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Review of Heavy Quark Effective Theory

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Abstract

A short review of a few selected topics in Heavy Quark Effective Theory is given. Applications to exclusive decays are discussed.

1 Introduction

Heavy Quark Effective Theory (HQET) or, more generally, the expansion in inverse powers of the heavy quark mass m_Q, has become a generally accepted and widely used tool in heavy quark physics[1]. Based on the infinite mass limit $m_Q \to \infty$ of QCD it provides a model independent starting point for the description of weak transitions involving heavy quarks.

The idea to exploit the fact that the mass of a heavy quark is large compared to the typical scale Λ of the light QCD degrees of freedom (e.g. the constituent mass of a light quark or the scale of the QCD coupling constant Λ_{QCD}) is in fact quite old[2]. However, in the late eighties a breakthrough was achieved by mainly two observations. First, in the infinite mass limit QCD exhibits an additional flavour symmetry and a spin symmetry[3], the group theory of which allow model independent statements concerning weak decays of heavy hadrons. Second, it was noted that the $1/m_Q$ expansion of QCD can be formulated as an effective field theory [4], which allows to access the corrections to the infinite mass limit in a systematic way.

Since then the field of heavy quark physics has attracted a lot of attention, documented by an enormous number of papers that have been published using these methods. In addition, from the experimental side a large effort is made to investigate the decays of bottom hadrons in order to pin down the origin of quark mixing and CP violation. In the standard model (SM) all of this is encoded in the CKM matrix, and the measurement of those elements of this matrix, which are only poorly known, involves mainly weak processes of b quarks.

HQET, or more generally the systematic application of the $1/m_Q$ expansion in QCD, has brought some progress in the determination of these CKM matrix elements, since heavy quark symmetries allow a drastic reduction of the hadronic uncertainties which enter the game through our ignorance to deal with the QCD bound state problem from first principles. In particular, the $b \to c$ semileptonic decay may considered as a heavy $\to$ heavy transition, where heavy quark symmetries work very efficiently; consequently, the $1/m_Q$ expansion allows for an almost model independent determination of the CKM matrix element V_{cb}.

In the next section a brief summary on HQET and heavy quark symmetries is given. Section 3 deals with the "picture book application", namely the exclusive $b \to c$ semileptonic decays. Heavy quark symmetries are also of some use in the case of a heavy hadron decaying into something light, and in section 4 this is considered for the case of heavy to light transitions. Finally a summary and a few conclusions are given.

2 Synopsis of HQET and heavy quark symmetries

2.1 Lagrangian and Fields

HQET is an effective field theory which may be obtained from QCD by performing a $1/m_Q$ expansion. The leading term corresponds to the infinite mass limit in which the heavy quark acts as a static color source. The momentum p_Q of the heavy quark scales with its mass and in order to perform the infinite mass limit it is convenient to use the velocity v of the heavy quark as the basic kinematic quantity. To this end the heavy quark momentum is split into a large part $m_Q v$ and a residual part k, which is assumed not to scale with the heavy mass. Thus

$$p_Q = m_Q v + k = m_Q \left(v + \frac{k}{m_Q} \right) \tag{1}$$

We shall consider exclusively hadrons containing only a single heavy quark such that in the infinite mass limit the velocity v of the heavy quark becomes simply the velocity of the heavy hadron.

In order to write down a field theory which describes the static heavy quark one may go through the usual steps of the construction of an effective field theory, namely to integrate out the heavy degrees of freedom [5]. An alternative method [6] is to perform a Foldy Wouthuysen Transformation as it is used in the standard non-relativistic reduction of the Dirac equation. Although the $1/m_Q$ expansion of the Lagrangian and the corresponding expansion of the fields look completely different, the results for physical matrix elements will be the same since the $1/m_Q$ expansion of the QCD Greens functions constructed from this effective theory has to be unique.

In the notation of [5] one obtains

$$Q(x) \;=\; e^{-im_Q vx} \left[1 + \left(\frac{1}{2m + ivD} \right) i \not{D}_\perp \right] h_v \tag{2}$$

$$
\begin{aligned}
&= e^{-im_Q vx}\left[1+\frac{1}{2m_Q}\not{D}_\perp+\left(\frac{1}{2m_Q}\right)^2(-ivD)\not{D}_\perp+\cdots\right]h_v \\
\mathcal{L} &= \bar{h}_v(ivD)h_v+\bar{h}_v i\not{D}_\perp\left(\frac{1}{2m+ivD}\right)i\not{D}_\perp h_v \qquad (3)\\
&= \bar{h}_v(ivD)h_v+\frac{1}{2m}\bar{h}_v(i\not{D}_\perp)^2 ih_v+\left(\frac{1}{2m}\right)^2\bar{h}_v(i\not{D}_\perp)(-ivD)(i\not{D}_\perp)h_v+\cdots
\end{aligned}
$$

where D is the covariant derivative of QCD and $Q(x)$ is the heavy quark field in full QCD, while h_v is the static heavy quark moving with the velocity v. Note that h_v corresponds to the upper components of the full field since

$$
P_+h_v=h_v,\quad P_-h_v=0,\quad P_\pm=\frac{1}{2}(\not{v}\pm 1) \tag{4}
$$

The leading terms of these expansions define the static limit; the static lagrangian

$$
\mathcal{L}_{stat}=\bar{h}_v(ivD)h_v \tag{5}
$$

is a dimension-four operator and defines (in combination with the usual lagrangian for the light degrees of freedom) a renormalizable field theory.

2.2 Heavy Quark Symmetries

In the case in which the bottom and the charam quark are assumed to be heavy one would write a static lagrangian for both quarks

$$
\mathcal{L}_{stat}=\bar{b}_v(v\cdot D)b_v+\bar{c}_{v'}(v\cdot D)c_{v'}, \tag{6}
$$

where b_v ($c_{v'}$) is the field operator for the b (c) quark moving with velocity v (v'). In particular, the masses of the heavy quarks do not appear in the Lagrangian (6), and as a consequence (6) in the case $v=v'$ exhibits an $SU(2)$ Heavy Flavour Symmetry which rotates the b_v field into the c_v field.

