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An estimate of the QCD corrections to the decay $Z \to W u \bar{d}$

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Abstract

We present an estimate of perturbative QCD corrections to the decay $Z \to W u \bar{d}$. A simple approximate approach is described in detail. The difference of masses of M_Z and M_W is used as an expansion parameter. A complete analytical formula for a part of the corrections is also presented.

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1 Introduction

The decay channel $Z \to WX$ was suggested in the early days of the Standard Model (SM) as a possible source of the W–bosons [1]. A number of detailed studies arrived at the conclusion that the branching ratio of this decay mode is extremely small. There are several factors which contribute to this strong suppression. First, it is a higher order electroweak process; second, it is suppressed due to the small difference in masses of the Z and W bosons; and third, it suffers from a destructive interference among contributions of different diagrams. On the other hand it was also pointed out that such a decay process would lead to an exceptional signature (in the case when W boson decays leptonically). Namely, there will be one charged lepton with the transverse energy $E_\perp \sim 40$ GeV accompanied by the same amount of missing energy. If the W decays hadronically it will be more difficult to isolate the signal from the background.

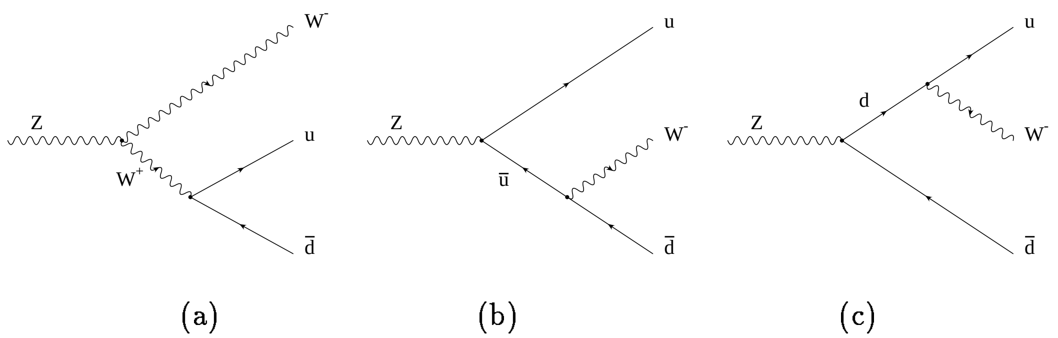

Figure 1: Contributions to the decay $Z \to Wu\bar{d}$ in the Born approximation

An interesting feature of this process is that it involves the three gauge boson coupling ZWW (see fig. 1a). This coupling gives the largest contribution to the decay rate. Unfortunately, there are also large gauge cancellations from the interference of the diagram with three gauge boson coupling with the other diagrams involved. In the SM the coupling ZWW is fixed. It is likely that effects of physics beyond the SM can modify this coupling and lead to the deviations from its SM value. In the context of the $Z \to WX$ decay channel this possibility has been analyzed in Ref. [2]. It has been concluded that in a special case the introduction of anomalous couplings the rate can be increased by up to one order of magnitude.

Let us present some numbers from Ref. [3] concerning the number of events one can expect per 10^7 Z events in the Standard Model:

$$N(Z \to Wl\nu \to l\nu l\nu) = 0.19,$$

$$N(Z \to Wl\nu \to q\bar{q}l\nu) = 0.38,$$

$$N(Z \to Wq\bar{q} \to l\nu q\bar{q}) = 0.35,$$

$$N(Z \to Wq\bar{q} \to q\bar{q}q\bar{q}) = 0.70.$$

Hence to observe the decay $Z \to WX$ within the SM requires more than 10^8 Z events. The largest sample of Z events has been collected by the four experiments at LEP1. Though the number of Z's is huge ($\sim 16 \cdot 10^6$) it is still not sufficient to observe the decay $Z \to WX$. On the other hand, the designed luminosity of a future e^+e^- collider will allow to produce 10^7 Z bosons per day, if the machine operates on the Z resonance. Such experiments are indeed being planned at the JLC and will allow a study of the rare decay channels of Z and in particular of $Z \to WX$.

