rho^{(3m)}(Q^2)(\sqrt{2}-1)\left[\sqrt{2}T_\omega(s_2)+T_{K^\star}^{(2m)}(s_1)\right]$	
$K_S\pi^-K_S$	$\frac{-\cos\theta_c}{2}$	$T_\rho^{(3m)}(Q^2)(\sqrt{2}-1)\left[T_{K^\star}^{(2m)}(s_1)-T_{K^\star}^{(2m)}(s_3)\right]$	
$K_S\pi^-K_L$	$\frac{\cos\theta_c}{2}$	$T_\rho^{(3m)}(Q^2)(\sqrt{2}-1)\left[2\sqrt{2}T_\omega(s_2)+T_{K^\star}^{(2m)}(s_1)+T_{K^\star}^{(2m)}(s_3)\right]$	
$K^-\pi^0K^0$	$\frac{-\cos\theta_c}{\sqrt{2}}$	$T_\rho^{(3m)}(Q^2)(\sqrt{2}-1)\left[T_{K^\star}^{(2m)}(s_3)-T_{K^\star}^{(2m)}(s_1)\right]$	

Table 4
Parametrization of the form factors F_1 F_2 and F_3 in Eqs. (17,18,19) for $K\pi\pi$ decay modes.

channel (abc)	$A^{(abc)}$	$G_1^{(abc)}(Q^2,s_2,s_3)$	$G_2^{(abc)}(Q^2,s_1,s_3)$
$\pi^0\pi^0K^-$	$\frac{\sin\theta_c}{4}$	$T_{K_1}^{(a)}(Q^2)T_{K^\star}^{(2m)}(s_2)$	$T_{K_1}^{(a)}(Q^2)T_{K^\star}^{(2m)}(s_1)$
$K^-\pi^-\pi^+$	$\frac{-\sin\theta_c}{2}$	$T_{K_1}^{(a)}(Q^2)T_{K^\star}^{(2m)}(s_2)$	$T_{K_1}^{(b)}(Q^2)T_\rho^{(1)}(s_1)$
$\pi^-\overline{K^0}\pi^0$	$\frac{3\sin\theta_c}{2\sqrt{2}}$	$\frac{2}{3}T_{K_1}^{(b)}(Q^2)T_\rho^{(2m)}(s_2)$ $+\frac{1}{3}T_{K_1}^{(a)}(Q^2)T_{K^\star}^{(2m)}(s_3)$	$\frac{1}{3}T_{K_1}^{(a)}(Q^2)\left[T_{K^\star}^{(2m)}(s_1)-T_{K^\star}^{(2m)}(s_3)\right]$
		$G_3^{(abc)}(Q^2,s_1,s_2,s_3)$	
$\pi^0\pi^0K^-$	$\sin\theta_c$	$\frac{1}{4}T_{K^\star}^{(3m)}(Q^2)\left[T_{K^\star}^{(2m)}(s_1)-T_{K^\star}^{(2m)}(s_2)\right]$	
$K^-\pi^-\pi^+$	$\sin\theta_c$	$\frac{1}{2}T_{K^\star}^{(3m)}(Q^2)\left[T_\rho^{(2m)}(s_1)+T_{K^\star}^{(2m)}(s_2)\right]$	
$\pi^-\overline{K^0}\pi^0$	$\sqrt{2}\sin\theta_c$	$\frac{1}{4}T_{K^\star}^{(3m)}(Q^2)\left[2T_\rho^{(2m)}(s_2)+T_{K^\star}^{(2m)}(s_1)+T_{K^\star}^{(2m)}(s_3)\right]$	