The static heavy quark field h_v is still a two component object corresponding to the upper component of the full heavy quark field Q. However, both spin directions couple in the same way to the gluons; we may rewrite the leading-order Lagrangian as

$$
\mathcal{L}=\bar{h}_v^{+s}(ivD)h_v^{+s}+\bar{h}_v^{-s}(ivD)h_v^{-s}, \tag{7}
$$

where $h_v^{\pm s}$ are the projections of the heavy quark field on a definite spin direction s

$$h_v^{\pm s} = \frac{1}{2}(1 \pm \gamma_5 \not{s}) h_v, \quad s \cdot v = 0. \tag{8}$$

This Lagrangian has a symmetry under the rotations of the heavy quark spin and hence all the heavy hadron states moving with the velocity v fall into spin-symmetry doublets as $m_Q \to \infty$. The simplest spin-symmetry doublet in the mesonic case consists of the pseudoscalar meson $H(v)$ and the corresponding vector meson $H^*(v, \epsilon)$, since a 90°-spin rotation $R(\epsilon)$ around the rotation axis ϵ ($v\epsilon = 0$) yields

$$R(\epsilon)|H(v)\rangle = (-i)|H^*(v, \epsilon)\rangle, \tag{9}$$

In the heavy-mass limit the spin symmetry partners have to be degenerate and their splitting has to scale as $1/m_Q$. In other words, the quantity

$$\lambda_2 = \frac{1}{4}(M_{H^*}^2 - M_H^2) \tag{10}$$

has to be the same for all spin symmetry doublets of heavy ground state mesons. This is well supported by data: For both the (B, B^*) and the (D, D^*) doublets one finds a value of $\lambda_2 \sim 0.12$ GeV^2. This shows that the spin-symmetry partners become degenerate in the infinite mass limit and the splitting between them scales as $1/m_Q$.

In the infinite mass limit the symmetries imply relations between matrix elements involving heavy quarks. For a transition between heavy ground-state mesons H (either pseudoscalar or vector) with heavy flavour f (f') moving with velocities v (v'), one obtains in the heavy-quark limit

$$\langle H^{(f')}(v')|\bar{h}_{v'}^{(f')} \Gamma h_v^{(f)}|H^{(f)}(v)\rangle = \xi(vv') \text{ Tr } \left\{ \overline{\mathcal{H}(v)} \Gamma \mathcal{H}(v) \right\}, \tag{11}$$

where Γ is some arbitrary Dirac matrix and $H(v)$ are the representation matrices for the two possibilities of coupling the heavy quark spin to the spin of the light degrees of freedom, which are in a spin-1/2 state for ground state mesons

$$\mathcal{H}(v) = \frac{\sqrt{M_H}}{2} \begin{cases} (1 + \not{v})\gamma_5 & 0^-, \ (\bar{q}Q) \text{ meson} \\ (1 + \not{v})\not{\epsilon} & 1^-, \ (\bar{q}Q) \text{ meson} \\ & \text{with polarization } \epsilon. \end{cases} \tag{12}$$

Due to the spin and flavour independence of the heavy mass limit the Isgur–Wise function ξ is the only non-perturbative information needed to describe all heavy to heavy transitions within a spin-flavour symmetry multiplet.

Similar statements may be derived for the spin symmetry doublets of excited heavy mesons[7] and also for baryons[8, 9, 10].

2.3 Corrections to the infinite mass limit

Corrections to the infinite mass limit may be considered in a systematic way. They fall into two classes: The recoil or $1/m_Q$ corrections and the QCD radiative corrections.

In order to discuss the corrections we shall consider a specific example, namely the matrix element of a current $\bar{q}\Gamma Q$ mediating a transition between a heavy meson and some arbitrary state $|A\rangle$. Using the expansions (3) and (2) one obtains up to order $1/m_Q$

$$\begin{aligned}\langle A|\bar{q}\Gamma Q|M(v)\rangle &= \langle A|\bar{q}\Gamma h_v|H(v)\rangle \\ &+\frac{1}{2m_Q}\langle A|\bar{q}\Gamma P_- i\not{D} h_v|H(v)\rangle - i\int d^4x\langle A|T\{L_1(x)\bar{q}\Gamma h_v\}|H(v)\rangle + \mathcal{O}(1/m^2)\end{aligned} \tag{13}$$

where L_1 are the first-order corrections to the Lagrangian as given in (3). Furthermore, $|M(v)\rangle$ is the state of the heavy meson in full QCD, including all its mass dependence, while $|H(v)\rangle$ is the corresponding state in the infinite mass limit.

Expression (13) displays the generic structure of the higher-order corrections as they appear in any HQET calculation. There will be local contributions coming from the expansion of the full QCD field; these may be interpreted as the corrections to the currents. The non-local contributions, i.e. the time-ordered products, are the corresponding corrections to the states and thus in the r.h.s. of (13) only the states of the infinite-mass limit appear.

Although the $1/m_Q$ corrections need in general additional input beyond HQET, there is one important result on the corrections linear in $1/m_Q$, which is called Luke's theorem[11] and which is the application of the Ademollo Gatto theorem[12] to the case of heavy flavour symmetry. In its general form the theorem states that in the presence of explicit symmetry breaking the matrix elements of the symmetry generating currents, which are normalized due to the symmetry, do not receive corrections linear in the symmetry breaking. Applied to the case at hand this means that some of the form factors in weak decays, namely the ones proportional to the Isgur Wise function, receive only corrections quadratic in $1/m_Q$. This result has important phenomenological consequences, which we shall discuss below.

All the relations given up to now are tree level relations. Going beyond tree level will induce QCD radiative corrections of order $\alpha_s^n(m_Q)$, $n = 1, \ldots$. As in any field theory these corrections are perturbatively calculable in terms of Feynman diagramms. The effective theory has two additional Feynman rules (the propagator of the heavy quark and the coupling of the heavy quark to the gluons) which may be read off from the static Lagrangian (5).