In Ref. [4] the QCD corrections to this decay mode were discussed. The authors of that paper mainly concentrated on the non–perturbative and logarithmic perturbative corrections. They concluded that the considered corrections are small and can not strongly shift the lowest order branching ratio. Nevertheless, pure perturbative corrections $\mathcal{O}(\alpha_s(M_Z - M_W))$ remained unknown.

In this paper we present a simple estimate of these corrections, based on the expansion of the rate with respect to the small ratio $(M_Z - M_W)/M_Z$. In the next section we demonstrate the basic idea of our approach with the example of the lowest order decay rate. Then we discuss the QCD corrections; our results are summarized in the Conclusions.

2 Lowest order decay rate

Let us describe the idea of the calculation with the example of the Born approximation. We consider M_Z and M_W as large parameters (compared to $M_Z - M_W$). Then the quarks in the final state will have relatively small energies and momenta. Hence we can try to expand the amplitude for $Z \to Wu\bar{d}$ in the quantities of the order of $(M_Z - M_W)/M_Z$.

We adopt the following notation: p and k are momenta of Z and W, p_1 and p_2 are momenta of the quarks in the final state.

We begin with the diagram of Fig. 1b where the W–boson is emitted from a quark line. The virtual quark is far off-shell. Contracting its propagator to a point and neglecting momenta of the final quarks we obtain the following expression for the amplitude:

$$\mathcal{M}_2 = \frac{ie^2 g_-^u}{\sqrt{2} s_W M_W^2} \bar{u}(p_1) \hat{O} v(p_2), \qquad \hat{O} = \hat{\epsilon}_Z \hat{p} \hat{\epsilon}_W L. \tag{1}$$

Here $L = (1 - \gamma_5)/2$ and g_-^q describes Z coupling to the left-handed quarks, $g_-^q = I_3^q / s_W$. s_W denotes the sine of the weak mixing angle. We note that according to our approximation we put its cosine equal to 1.

Using Dirac algebra identities we can rewrite this as

$$\mathcal{M}_2 = -\frac{ie^2 g_-^u}{\sqrt{2} s_W M_W^2} (C_\mu + A_\mu) \bar{u}(p_1) \gamma^\mu L v(p_2) \tag{2}$$

where

$$C_\mu = p_\mu \epsilon_W \cdot \epsilon_Z, \quad A_\mu = i \epsilon_{\mu\alpha\beta\sigma} p^\alpha {\epsilon_W}^\beta {\epsilon_Z}^\sigma. \tag{3}$$

In deriving this equation we have used the fact that:

$$\epsilon_W \cdot p = \epsilon_Z \cdot p = 0, \tag{4}$$

which is exactly valid if $M_W = M_Z$.

The interference of the two pieces of the amplitudes, proportional to C_μ and A_μ, vanishes after the sum over W and Z polarizations. Note also that the amplitude $\mathcal{M}_3$ is described by the same equation, only the sign of C_μ is opposite.

The amplitude $\mathcal{M}_1$ involves a three gauge boson coupling. Taking there the limit $p_1+p_2 \to 0$ we arrive at the approximate formula:

$$\mathcal{M}_1 = \frac{ie^2\sqrt{2}}{s_W^2 M_W^2}(\epsilon_W \cdot \epsilon_Z) p_\mu \bar{u}(p_1)\gamma_\mu L v(p_2). \tag{5}$$

To calculate the leading asymptotics of the Born width we integrate over the light quark phase space and then over W three–momenta. Because of the approximate equality $M_Z \approx M_W$ the W boson is always non–relativistic. We approximate its phase space element by

$$\frac{d^3k}{2E_k} \approx \frac{4\pi}{2M_W}k^2 dk, \qquad k_{max} = \frac{M_Z}{2}x, \qquad x \equiv \frac{M_W^2}{M_Z^2}. \tag{6}$$

By counting the powers of $(M_Z - M_W)$ we see that the leading order behavior of the tree–level width is given by $(1-x)^5$.