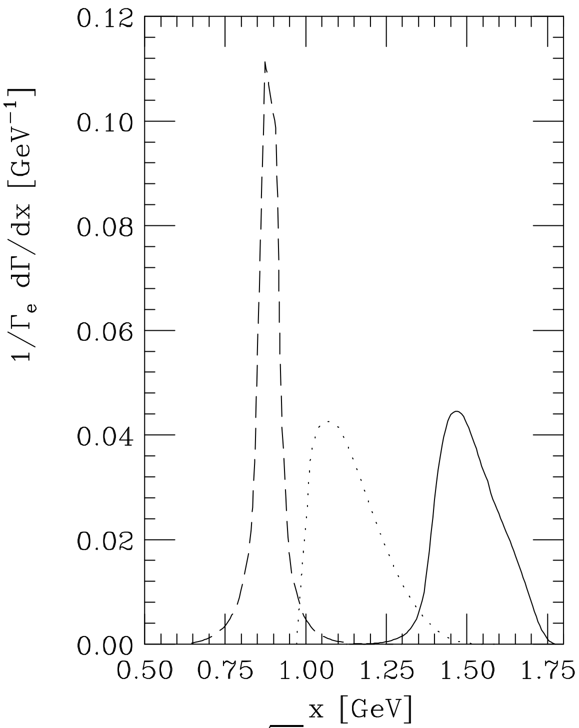

Figure 4. $x = \sqrt{Q^2} = m(K^-\pi^-K^+)$ (solid), $x = \sqrt{s_1} = m(K^+\pi^-)$ (dashed) and $x = \sqrt{s_2} = m(K^-K^+)$ (dotted) invariant mass distributions for the $\tau \to K^-\pi^-K^+\nu_\tau$ decay mode normalized to Γ_e.

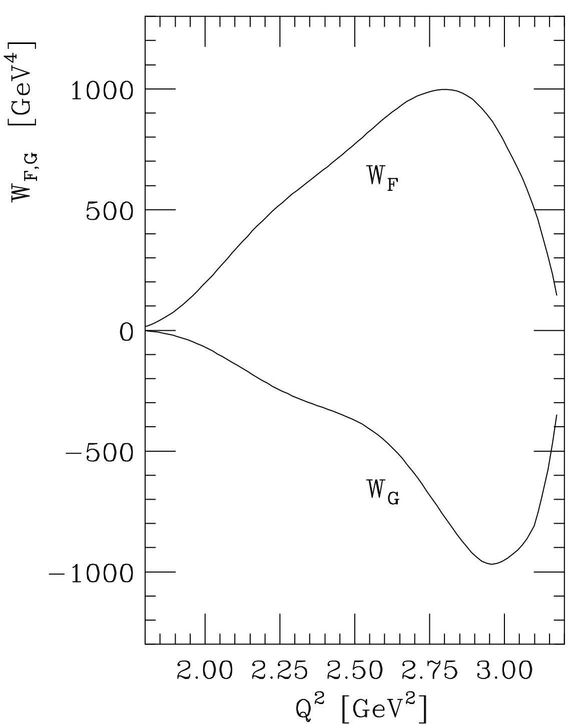

Figure 5. Axial vector-vector interference structure functions w_F and w_G for the $\tau \to K^-\pi^-K^+\nu_\tau$ decay mode.

modes are listed in table 4.

The form factors F_1 and F_2 are governed by the $J^P = 1^+$ three particle resonances with strangeness

$$T^{(a)}_{K_1}(Q^2) = \frac{1}{1+\xi}\Big[\mathrm{BW}_{K_1(1400)}(Q^2) + \xi\mathrm{BW}_{K_1(1270)}(Q^2)\Big] , \tag{23}$$

$$T^{(b)}_{K_1}(Q^2) = \mathrm{BW}_{K_1(1270)}(Q^2) \tag{24}$$

with $\xi = 0.33$ [4] and [21] (all numbers in GeV)

$$\begin{array}{ll} m_{K_1}(1400) = 1.402, & \Gamma_{K_1}(1400) = 0.174 \ , \\ m_{K_1}(1270) = 1.270 \ , & \Gamma_{K_1}(1270) = 0.090 \, . \end{array} \tag{25}$$