For the sake of clarity we shall stick to our example of a heavy light current considered above. To leading order in the $1/m_Q$ expansion one may evaluate the radiative corrections to such a matrix element using the above Feynman rules and finds a divergent result with a divergence related to the short distance behavior. Since HQET is an effective theory, the machinery of effective theory guarantees the factorization of long distance effects from the short distance ones, which are related to the large mass m_Q. Neglecting $1/m_Q$ corrections, this factorization takes the form

$$\langle A|\bar{q}\Gamma Q|M(v)\rangle = Z\left(\frac{m_Q}{\mu}\right)\langle A|\bar{q}\Gamma h_v|H(v)\rangle|_\mu + \mathcal{O}(1/m_Q) \tag{14}$$

From Feynman rule calculation one obtains the perturbative expansion of the renormalization constant Z which generically looks like

$$\begin{aligned} Z\left(\frac{m_Q}{\mu}\right) &= a_{00} \\ &+ a_{11}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{10}\alpha_s \\ &+ a_{22}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{21}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) + a_{20}\alpha_s^2 \\ &+ a_{33}\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^3 + a_{32}\alpha_s\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right)^2 + a_{31}\alpha_s^2\left(\alpha_s \ln\left(\frac{m_Q}{\mu}\right)\right) \\ &\qquad + a_{30}\alpha_s^3 + \cdots \end{aligned} \tag{15}$$

where $\alpha_s = g^2/(4\pi)$.

This factorization theorem corresponds to the statement that the ultraviolet divergencies in the effective theory have to match the logarithmic mass dependences of full QCD. The factorization scale μ is an arbitrary parameter, and the physical quantity $\langle A|\bar{q}\Gamma Q|M(v)\rangle$ does not depend on this parameter. However, calculating the matrix element of this operator in the effective theory and studying its ultraviolet behavior allows us to access the mass dependence of the matrix element $\langle A|\bar{q}\Gamma Q|M(v)\rangle$.

The ultraviolet behavior of the effective theory is investigated by the renormalization group equation (RGE), which is obtained in the usual way from differentiating (14) with respect to the factorization scale μ. The RGE for the short distance coefficient Z becomes

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g} + \gamma_J(g)\right) Z\left(\frac{m_Q}{\mu}, g\right) = 0. \tag{16}$$

where $\gamma_J(g)$ is the anomalous dimension of the current J which is related to the ultraviolet behavior of the matrix elements of J. The function $\beta(g)$ defines the running of the coupling constant

$$\frac{d}{d\ln\mu} g(\mu) = \beta(\mu). \tag{17}$$

Both functions $\gamma_J(g)$ and $\beta(g)$ are calculable in perturbation theory using a loopwise expansion, where the first term of the β function of QCD is well known

$$\beta(g) = -\frac{1}{(4\pi)^2}\left(11 - \frac{2}{3}n_f\right) g^3 + \cdots, \tag{18}$$

where n_f is the number of flavors with a mass less than m_Q.

With this input the renormalization group equation may be solved to yield

$$Z\left(\frac{m_Q}{\mu}\right) = a_{00}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{48\pi^2}{33-2n_f}\gamma_1} \tag{19}$$

where γ_1 is the first coefficient in the perturbative expansion of the anomalous dimension $\gamma_J = \gamma_1 g^2 + \cdots$ and $\alpha_s(\mu)$ is the one loop expression for the running coupling constant of QCD

$$\alpha_s(\mu) = \frac{12\pi}{(33-2n_f)\ln(\mu^2/\Lambda^2_{QCD})} \tag{20}$$

which is obtained from solving (17) using (18).

This expression corresponds to a summation of the leading logarithms $(\alpha_s \ln m_Q)^n$ which is achieved by a one-loop calculation of the renormalization group functions β and γ_Q; in other words, in this way a resummation of the first column of the expansion (15) is achieved.

In a similar way one may also resum the second column of (15), if the renormalization group functions β and γ are calculated to two loops and the non-logarithmic terms of the one loop expression are included.

Eq.(16) describes the renormalization group scaling in the effective theory. It allows to shift logarithms of the large mass scale from the matrix element of J into the coefficient Z: If the matrix element is renormalized at the large scale m_Q the logarithms of the type $\ln m_Q$ will apear in the matrix element of J while the coefficient Z at this scale will simply be

$$Z(1) = a_{00} + a_{10}\alpha_s(m_Q) + a_{20}\alpha_s^2(m_Q) + a_{30}\alpha_s^3(m_Q) + \cdots \qquad (21)$$

The renormalization group equation (16) allows to lower the renormalization point from m_Q to μ; the matrix element renormalized at μ will not contain any logarithms of m_Q any more, they will appear in the coefficient Z in the way shown in (15).

In all cases relevant in the present context the matrix elements will be matrix elements involving hadronic states, which are in most cases impossible to calculate from first principles. However, eq.(16) allows to extract the short distance piece, i.e. the logarithms of the large mass m_Q and to separate it into the Wilson coefficients.

Finally, the case we have considered as an example is indeed very simple; in general all operators of a given dimension may mix under renormalization, i.e. instead of a simple anomalous dimension a matrix of anomalous dimensions may occur and the renormalization group equation 16 becomes a system of differential equations.

3 Exclusive semileptonic $b \to c$ transitions

In this section we shall discuss the transitions of the heavy to heavy type, i.e. the $b \to c$ decays. The implications of heavy quark symmetry have been given already in the form of the Wigner Eckart theorem (11) in the last section, so we shall focus here on the status of the corrections to the infinite mass limit and give the phenomenological applications of the results.

3.1 QCD Radiative Corrections

The matrix elements relevant for the processes under consideration are

$$V_\mu = \langle D^{(*)}(v')|\bar{c}\gamma_\mu b|B(v)\rangle, \qquad A_\mu = \langle D^*(v')|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle \tag{22}$$

where the point $v = v'$ is of particular interest, since the absolute normalization of the matrix elements is known at this point due to heavy quark symmetries.

In order to evaluate the QCD radiative corrections to these matrix elements we shall make use of the renormalization group (RG) machinery as outlined in the last section. At a large scale $\mu \sim M_W$ both currents are conserved in the limit of vanishing b and c quark masses and hence their anomalous dimension vanishes. Running down from M_W to m_b, i.e. lowering the renormalization scale of the matrix elements (22) from M_W to m_b induces no large logarithms of the form $\alpha_s(M_W)\ln(M_W/m_b)$, rather the corrections will be small, of the order $\alpha_s(M_W)/\pi$.