The decay width for $Z \to Wu\bar{d}$ can be written as

$$\Gamma(Z \to Wu\bar{d}) = \Gamma_0 \left[\frac{g_-^{u\,2} + g_-^{d\,2}}{2} H_1(x) + \frac{1}{s_W^2} H_2(x) + g_-^u g_-^d H_3(x) - \frac{(g_-^u - g_-^d)}{s_W} H_4(x) \right]. \tag{7}$$

In the above equation we denote

$$\Gamma_0 = \frac{N_c M_Z \alpha^2}{16\pi s_W^2} \tag{8}$$

and $H_i(x)$ are the contributions of various diagrams to the width. Namely, $H_1(x)$ is the contribution of the squares of the graphs $1b$ or $1c$, $H_2(x)$ is the contribution of the graph $1a$ squared, $H_3(x)$ is $1b-1c$ interference and $H_4(x)$ is determined by $1a-1b$ and $1a-1c$ interferences.

The leading asymptotics of these functions are

$$H_i(x) = (1-x)^5 h_i^{(0)}\left(1 + C_F\frac{\alpha_s}{\pi}\delta_i\right); \tag{9}$$

$$h_1^{(0)} = \frac{1}{20}, \quad h_2^{(0)} = \frac{1}{30}, \quad h_3^{(0)} = \frac{1}{60}, \quad h_4^{(0)} = \frac{1}{30}.$$

The results of a complete Born level calculation can be found in [1]. Let us note that since we use the values $g_-^u = -g_-^d = \frac{1}{2s_W}$ we obtain at Born level complete cancellation between H_2 and H_4, potentially the largest contributions to the width. This is the destructive interference which makes the rate additionally suppressed.

Our aim in this paper is to discuss the QCD corrections to this decay width which are parametrized by the coefficients δ_i.

3 QCD corrections

First, consider QCD corrections to the square of the amplitude $\mathcal{M}_1$. Since the gluons couple to the external quarks the only QCD correction to $|\mathcal{M}_1|^2$ will be the correction to the correlator of two $V-A$ currents, which can be taken from W–decay. This has been calculated for the first time in Ref. [5]. The ratio of the one–loop correction to the Born contribution is[3] α_s/π.

Next, we analyze the QCD corrections to the remaining amplitudes. As an example let us take the graph were W–boson is emitted from the down quark line ($\mathcal{M}_2$). We discuss first the radiation of real gluons.

There are three graphs describing real gluon emission. In two of them the gluon is emitted from the external fermion line and in the third one from the fermion line with high ($\mathcal{O}(M_W^2)$) virtuality. The energy of the gluon is restricted due to the phase space suppression, therefore the third graph will have an additional suppression factor $\mathcal{O}((M_Z - M_W)^2/M_Z^2)$ relative to the Born one. It is therefore of no interest for us.

The leading contribution of the two remaining graphs can be found by contracting the virtual fermion propagator carrying the momentum $\mathcal{O}(M_{W,Z})$ to a point. The effective vertex which appears as the result of such contraction is equivalent to the effective vertex presented in the eq. (1). Unfortunately, the transformation which has been used in the transition from eq. (1) to eq. (2) can not be applied for the graphs which describe radiative corrections. The reason for this is the use of dimensional regularization and the presence of infra–red divergences in the graphs with emission of real gluons. This problem makes it necessary to perform an "honest" calculation of the $\mathcal{O}(\alpha_s)$ correction to the production of hadrons by the operator $\hat{O}$ defined in eq. (1).

Next, it is necessary to analyze virtual corrections. There are four of them. Note that there are no self energy corrections to the external quark lines because they are represented by no-scale diagrams which vanish in dimensional regularization. If we perform on–shell renormalization for the external quark lines this also means that there is no wave–function renormalization at all. Therefore the sum of virtual and real corrections must be finite on its own.

There is a simple way to rearrange virtual contributions to make them more transparent. First let us note that in the vertex corrections to $Z \to \bar{u}^* u$ or $\bar{u}^* \to Wd$ we can simply put the momenta of the "soft" quark equal to zero; this will not introduce any new divergences. The same also applies to the self energy correction to the virtual fermion line.

