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EFFECTIVE FIELD THEORY FOR A HEAVY HIGGS BOSON: MATCHING AND GAUGE INVARIANCE [a]

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For large values of the Higgs mass the low energy structure of the gauged linear sigma model in the spontaneously broken phase can adequately be described by an effective field theory. We present a manifestly gauge-invariant functional technique to explicitly evaluate the corresponding effective Lagrangian from the underlying theory.

1 Introduction

The method of effective field theory has repeatedly been used in the analysis of the symmetry breaking sector of the Standard Model. It provides a model independent parametrization of various scenarios for the spontaneous breakdown of the electroweak symmetry. The unknown physics is then hidden in the low energy constants of an effective Lagrangian.

The physics in the low energy region of a full theory is adequately described by an effective field theory if corresponding Green functions in both theories have the same low energy structure. One can take this matching requirement as the definition of the effective field theory. It determines functional relationships between the low energy constants of the effective Lagrangian and the parameters of the underlying theory.

The special role of gauge theories is readily understood. The effective field theory analysis should not make any particular assumptions about the

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underlying theory – apart from symmetry properties and the existence of a mass gap. This also requires parametrizing low energy phenomenology by a gauge-invariant effective Lagrangian. The definition of Green's functions, on the other hand, usually does not reflect the symmetry properties of a gauge theory. Gauge invariance is broken, and the off-shell behaviour of Green's functions is gauge-dependent. Therefore, if the Green functions which enter the matching relations do not reflect the symmetry properties of the full theory, the effective field theory will also include the corresponding gauge artifacts.

Without resolving these issues, different approaches to determine the low energy constants at order p^4 for the Standard Model with a heavy Higgs boson where presented in several recent articles [1]. For Higgs masses below about $1TeV$ the effective Lagrangian can be evaluted explicitly with perturbative methods. These works show clearly that the matching of gauge-dependent quantities causes all kinds of trouble. This calls for a new manifestly gauge-invariant technique which avoids these problems, yet maintains the simplicity and elegance of matching Green's functions as in the ungauged case.

2 A gauge-invariant approach

Any approach to determine the effective Lagrangian for a given underlying gauge theory should match only gauge-invariant quantities. Then one does not have to worry about any gauge artifacts which otherwise might enter the effective field theory. Perhaps the most straightforward idea that comes to mind is to match only S-matrix elements. However, this approach is quite cumbersome. In particular, it involves a detailed treatment of infrared physics.

Functional techniques like those described in Ref. [2] provide a much easier approach. In this case one matches the generating functionals of Green's functions in the full and the effective theory. Infrared physics drops out at a very early step of the calculation. The remaining contributions all involve the propagation of heavy particles over short distances. Hence, they can be evaluated with a short distance expansion. The computation of loop-integrals is not necessary.

Gauge invariance is broken as soon as Green's functions of gauge-dependent operators are considered. Hence, any manifestly gauge-invariant approach must confine itself to analyze Green's functions of gauge-invariant operators, such as the field strength of an Abelian gauge field or the density of the Higgs field. In the following we summarize a manifestly gauge-invariant technique to evaluate the effective Lagrangian describing the low-energy region of the gauged linear sigma model in the spontaneously broken phase. It involves only Green's functions of gauge-invariant operators. For any details the reader is referred to Ref. [3] where it has been applied to the Abelian case. The Higgs sector of the Standard model with the non-abelian group $SU_L(2) \times U_Y(1)$ can be treated in the same way [4]. We would like to point out, that all S-matrix elements of the theory can be evaluated from these Green functions as well.

At tree-level the generating functional is given by the classical action. Since the external sources are gauge invariant, i.e., do not couple to the gauge degrees of freedom, the equations of motion can be solved without gauge-fixing. As a result they have a whole class of solutions. Every two representatives are related to each other by a gauge transformation. In order to determine the leading contributions to the low energy constants one merely has to solve the equation of motion for the Higgs boson field.

To incorporate higher order corrections one may evaluate the path-integral representation of the generating functional with the method of steepest descent. In this case they are described by Gaussian integrals. Since gauge invariance is manifest, these integrals can also be evaluated without gauge-fixing. As a consequence the gauge degrees of freedom manifest themselves through zero-modes of the quadratic form in the exponent of the gaussian factor. The integration over these modes yields the volume of the gauge group, which can be absorbed by the normalization of the path-integral. The remaining integral over the non-zero modes contains all the physics.

We would like to point out one difference between this approach to evaluate a path-integral and the method of Faddeev and Popov: if the gauge is not fixed the evaluation of the path-integral does not involve ghost fields. Hence, the number of diagrams to compute is reduced.

The effective Lagrangian of the linear sigma model is a sum of gauge-invariant terms with an increasing number of covariant derivatives and gauge-boson mass factors, corresponding to an expansion in powers of the momentum and the masses. Note that the covariant derivative, the gauge-boson masses and the gauge couplings all count as quantities of order p. Thus, the low energy expansion is carried out such that all light-particle singularities are correctly reproduced. Furthermore, if the coupling λ of the scalar field is small enough, the low energy constants in the effective Lagrangian admit an expansion in powers of this quantity, corresponding to the loop expansion in the full theory. n-loop Feynman diagrams in the Abelian Higgs model yield corrections of order λ^{n-1} to the low energy constants.

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