Similarly, at scales μ below the charm quark mass m_c both quarks may be taken to be infinitely heavy, and at the non-recoil point $v = v'$ again the two currents are conserved and thus their anomalous dimension vanishes. Running below the charm quark mass will thus induce only small corrections of the order $\alpha_s(m_c)$.

Thus the main corrections originate from scales μ between m_b and m_c. In the effective theory where the b is taken to be infinitely heavy and the c is still light the one[13] and two loop[14] anomalous dimensions have been calculated and allow a resummation of terms of order $(\alpha_s(m_b)\ln(m_b/m_c))^n$, and $\alpha_s(m_b)(\alpha_s(m_b)\ln(m_b/m_c))^{n-1}$ respectively. Furthermore, in this theory the subleading terms of order $1/m_b$ [15] and $1/m_b^2$ [16] have been considered at the one loop level, and the matching at the scale m_c yields a resummation of terms of order $(m_b/m_c)(\alpha_s(m_b)\ln(m_b/m_c))^n$ and $(m_b/m_c)^2(\alpha_s(m_b)\ln(m_b/m_c))^n$.

The procedure described here has the disadvantage that numerically it is useful only in the limit $m_b \gg m_c$ such that $\ln(m_b/m_c) \gg 1$ is a large logarithm and terms of order $(m_c/m_b)^n\alpha_s(m_b)\ln(m_b/m_c))$ may be neglected. In real life we have $m_b/m_c \sim 3$ and one may think of simply performing the one loop calculation including the masses in full QCD and use this to match directly QCD to a theory with two static quarks, where heavy quark symmetries hold. In this way one obtains all terms of order $(m_c/m_b)^n \alpha_s \ln(m_b/m_c))$. However, the price to pay is a scale ambiguity in the scale of α_s which has to be taken at some scale $\bar{m}$ between m_b and m_c. Numerically this is not a problem since α_s does not run much between m_c and m_b.

Recently the latter procedure has ben applied at the two loop level [17] such that the terms of order $(m_b/m_c)^n \alpha_s^2(\bar{m}) \ln(m_b/m_c))$ are now known. The usual way to parametrize the corrections is by introducing corrections factors

$$V_\mu \to \eta_V V_\mu, \qquad A_\mu \to \eta_A A_\mu \tag{23}$$

which are known up to terms of order $\alpha_s^3(\bar{m})$. Taking the numbers of [17] one has

$$\eta_V = 1 + 0.018 + 0.004 + \mathcal{O}(\alpha_s^3(\bar{m})) = 1.022 \pm 0.004 \tag{24}$$
$$\eta_A = 1 - 0.033 - 0.007 + \mathcal{O}(\alpha_s^3(\bar{m})) = 0.960 \pm 0.007 \tag{25}$$

The uncertainty given here is the uncertainty due to the terms of order $\alpha_s^3(\bar{m})$ and is conservatively estimated by the size of the calculated $\alpha_s^2(\bar{m})$ corrections.

Given the fact that the calculated QCD corrections are already below the level of 1% one needs to worry also about QED corrections. In fact, the QED corrections to processes of this type have been calculated already some time ago in the context of μ decay[18]. Similar to the QCD corrections they may be factorized and in principle one could apply also renormalization group methods. However, the running of the QED coupling is negligible at this level and the QED corrections factor is given by

$$\eta_{A/V} \to \eta_{A/V} \left[1 + \frac{\alpha_{QED}}{\pi} \ln \frac{M_Z}{m_b}\right] \sim 1.013\ \eta_{A/V} \tag{26}$$

Hence short distance QED corrections enhance the matrix elements by 1.3%.

3.2 Recoil Correction

In general the recoil corrections are more complicated, since they may not be calculated from the effective theory, rather they can only be parametrized in terms of new matrix elements. The calculation of these matrix elements in any case needs additional input (such as models or a lattice calculation) beyond the framework of HQET.

However, in some cases Luke's theorem applies. In particular, the axial vector current taken at the non-recoil point $v = v'$ is protected by this theorem, such that

$$\langle D^*(v')|\bar{c}\gamma_5\gamma_\mu b|B(v)\rangle = 2\sqrt{M_{D^*}M_B}\epsilon_\mu\left(1+\delta_{1/m^2}\right), \quad \delta_{1/m^2} = \mathcal{O}\left(\frac{1}{m_b^2}, \frac{1}{m_c^2}\frac{1}{m_c m_b}\right) \tag{27}$$

Among the first non-trivial corrections the ones of the order $1/m_c^2$ will be the most important, and HQET allows only to parametrize them in terms of new matrix elements. One obtains

$$\begin{aligned}
\delta_{1/m^2} &= -\left(\frac{1}{2m_c}\right)^2 \frac{1}{2}\left(-\lambda_1 + \lambda_2\right. \\
&+ \left.(-i)^2 \frac{1}{2\sqrt{M_B M_D}} \int d^4x\, d^4y\, \langle B^*(v,\epsilon)|T\left[\mathcal{L}_b^{(1)}(x)\bar{b}_v c_v \mathcal{L}_c^{(1)}(y)\right]|D^*(v,\epsilon)\rangle\right) \\
&+ \mathcal{O}(1/m_c^3, 1/m_b^2, 1/(m_c m_b)),
\end{aligned} \tag{28}$$

where $\mathcal{L}_Q^{(1)}$ is the first order Lagangian for the quark Q as given in (3). Furthermore, the parameters λ_1 and λ_2 are the kinetic energy and the chromomagnetic moment of the heavy quark. They also appear in the $1/m_Q$ expansion of the heavy meson mass

$$m_H = m_Q\left(1 + \frac{\bar{\Lambda}}{m_Q} + \frac{1}{2m_Q^2}\left(\lambda_1 + d_H\lambda_2\right) + \mathcal{O}(1/m_Q^3)\right) \tag{29}$$

where $d_H = 3$ for the 0^- and $d_H = -1$ for the 1^- meson, and may be related to the matrix elements

$$\bar{\Lambda} = \frac{\langle 0|q\, iv\overleftarrow{D}\,\gamma_5 h_v|H(v)\rangle}{\langle 0|q\gamma_5 h_v|H(v)\rangle} \tag{30}$$

$$\lambda_1 = \frac{\langle H(v)|\bar{h}_v(iD)^2 h_v|H(v)\rangle}{2M_H} \tag{31}$$

$$\lambda_2 = \frac{\langle H(v)|\bar{h}_v\sigma_{\mu\nu}iD^\mu iD^\nu h_v|H(v)\rangle}{2M_H} \tag{32}$$