The remaining contribution is due to the box diagram. Here the situation is slightly more delicate — we can not put the momenta of the light quarks equal to zero because this leads to additional divergences. The solution is to apply the ideas of asymptotic expansions [6]. In that approach the contribution of the box graph can be obtained as a sum of two pieces: in the first one the large momentum flows through the fermion propagator which connects the Z and W vertices. To get the leading contribution we must contract it to the point. The second piece is the expansion of the whole box in Taylor series in the small external momenta. In both of these pieces one gets additional divergences (ultraviolet in

[3]This result is true not only for the leading asymptotics in $M_Z - M_W$.

the "contracted" part and infrared in the part originating from the Taylor series). These divergences are canceled in the sum. However, it is very useful to separate these pieces because of the following observation: The "contracted" part of the box taken together with the real emission graphs will give a complete $\mathcal{O}(\alpha_s)$ correction to the production of hadrons by an operator $\hat{O}$ in eq. (1). Therefore, this sum should be ultraviolet and infrared finite. For obvious reasons we will call this contribution "soft".

The remaining ("hard") virtual corrections, i.e. corrections to the $Z \to \bar{u}^* u$ vertex, $\bar{u}^* \to W\bar{d}$ vertex, self energy correction to the $\bar{u}^*$ propagator, and the box graph, all taken at zero momenta of external fermions, must also be finite. Note, however, that individual pieces are divergent and hence one has to perform the Dirac algebra in D–dimensions. The final expression for the hard correction to the amplitude which can be obtained in this way is remarkably simple:

$$\mathcal{M}_2^{hard} = \frac{ie^2 g_-^u}{\sqrt{2} s_W M_W^2} \frac{C_F \alpha_s}{4\pi} (2\, C_\mu + 7\, A_\mu) \bar{u}(p_1) \gamma_\mu L v(p_2). \tag{10}$$

To get an expression for $\mathcal{M}_3^{hard}$ it is sufficient to reverse the sign in front of C_μ in eq. (10). One gets

$$\mathcal{M}_3^{hard} = \frac{ie^2 g_-^u}{\sqrt{2} s_W M_W^2} \frac{C_F \alpha_s}{4\pi} (-\, 2\, C_\mu + 7\, A_\mu) \bar{u}(p_1) \gamma_\mu L v(p_2). \tag{11}$$

We remind that there is no interference between C_μ and A_μ structures, hence if we know their relative contributions to the Born width it is a trivial exercise to find a value of the QCD correction.

Let us demonstrate how this works by considering the QCD corrections to the interference of the diagrams $\mathcal{M}_1$ and $\mathcal{M}_3$. Consider first the correction due to the soft part of the diagram $\mathcal{M}_3$. The operator which "produces" final hadronic state can be written as

$$\hat{\epsilon}_W \hat{p} \hat{\epsilon}_Z L = -\hat{p} \left(\epsilon_W \cdot \epsilon_Z + \frac{1}{2} [\hat{\epsilon}_W, \hat{\epsilon}_Z] \right) L. \tag{12}$$

Here we have used Eq. (4). Note that because of the sum over polarizations of the Z and W bosons there is no interference between the first and the second structure in the above equation. The amplitude $\mathcal{M}_1$ is always proportional to the product $\epsilon_W \cdot \epsilon_Z$. Therefore in the soft correction only the first structure in eq. (12) contributes. The correction to it is again just the correction to the W decay width. As a result we find

$$\delta_4^{soft} = \frac{3}{4}. \tag{13}$$

Now the "hard" part of the correction comes from the interference of the C_μ structure of eq. (11) and eq. (3). Comparing coefficients and signs of the "hard" correction with the corresponding Born one we conclude that the hard correction to $H_4(x)$ is

$$\delta_4^{hard} = -\, \frac{1}{2} \tag{14}$$

and

$$\delta_4 = \delta_4^{soft} + \delta_4^{hard} = \frac{1}{4} \tag{15}$$

Clearly, the contribution of the diagram with the down-type quarks coupling to W–boson is the same up to trivial redefinitions of the coupling constant.