The three meson vector resonance in the 1^- configuration in the form factor F_3, denoted by $T^{(3m)}_{K^\star}$, include the higher radial excitations $K^{\star\prime}$ and $K^{\star\prime\prime}$

$$T^{(3m)}_{K^\star} = \frac{\mathrm{BW}_{K^\star} + \lambda\mathrm{BW}_{K^{\star\prime}} + \mu\mathrm{BW}_{K^{\star\prime\prime}}}{1+\lambda+\mu} \tag{26}$$

with

$$\lambda = -0.25, \quad \mu = -0.038 \tag{27}$$

and [21]

$$\begin{array}{ll} m_{K^\star} = 0.892\,\mathrm{GeV}\,, & \Gamma_{K^\star} = 0.050\mathrm{GeV}\,, \\ m_{K^{\star\prime}} = 1.412\,\mathrm{GeV}\,, & \Gamma_{K^{\star\prime}} = 0.227\,\mathrm{GeV}\,, \\ m_{K^{\star\prime\prime}} = 1.714\,\mathrm{GeV}\,, & \Gamma_{K^{\star\prime\prime}} = 0.323\,\mathrm{GeV}\,. \end{array} \tag{28}$$

The parameters λ and μ in Eq. (27) are those from [4]. Based on $SU(3)$ symmetry one should rather use $\lambda = -0.22$ and $\mu = -0.1$ as used in Eq. (20). However, the numerical significance of these details is fairly small, because of the small vector channel contribution in the relevant decay modes (about 10 % or less) [4]. We will therefore not discuss this problem here.

Our predicitons for the branching ratios of the various $K\pi\pi$ final states based on these parameters are listed in the second column of table 5. The results are about a factor of two larger than the experimental values (see the vertical dotted lines in Fig. 6.)

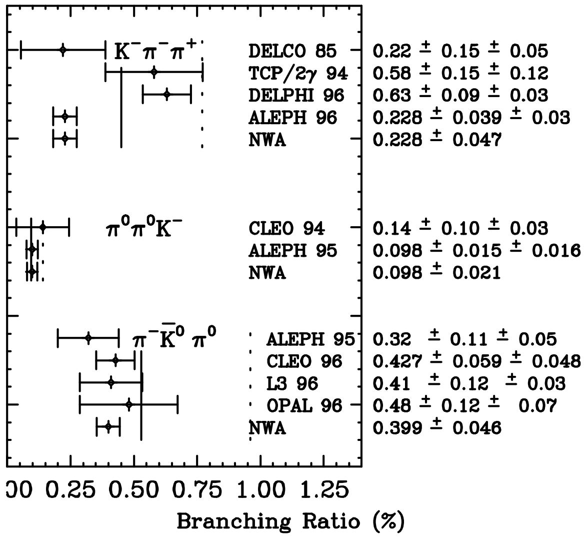

Figure 6. $\tau \to [K\pi\pi]^- \nu_\tau$ branching ratio measurements. The vertical dotted (solid) lines are the theoretical predictions from the second (third) column in table 5.

Moreover, QCD predicts [26]

$$\mathcal{B}(\tau \to \nu_\tau + \text{hadrons}(J^P = 1^+/0^-,\, S = -1)) = (1.30 \pm 0.06)\% \quad (29)$$

for the inclusive decay rate into axial vector or pseudoscalar hadronic states with strangeness -1. Subtracting from this the prediction for the branching ratio into a single kaon [3]

$$\mathcal{B}(\tau \to K\nu_\tau) = 0.72\% \quad (30)$$

we find that the axial vector contribution to the three $K\pi\pi$ final states must be less than 0.58%. Our prediction (second column in table 5), however, for this contribution is $\mathcal{B}(\tau \to (K\pi\pi)_A\nu_\tau) = 1.68\%$. So there is some strong indication that at least some of our predictions are by about a factor three too large.

We believe that we have identified the widths of the K_1 particles as the culprit. As mentioned before, the results in the second column in table 5 are based on the particle data group values for the widths of the two K_1 resonances (see Eq. (25)). We believe that these numbers are considerably too small, maybe by factors of two or three. We have three independent reasons to justify this statement.

1.) This is the only natural explanation we can find for the factor three discrepancy between the inclusive QCD constraint and our prediction in [4] for the rate of $\tau \to (K\pi\pi)_A\nu_\tau$. Note that most of the other predictions of the chirally normalized vector meson dominance model agree fairly well with data.

2.) From $SU(3)$ flavour symmetry and $\Gamma_{a_1} \approx 400 \cdots 600\,\text{MeV}$, the widths given in [21] seem unnaturally small.