The only parameter which is easy to access is λ_2, since it is related to the mass splitting between $H(v)$ and $H^*(v,\epsilon)$. From the B-meson system we obtain

$$\lambda_2(m_b) = \frac{1}{4}(M_{H^*} - M_H) = 0.12 \text{ GeV}^2; \tag{33}$$

and from the charm system (including the renormalization group scaling of λ_1) the same value is obtained. This shows that indeed the spin-symmetry partners are degenerate in the infinite mass limit and the splitting between them scales as $1/m_Q$.

The other parameters appearing in (29) are not simply related to the hadron spectrum. Furthermore, they exhibit renormalon ambiguities, which imply that a proper prescription has to be given how to extract these quantities from data using renormalized perturbation theory.

Recently there has been such an attempt [19], namley to extract $\bar{\Lambda}$ and λ_1 from the shape of the lepton energy spectrum in inclusive semileptonic B decays. The values obtained from this analysis are $\bar{\Lambda} = 0.39 \pm 0.11$ GeV and $-\lambda_1 = 0.19 \pm 0.10$ GeV^2, where the $\overline{\text{MS}}$ definition of the mass has been used. The uncertainties quoted are only the 1σ statistical ones; the systematical uncertainties of this approach are difficult to estimate.

These considerations only fix the local matrix elements in (28), while the nonlocal terms involving the time-ordered products are much harder to estimate. The estimates found in the literature have to rely on some model estimates and span a range which implies roughly a 3% theoretical uncertainty. A commonly accepted number was given in [20]

$$\delta_{1/m^2} = -0.0055 \pm 0.025 \tag{34}$$

3.3 Phenomenology

3.3.1 Differential rates

In the last few years a lot of data has been accumulated which may be used to test heavy quark symmetry and to extract CKM matrix elements with the use of HQET. The exclusive decays of prime interest are the transitions $B \to D\ell\nu_\ell$ and $B \to D^*\ell\nu_\ell$. The relevant matrix elements are

$$\langle D(v')|\bar{c}\gamma_\mu b|B(v)\rangle = \sqrt{m_B m_D}\left[\xi_+(y)(v_\mu + v'_\mu) + \xi_-(y)(v_\mu - v'_\mu)\right] \tag{35}$$

$$\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu b|B(v)\rangle = i\sqrt{m_B m_{D^*}}\xi_V(y)\varepsilon_{\mu\alpha\beta\rho}\epsilon^{*\alpha}v'^\beta v^\rho \tag{36}$$

$$\langle D^*(v',\epsilon)|\bar{c}\gamma_\mu\gamma_5 b|B(v)\rangle = \sqrt{m_B m_{D^*}}\Big[\xi_{A1}(y)(vv'+1)\epsilon^*_\mu - \xi_{A2}(y)(\epsilon^* v)v_\mu - \xi_{A2}(y)(\epsilon^* v)v'_\mu\Big]\,, \tag{37}$$

where we have defined $y = vv'$. Thus in general there are six form factors, which in the heavymass limit for both the b and the c quark may be related to the Isgur Wise function as introduced in (11)

$$\xi_i(y) = \xi(y) \text{ for } i = +, V, A1, A3, \qquad \xi_i(y) = 0 \text{ for } i = -, A2. \tag{38}$$

In particular, at the non-recoil point $v = v'$ we have due to heavy quark symmetry and Lukes theorem

$$\xi_i(1) = 1 + \mathcal{O}(1/m_Q^2) \text{ for } i = +, V, A1, A3, \quad \xi_i(1) = \mathcal{O}(1/m_Q) \text{ for } i = -, A2. \tag{39}$$

Note that the form factors for which there is no normalization at $v = v'$ there is also no protection against corrections of lienar order in $1/m_Q$.

The differential rates for the exclusive semiletonic $b \to c$ transitions may be expressed in terms of the six form factors as

$$\frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \tag{40}$$
$$= \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2\left(m_D\sqrt{y^2-1}\right)^3\left|\xi_+(y) - \frac{m_B - m_D}{m_B + m_D}\xi_-(y)\right|^2$$
$$\frac{d\Gamma}{dy}(B \to D^*\ell\nu_\ell) \tag{41}$$
$$= \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B - m_{D^*})^2 m_{D^*}^2\left(m_{D^*}\sqrt{y^2-1}\right)(y+1)^2|\xi_{A1}(y)|^2\sum_{i=0,\pm}|H_i(y)|^2$$

with

$$|H_\pm(y)|^2 = \frac{m_B^2 - m_{D^*}^2 - 2y m_B m_{D^*}}{(m_B - m_{D^*})^2}\left[1 \mp \sqrt{\frac{y-1}{y+1}}R_1(y)\right]^2 \tag{42}$$
$$|H_0(y)|^2 = \left(1 + \frac{m_B(y-1)}{m_B - m_{D^*}}\left[1 - R_2(y)\right]\right)^2 \tag{43}$$

where we have defined the form factor ratios

$$R_1(y) = \frac{\xi_V(y)}{\xi_{A1}(y)}, \quad R_2(y) = \frac{\xi_{A3}(y) + \frac{m_B}{m_{D^*}}\xi_{A2}(y)}{\xi_{A1}(y)} \tag{44}$$