In a similar manner we obtain the QCD correction to the square of the amplitudes $\mathcal{M}_2$ or $\mathcal{M}_3$ and for their interference. The hard correction can be immediately determined from $\mathcal{M}_{2,3}^{hard}$. For the soft part, however, one needs a full calculation. This is by no means difficult and technically is equivalent to the calculation of the QCD correction to the e^+e^- annihilation to massless quarks. All together this gives

$$\delta_1 = -\frac{7}{12}. \tag{16}$$

We find also

$$\delta_3 = -\frac{5}{4}. \tag{17}$$

Let us summarize the results of the QCD corrections to the quantities $H_i(x)$:

$$\begin{array}{llll} \delta_1^{hard} = -\frac{8}{3}, & \delta_2^{hard} = 0, & \delta_3^{hard} = -6, & \delta_4^{hard} = -\frac{1}{2}, \\ \delta_1^{soft} = \frac{25}{12}, & \delta_2^{soft} = \frac{3}{4}, & \delta_3^{soft} = \frac{19}{4}, & \delta_4^{soft} = \frac{3}{4}. \end{array} \tag{18}$$

The division of the QCD corrections into the soft and hard parts helps us to determine the proper energy scale of the strong coupling constant. From the explicit calculations which we have described it is clear that the characteristic virtualities of the gluons are $(M_Z - M_W)/2$ and M_Z in the soft and hard parts, respectively. Therefore, instead of eq. (9) we use

$$H_i(x) = (1-x)^5 h_i^{(0)} \left(1 + C_f \frac{\alpha_s\left(M_Z^2\right)}{\pi} \delta_i^{hard} + C_f \frac{\alpha_s\left(\left[\frac{M_Z - M_W}{2}\right]^2\right)}{\pi} \delta_i^{soft} \right). \tag{19}$$

Among the four correction factors $\delta_i \equiv \delta_i^{hard} + \delta_i^{soft}$ two can be compared with previously known results. δ_2 is the well known QCD correction to (axial)vector currents. δ_1, on the other hand, is the first term of the expansion of the exact result for the square of the diagram 1b (or 1c) which was recently found in a rather unusual way [7]. The complete mixed electroweak and QCD corrections to the Z boson decay width were computed in the limit of very small and very large W boson mass. Their difference gives precisely the QCD corrections to the emission of real W. From the several known terms of both expansions the general (very simple) formula for the coefficients was guessed; the exact sum of this series is, however, rather complicated:

$$\begin{aligned} \Gamma^{1b+1c} = C_F \frac{\alpha_s}{\pi} \Gamma_0 \frac{g_-^{u2} + g_-^{d2}}{2} & \left[-\frac{7}{4} + \frac{1}{2}\ln x + \frac{3}{32x} - \frac{9x}{8}\ln x + x^2\left(\frac{7}{4} + \frac{1}{2}\ln x\right) \right. \\ & \left. -\frac{3x^3}{32} + \frac{2x^2}{3} S(x) - \frac{2}{3} S\left(\frac{1}{x}\right) \right]. \end{aligned} \tag{20}$$

Here

$$S(x) \;=\; -\frac{4}{x^2}\left(x^2+4x+1\right)\mathrm{Li}_4(x) + \frac{2}{x^2}\left(7x^2+16x+7\right)\mathrm{Li}_2(x)$$

$$
-\frac{30}{x^2}\left(1-x^2\right)\ln(1-x)-\frac{137}{4}-\frac{40}{x}
+\ln(x)\left[\frac{4}{x^2}\left(x^2+4x+1\right)\mathrm{Li}_3(x)+\frac{6}{x^2}\left(1-x^2\right)\mathrm{Li}_2(x)-18-\frac{10}{x}\right]
+\ln^2(x)\left[-\frac{1}{x^2}\left(x^2+4x+1\right)\mathrm{Li}_2(x)+\frac{3}{x^2}\left(1-x^2\right)\ln(1-x)+\frac{23}{4}+\frac{4}{x}\right] \quad (21)
$$

where $\mathrm{Li}_{2,3,4}(x)$ are polylogarithms [8]. An expansion of this formula around $x = 1$ confirms our result for δ_1. One can expect that the exact result for the correction to the interference of diagrams 1b and 1c would be even more complicated and its full calculation is a rather daunting task. Therefore it is appropriate to use an expansion method described in this section.