3.) Until now, the widths of the K_1's have only been measured in hadronic production. These measurements have strong backgrounds, and results for the resonance parameters depend on the model used for the background. Remember that hadronic production of a_1 yielded small values for its widths, of about 300 MeV, incompatible with results from τ decays. In fact, it has been shown in [27] that by a modification of the coherent background in the diffractive hadronic prodution of a_1, a considerably larger width can be extracted which is compatible with τ data. In the K_1 measurements in [28], the same assumptions have been made as in [29], which yielded those small values for the a_1 width.

Table 5
Predictions for the branching ratios $\mathcal{B}(abc)$ in % for the $K\pi\pi$ decay modes. Results are shown for K_1 paramaters in Eq. (25) (second column, vector contribution in parentheses) and for $\Gamma_{K_1}(1400) = \Gamma_{K_1}(1270) = 0.250$ GeV (third column).

channel (abc)	Γ_{K_1} [Eq. (25)]	$\Gamma_{K_1} = 0.250$GeV
$\pi^0\pi^0K^-$	0.14 (0.012)	0.095
$K^-\pi^-\pi^+$	0.77 (0.077)	0.45
$\pi^-\overline{K^0}\pi^0$	0.96 (0.010)	0.53

The strong sensitivity to the K_1 width is demonstrated by the numbers in the third column of table 5, where predictions for the branching ratios based on $\Gamma_{K_1} = 0.250$ GeV are shown. The results are now much closer to the measured values (see the vertical solid lines in Fig. 6). A direct measurement of the K_1 parameters in τ decays, in particular a measurement of the widths, would be very interesting in view of these results.

Finally, Figs. 7 (8) show the resonance structure in the $K^-\pi^-\pi^+$ decay mode based on the two choices for the K_1 widths. The results in Fig. 7 are in good agreement with [22]. Fig. 4 in [25] shows the structure function w_B, which is

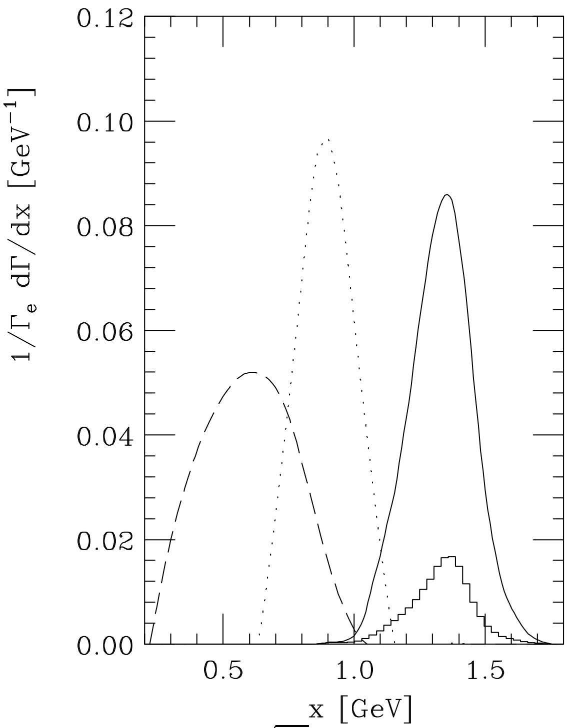

Figure 7. $x = \sqrt{Q^2} = m(K^-\pi^-\pi^+)$ (solid), $x = \sqrt{s_1} = m(\pi^+\pi^-)$ (dashed) and $x = \sqrt{s_2} = m(K^-\pi^+)$ (dotted) invariant mass distributions for the $\tau \to K^-\pi^-\pi^+\nu_\tau$ decay mode normalized to Γ_e. The results are based on $\Gamma_{K_1}(1400) = \Gamma_{K_1}(1270) = 0.250\,\text{GeV}$. The contribtion from the vector part ($\sim |F_3|^2$) to the $K^-\pi^-\pi^+$ invariant mass distribution is shown as the histogram.

very sensitive to the two choices of the K_1 widths.

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Figure 8. Same as Fig. 7 but with the K_1 parameters from Eq. (25).

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