In the heavy mass limit these differential rates depend only on the Isgur-Wise function

$$\frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \overset{m_b,m_c\to\infty}{=} \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2\left(m_D\sqrt{y^2-1}\right)^3|\xi(y)|^2 \quad (45)$$

$$\frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \overset{m_b,m_c\to\infty}{=} \quad (46)$$

$$\frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B-m_{D^*})^2 m_{D^*}^2\left(m_{D^*}\sqrt{y^2-1}\right)(y+1)^2 \quad (47)$$

$$\left[1+\frac{4y}{y+1}\frac{m_B^2-m_{D^*}^2-2ym_Bm_{D^*}}{(m_B-m_{D^*})^2}\right]|\xi(y)|^2$$

These relations allow for a test of heavy quark symmetry, since the ratios of the differential rates do not depend on any unknown form factor any more. In particular the ratios R_1 (R_2) measures the ratio of the differential transverse (longitudinal) rate and the total differential rate. In the heavy mass limit both R_1 and R_2 are unity; this has to be compared to the measurements by CLEO [21, 22]

$$R_1 = 1.24 \pm 0.26 \pm 0.12 \quad (48)$$

$$R_2 = 0.72 \pm 0.18 \pm 0.07 \quad (49)$$

3.3.2 The determination of V_{cb}

From the measured lepton invariant mass spectrum one may determine V_{cb} in a model independent way by extrapolating to the kinematical endpoint of maximal momentum transfer to the leptons, corresponding to the point $v = v'$. At this point heavy quark symmetries determine the absolute normalization of some of the form factors and the corrections to this normalization have been discussed in section 2.

The mode $B \to D^*\ell\nu_\ell$ has the advantage of a higher branching fraction and hence we shall start the discussion with this decay. The relevant formula may be derived from (41) and reads

$$\lim_{y\to 1}\frac{1}{\sqrt{y^2-1}}\frac{d\Gamma}{dy}(B\to D^*\ell\nu_\ell) = \frac{G_F^2}{4\pi^3}(m_B-m_{D^*})^2 m_{D^*}^3|V_{cb}|^2|\xi_{A1}(1)|^2 \quad (50)$$

The form factor ξ_{A1} is normalized due to heavy quark symmetries and is hence protected against $1/m_Q$ corrections at $v = v'$ by Lukes theorem. Hence we have

$$\xi_{A1}(1) = \eta_A(1 + \delta_{1/m^2}) \tag{51}$$

Including QED corrections and the estimate of the $1/m_Q^2$ corrections in the way discussed in section 2 one obtains

$$\xi_{A1}(1) = 0.92 \pm 0.03 \tag{52}$$

From this value one may extract from the extrapolations shown in figs.1 and 2 a value for V_{cb}

$$|V_{cb}| = 0.0362 \pm 0.0019 \pm 0.0020 \pm 0.0014 \text{ (CLEO)[23]} \tag{53}$$

$$|V_{cb}| = 0.0345 \pm 0.0025 \pm 0.0027 \pm 0.0015 \text{ (ALEPH)[24]} \tag{54}$$

where the last error reflects the theoretical uncertainty of the $1/m_Q$ corrections.

Recently CLEO has also measured the leptonic invariant mass spectrum for the decay $B \to D\ell\nu_\ell$. In a similar way one may perform an extrapolation to obtain V_{cb}. Here one gets from (40)

$$\begin{aligned} \lim_{y\to 1} \quad & \left(\frac{1}{\sqrt{y^2-1}}\right) \frac{d\Gamma}{dy}(B \to D\ell\nu_\ell) \\ & = \frac{G_F^2}{48\pi^3}(m_B + m_D)^2 m_D^3 |V_{cb}|^2 \left|\xi_+(1) - \frac{m_B - m_D}{m_B + m_D}\xi_-(1)\right|^2 \end{aligned} \tag{55}$$

In this case a form factor enters which is not protected by Lukes theorem, $\xi_-(1) = \mathcal{O}(1/m_Q)$. However, this does not spoil the possibility to determine $|V_{cb}|$ from this mode, since the $1/m_Q$ corrections are kinematically suppressed by the factor $(m_B - m_D)/(m_B + m_D)$. Here we have

$$\left|\xi_+(1) - \frac{m_B - m_D}{m_B + m_D}\xi_-(1)\right| = \eta_V(1 + \Delta_{1/m_Q}) \tag{56}$$

where Δ_{1/m_Q} are the $1/m_Q$ corrections induced by $\xi_-(1)$. These corrections have been estimated recently [25]

$$\left|\xi_+(1) - \frac{m_B - m_D}{m_B + m_D}\xi_-(1)\right| = 0.98 \pm 0.07 \tag{57}$$

The data from CLEOcleovcb1 are shown in fig.3. From the extrapolation shown in fig.?? one obtain the value

$$|V_{cb}| = 0.0353 \pm 0.0046 \pm 0.0044 \pm 0.0025 \tag{58}$$

where the last error again reflects the theoretical uncertainty. Both values of V_{cb} are in very good agreement and are also consistent with values obtained form other methods, in particular with the results from inclusive semileptonic decays[34].

3.3.3 The slope of the form factors

The data on the leptonic invariant mass spectrum extend over the whole rage of y and the extrapolation to $y = 1$ has to rely on some ansatz for the Isgur Wise function. This may be used in turn to extract a value also for the slope of the Isgur Wise function.

Close to the point $y = 1$ one thus parametrizes the Isgur Wise function as

$$\xi(y) = 1 - \rho^2(y - 1) + \cdots \tag{59}$$

The theoretical predictions[27] for the slope parameter ρ^2 depend on matrix elements involving excited D mesons which are hard to estimate; thus the theoretical value is quite uncertain and ranges between

$$0.5 \le \rho^2 \le 1.1 \tag{60}$$

Data on the slope parameter come from the LEP experiments as well as from ARGUS and CLEO, the results obtained from experiment are

$$\rho^2 = \begin{cases} 0.29 \pm 0.21 \pm 0.12 & \text{ALEPH[28]} \\ 0.84 \pm 0.12 \pm 0.08 & \text{CLEO[29]} \\ 1.17 \pm 0.22 \pm 0.06 & \text{ARGUS[30]} \\ 0.81 \pm 0.16 \pm 0.10 & \text{DELPHI[31]} \end{cases} \tag{61}$$

which are compatible with the theoretical expectation.