4 Conclusion

We have presented a simple estimate of the $\mathcal{O}(\alpha_s)$ correction to the decay rate $Z \to W^- u\bar{d}, W^+ \bar{u} d$. Our approach is based on the expansion of the event rate in powers of $(M_Z - M_W)/M_Z$. In principle, this expansion parameter is not very small and the leading term in the expansion is not expected to approximate the exact result with high accuracy. However, it gives us reasonable estimates of the lowest order and one–loop QCD corrections to the decay width almost without effort. Another important point is that the accuracy of this approximation can be improved by calculating further terms in the expansion, should need arise. Such a calculation is by no means impossible.

Combining all QCD corrections presented in eq. (18) we arrive at the following correction to the decay rate:

$$
\Gamma(Z \to W u\bar{d}) = \Gamma^{\mathrm{Born}}(Z \to W u\bar{d}) \left(1 + \frac{4}{3}\frac{\alpha_s\left(M_Z^2\right)}{\pi} + \frac{\alpha_s\left(\left[\frac{M_Z - M_W}{2}\right]^2\right)}{\pi}\right) \quad (22)
$$

where we have used $C_F = 4/3$.

For the values of the strong coupling constant we use $\alpha_s\left(M_Z^2\right) = 0.12$ and $\alpha_s\left(\left[\frac{M_Z - M_W}{2}\right]^2\right)$ $= (0.25..0.30)$. Numerically this gives the correction to the decay rate $Z \to W u\bar{d}$ of the order of 13–15 percent.

Let us have a closer look at the accuracy of our formulas. We can compare our approximate results (7–9) for the lowest order event rate with the results based on a complete calculation [1]. We conclude that our results based on the leading asymptotics to the event rate give a 30 percent accuracy. Note also that the accuracy with which the ratios H_i/H_j are reproduced is even better and amounts to approximately 10%. We can therefore expect that the QCD corrections derived in this paper represent the complete unknown QCD corrections to the decay rate $Z \to W u d$ with the accuracy 10–20 percent.

Our final result is that the QCD corrections increase the decay rate of the process $Z \to W u\bar{d}$ by approximately 14 ± 2 percent.

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References

[1] W. Marciano, D. Wyler, Z. Phys. **C3** (1979) 181; H. Pietschmann, H. Rupertsberg, K. Svozil, Z. Phys. **C12** (1982) 367; H. Rupertsberg, K. Svozil, Acta Physica Austriaca, **54** (1982) 255.

[2] S. C. Lee and W. C. Su, Phys. Lett. **B205** (1988) 569; Z. Phys. **C40** (1988) 547.

[3] E. W. N. Glover and J. J. van der Bij (conv.), in G. Altarelli, R. Kleiss, and C. Verzegnassi (eds.), *Z Physics at LEP 1*, CERN Yellow Report 89-08.

[4] E. Braaten, A. Kumar, Phys. Rev. **D37** (1988) 3349.

[5] D. Albert, W.J. Marciano, Z. Parsa, and D. Wyler, Nucl. Phys. **B166** (1980) 460.

[6] S.G. Gorishny, preprints JINR E2–86–176, E2–86–177 (Dubna 1986); Nucl. Phys. **B319** (1989) 633; K.G. Chetyrkin, Teor. Mat. Fiz. **75** (1988) 26; **76** (1988) 207; V.A. Smirnov, Commun. Math. Phys. **134** (1990) 109; K.G. Chetyrkin, preprint MPI-PAE/PTh 13/91 (Munich, 1991).

[7] A. Czarnecki and J.H. Kühn, preprint TTP96-28, hep-ph/9608366.

[8] L. Lewin, *Polylogarithms and associated functions* (North Holland, New York, 1981).