4 Exclusive leptonic and semileptonic $b \to u$ transitions

Heavy quark symmetries may also be used to restrict the independent form factors appearing in heavy to light decays. For the decays of heavy mesons into light 0^- and 1^- particles heavy quark symmetries restrict the number of independent form factors to six, which is just the number needed to parametrize the semileptonic decays of this type. Furthermore, no absolute normalization of form factors may be obtained from heavy quark symmetries in the heavy to light case; only the relative normalization of B meson decays heavy to light transitions may be obtained from the corresponding D decays.

In general we shall discuss matrix elements of a heavy to light current which have the following structure

$$J = \langle A|\bar{q}\Gamma h_v|H(v)\rangle, \tag{62}$$

where Γ is an arbitrary Dirac matrix, q is a light quark (u, d or s) and A is a state involving only light degrees of freedom.

Spin symmetry implies that the heavy quark index hooks directly to the heavy quark index of the Dirac matrix of the current. Thus one may write for the transition matrix element (68)

$$\langle A|\bar{q}\Gamma h_v|H(v)\rangle = \text{ Tr } (\mathcal{M}_A \Gamma H(v)) \tag{63}$$

where the matrix $H(v)$ representing the heavy meson has been given in (12). The matrix $\mathcal{M}_A$ describes the light degrees of freedom and is the most general matrix which may be formed from the kinematical variables involved. Furthermore, if the energies of the particles in the state A are small, i.e. of the order of Λ_{QCD}, the matrix $\mathcal{M}_A$ does not depend on the heavy quark; in particular it does not depend on the heavy mass m_H. In the following we shall discuss some examples.

The first example is the heavy meson decay constant, where the state A is simply the vacuum state. The heavy meson decay constant is defined by

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 h_v|H(v)\rangle = f_H m_H v_\mu, \tag{64}$$

and since $|A\rangle = |0\rangle$ the matrix $\mathcal{M}_0$ is simply the unit matrix times a dimensionful constant[1] and one has, using (63)

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 h_v|H(v)\rangle = \kappa \text{ Tr } (\gamma\gamma_5 H(v)) = 2\kappa\sqrt{m_H} v_\mu. \tag{65}$$

[1]Note that contributions proportional to $\not{v}$ may be eliminated using

$$H(v)\not{v} = -H(v).$$

As discussed above the constant κ does not depend on the heavy mass and thus one infers the well-known scaling law for the heavy meson decay constant from the last two equations

$$f_H \propto \frac{1}{\sqrt{m_H}} \tag{66}$$

Including the leading and subleading QCD radiative corrections one obtains a relation between f_B and f_D

$$f_B = \sqrt{\frac{m_c}{m_b}} \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25} \left[1 + 0.894\frac{\alpha_s(m_c) - \alpha_s(m_b)}{\pi}\right] f_D \sim 0.69 f_D. \tag{67}$$

The second example are transitions of a heavy meson into a light pseudoscalar meson, which we shall denote as π. The matrix element corresponding to (62) is

$$J_P = \langle \pi(p)|\bar{q}\Gamma h_v|H(v)\rangle, \tag{68}$$

where p is the momentum of the light quark,

The Dirac marix $\mathcal{M}_P$ for the light degrees of freedom appearing now in (63) depends on p and v. It may be expanded in terms of the sixteen independent Dirac matrices 1, γ_5, γ_μ, $\gamma_5\gamma_\mu$, and $\sigma_{\mu\nu}$ taking into account that it has to behave like a pseudoscalar. The form factors appearing in the decomposition of $\mathcal{M}_P$ depend on the variable $v \cdot p$, the energy of the light meson in the rest frame of the heavy one. In order to compare different heavy to light transition by employing heavy flavor symmetry this energy must be sufficiently small, since the typical scale for the light degrees of freedom has to be of the order of Λ_{QCD} to apply heavy quark symmetry[2]. For the case of a light pseudoscalar meson the most general decomposition of $\mathcal{M}_P$ is

$$\mathcal{M}_P = \sqrt{v \cdot p} A(\eta)\gamma_5 + \frac{1}{\sqrt{v \cdot p}} B(\eta)\gamma_5 \not{p}, \tag{69}$$

where we have defined the dimensionless variable

$$\eta = \frac{v \cdot p}{\Lambda_{QCD}}. \tag{70}$$

[2]Note that in this case the variable $v \cdot p$ ranges between 0 and $m_H/2$ where we have neglected the pion mass. Thus at the upper end of phase space the variable $v \cdot p$ scales with the heavy mass and heavy quark symmetries are not applicable any more.

The form factors A and B are universal in the kinematic range of small energy of the light meson, i.e. where the momentum transfer to the light degrees of freedom is of the order Λ_{QCD}; in this region η is of order unity. This universality of the form factors may be used to relate various kinds of heavy to light transitions, e.g. the semileptonic decays like $D \to \pi e\nu$, $D \to K e\nu$ or $B \to \pi e\nu$ and also the rare decays like $B \to K\ell^+\ell^-$ or $B \to \pi\ell^+\ell^-$ where ℓ denotes an electron or a muon.

As an example we give the relations between exclusive semileptonic heavy to light decays. The relevant hadronic current for this case may be expressed in terms of two form factors

$$\begin{aligned} \langle\pi(p)|\bar{q}\gamma(1-\gamma_5)h_v|H(v)\rangle &= F_1(v\cdot p)m_H v_\mu + F_2(v\cdot p)p_\mu \\ &= F_+(v\cdot p)(m_H v_\mu + p_\mu) + F_-(v\cdot p)q_\mu \end{aligned} \tag{71}$$

where

$$F_\pm(v\cdot p) = \frac{1}{2}\left(F_1(v\cdot p) \pm F_2(v\cdot p)\right) \tag{72}$$

Inserting this into (68) one may express $F_\pm$ in terms of the universal form factors A and B

$$F_1(v\cdot p) = F_+(v\cdot p) + F_-(v\cdot p) = -2\sqrt{\frac{v\cdot p}{m_H}}A(\eta) \tag{73}$$

$$F_2(v\cdot p) = F_+(v\cdot p) - F_-(v\cdot p) = -2\sqrt{\frac{m_H}{v\cdot p}}B(\eta) \tag{74}$$

From these relations one may read off the scaling of the form factors with the heavy mass which was already derived in [32].

This may be used to normalize the semileptonic B decays into light mesons relative to the semileptonic D decays. One obtains

$$F_\pm^B(v{\cdot}p) = \frac{1}{2}\left(\sqrt{\frac{m_D}{m_B}} \pm \sqrt{\frac{m_B}{m_D}}\right)F_+^D(v{\cdot}p) + \frac{1}{2}\left(\sqrt{\frac{m_D}{m_B}} \mp \sqrt{\frac{m_B}{m_D}}\right)F_-^D(v{\cdot}p) \tag{75}$$

Note that F_+ for the B decay is expressed in terms of F_+ *and* F_- for the D decays. In the limit of vanishing fermion masses only F_+ contributes, which means that the F_- contribution to the rate is of the order of m_{lepton}/m_H. Thus it will be extremely difficult to determine experimentally.

The case of a heavy meson decaying into a light vector meson may be treated similarly. The matrix element for the transition of a heavy meson into a light vector meson (denoted generically as ρ in the following) is given again by (62) and is in this case

$$J_V = \langle \rho(p,\epsilon)|\bar{q}\Gamma h_v|H(v)\rangle. \tag{76}$$

Using (63) one has

$$\langle \rho(p,\epsilon)|\bar{q}\Gamma h_v|H(v)\rangle = \text{ Tr } \left(\mathcal{M}_V \Gamma H(v)\right), \tag{77}$$

where now the Dirac matrix $\mathcal{M}_V$ has to be a linear function of the polarization of the light vector meson.

The most general decomposition is given in terms of four dimensionless form factors

$$\mathcal{M}_V = \sqrt{v\cdot p}C(\eta)(v\cdot\epsilon) + \frac{1}{\sqrt{v\cdot p}}D(\eta)(v\cdot\epsilon)\not{p} + \sqrt{v\cdot p}E(\eta)\not{\epsilon} + \frac{1}{\sqrt{v\cdot p}}F(\eta)\not{p}\not{\epsilon} \tag{78}$$

where the variable η has been defined in (70).

Similar to the case of the decays into a light pseudoscalar meson (77) may be used to relate various exclusive heavy to light processes in the kinematic range where the energy of the outgoing vector meson is small. For example, the semileptonic decays $D \to \rho e\nu$, $D \to K^* e\nu$ and $B \to \rho e\nu$ are related among themselves and all of them may be related to the rare heavy to light decays $B \to K^*\ell^+\ell^-$ and $B \to \rho\ell^+\ell^-$ with $\ell = e, \mu$.

Data on these decays are still very sparse; there are first measurements of the decays $B \to \pi\ell\nu$ and $B \to \rho\ell\nu$ from CLEO [33], from which total rates may be obtained. From this one may extract a value of V_{ub} by employing form factor models, and the value given by CLEO is

$$|V_{ub}| = (2.6 \text{ to } 4.0 \pm 0.2^{+0.3}_{-0.4}) \times 10^{-3} \tag{79}$$

where the errors are purely experimental, while the range in the central value indicates the span obtained from a representative set of models. In order to perform a model independent determination along the lines discussed above a good measurement of the lepton energy spectra in these decays is needed.

5 Conclusions

The standard model has turned out to be suprisingly successful and has passed many tests, in particular the very precise tests performed at the LEP collider. However, these tests mainly concern the coupling of the gauge bosons to the fermions while the CKM sector of the standard model has not been tested with comparable accuracy.

To test this part of the standard model one has to investigate weak processes among quarks which are in general plagued with strong interaction uncertainties. In this respect the heavy mass limit has brought an enormous success; HQET opens the unique possibility to determine some of the CKM matrix elements in a model independent way, thereby reducing uncertainties through models considerably. In particular, it allows to at least give the order of magnitude of the uncertainties involved, since HQET relies on an expansion in $\alpha_s(m_Q)$ and $1/m_Q$.

Since the discovery of the heavy quark symmetries their phenomenological applications as well as the theoretical background have been studied intensively. The most prominent example is the determination of V_{cb}, which corresponds to a heavy to heavy transition. Combining the method as described in this mini-review fo exclusive decays with the $1/m_Q$ expansion for inclusive decays [34] one may by now determine V_{cb} up to an uncertainty significantly less than ten percent.

In heavy to light decays heavy quark symmetries do not work as efficiently; in this case only the relative normalization of B decays versus the corresponding D decays may be obtained. From the experimental side there are first measurements of $B \to \pi \ell \nu$ and $B \to \rho \ell \nu$ from the CLEO collaboration and an extraction of the CKM matrix element V_{ub} from these processes is still to some extent model dependent. A more model independent extraction of this matrix element has to wait for more data, in particular a measurement of the lepton energy spectrum is needed to exploit the relative normalization from heavy quark symmetry.

HQET does not yet have much to say about exclusive non-leptonic decays; even for the decays $B \to D^{(*)} D_s^{(*)}$, which involves three heavy quarks, heavy quark symmetries are not sufficient to yield useful relations between the decay rates [35]. Of course, with additional assumptions such as factorization one can go ahead and relate the non-leptonic decays to the semileptonic ones; however, this is a very strong assumption and it is not clear in what sense factorization is an approximation. On the other side, the data on the non-leptonic B decays support factorization, and first attempts to understand this from QCD and HQET have been untertaken [36]; however, the problem of the exclusive non-leptonic decays still needs clarification and hopefully the heavy mass expansion will also be useful here.

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Figure Captions

Fig.1 CLEO data[23] and the extrapolation used to obtain V_{cb}. The upper figure corresponds to a linear extrapolation, the lower one includes also a curvature.

Fig.2 ALEPH data[24] and the extrapolation used to obtain V_{cb}.

Fig.3 Data and the extrapolation used to obtain V_{cb} from $B \to D\ell\nu_\ell$. The figure is taken from [26]