TTP96-23
hep-ph/96ffff299

Effective Theory for Heavy Quarkonia Decays

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Contribution to the *III German–Russian Workshop on Heavy Quark Physics*,
Dubna, Russia, May 20–22, 1996

Abstract

An effective theory approach to heavy quarkonia decays based on the $1/m_Q$ expansion is introduced. Its application to decays in which the two heavy quarks annihilate is discussed.

1 Introduction

Heavy Quark Effective Theory (HQET) [1] has turned out to be a very successful approach to describe systems with a single heavy quark. It is based on the infinite mass limit of QCD which serves as a starting point to perform a systematic expansion in Λ_{QCD}/m_Q and $\alpha_s(m_Q)$ using the methods of effective field theory.

Presently we are at the threshold of the discovery of flavoured "doubly heavy" systems, i.e. states consisting of two heavy quarks (such as the B_c's and also baryons with $b = c = 1 = \pm 1$, $b = \pm 2$ or $c = \pm 2$), while quarkonia like systems (the ψ's and the Υ's) are in the meantime quite well known.

Motivated by these expectations the question arises whether one may set up a similar effective theory approach for systems with two (or even more) heavy quarks based on the $1/m_Q$ expansion of QCD. It turns out [2] that one can not use the static limit for two heavy quarks if their velocities differ only by an amount of order $1/m_Q$, i.e. $vv' - 1 \sim \Lambda_{QCD}/m_Q$. The static limit breaks down and one is forced to include at least the kinetic energy into the leading order dynamics; in other words one has to use a non-relativistic approximation instead of the static limit. Such approximations have been formulated in the QED context (NRQED) some time ago; Two slightly different approaches (NRQCD [3] and HQQET [4]) have recently been studied in QCD. In this talk the general features of such an effective theory is outlined and its application to this class of decays is considered in which the two heavy quarks annihilate.

2 Structure of HQQET/NRQCD

In order to set up a heavy quarkonium effective theory one starts from the $1/m_Q$ expansion of the QCD Lagrangian and the corresponding expansion of the fields

$$Q(x) = e^{-im_Q v\cdot x}\left[1+\frac{1}{2m_Q}(i\not{D}_\perp)+\frac{1}{4m_Q^2}\left(v\cdot D\not{D}_\perp-\frac{1}{2}\not{D}_\perp^2\right)+\cdots\right]h_v(x)\,, \tag{1}$$

$$\mathcal{L} = \bar{h}_v\left[iv\cdot D-\frac{1}{2m_Q}\not{D}_\perp^2+\frac{i}{4m_Q^2}\left(-\frac{1}{2}\not{D}_\perp^2 v\cdot D+\not{D}_\perp v\cdot D\not{D}_\perp-\frac{1}{2}v\cdot D\not{D}_\perp^2\right)+\cdots\right]h_v\,.$$

The first terms of these two expansions define the static limit, which has been successfully applied to systems with a single heavy quark. In order to describe a system with more than one heavy (anti)quark one has to write down the same expansion (1) for each heavy quark. However, in the static limit for a state with two or more heavy quarks one runs into problems with diverging phases and "complex anomalous dimensions", which are considered in detail in [2].

In order to cure this problem one has to choose the unperturbed system such that these phases are already generated by the leading order dynamics, i.e. instead of the static limit one has to use the non-relativistic Lagrangian. For a system with a heavy quark and a heavy antiquark one then starts from

$$\mathcal{L}_0 = \bar{h}_v^{(+)}(ivD)h_v^{(+)}-\bar{h}_v^{(-)}(ivD)h_v^{(-)}+K_1\,,\quad K_1=\bar{h}_v^{(+)}\frac{(iD)^2}{2m_Q}h_v^{(+)}+\bar{h}_v^{(-)}\frac{(iD)^2}{2m_Q}h_v^{(-)} \tag{2}$$

where we have assumed for simplicity that the two quarks have the same mass; the case of unequal mass is obvious.

Most of the success of HQET is due to heavy quark flavour and spin symmetry. However, once one uses (2) the symmetries are somewhat different for HQQET. First of all, (2) depends on the mass through the kinetic energy term; consequently the states will depend on m_Q in a non-perturbative way and heavy flavour symmetry is lost. On the other hand, (2) does not depend on the spins of the two heavy quarks so there is a spin symmetry which is larger than in HQET because we have two heavy quark spins; the resulting symmetry is an $SU(2)\otimes SU(2)$ corresponding to separate rotations of the two spins.

For the case of heavy quarkonia all states fall into spin symmetry quartets which should be degenerate in the non-relativistic limit. In spectroscopic notation ${}^{2S+1}\ell_J$ these quartets consist of the states

$$[n^1\ell_\ell \quad n^3\ell_{\ell-1} \quad n^3\ell_\ell \quad n^3\ell_{\ell+1}] \tag{3}$$

For the ground states the spin symmetry quartet consists of the η_Q (the 0^- state) and the three polarization directions of the Υ_Q (the 1^- state).

The heavy quarkonia spin symmetry restricts the non-perturbative input to a calculation of processes involving heavy quarkonia. Of particular interest are decays in which the heavy quarks inside the heavy quarkonium annihilate. The annihilation is a short distance process that can be calculated perturbatively in terms of quarks and gluons, while the long distance contribution is encoded in certain matrix elements of quark operators. Logarithmic dependences on the heavy quark mass may be calculated by employing the usual renormalization group machinery.

3 Annihilation Decays of Heavy Quarkonia

The starting point to calculate processes like $\eta_Q \to$*light hadrons*, $\eta_Q \to \gamma+$*light hadrons*, or the corresponding decays of the Υ_Q states is the transition operator T for two heavy quarks which annihilate into light degrees of freedom. This will in general be bilinear in the heavy quark fields, such that

$$T(X,\xi) = (-i)\bar{Q}(X+\xi)K(X,\xi)Q(X-\xi) \tag{4}$$

where $K(X,\xi)$ involves only light degrees of freedom and X and ξ correspond to the cms and relative coordinate respectively. We identify the field Q with the quark and $\bar{Q}$ with the antiquark, so we shall make the large scale m_Q explicit by redefining the fields as

$$Q(x) = \exp(-im_Q(vx))Q_v^{(+)}(x), \quad \bar{Q}(x) = \exp(-im_Q(vx))\bar{Q}_v^{(-)}(x) \tag{5}$$

This corresponds to the usual splitting of the heavy quark momentum into a large part $m_Q v$ and a residual piece k. Inserting this into (4) this yields

$$T = (-i)\exp[-i2m_Q vX]\bar{Q}_v^{(-)}(X+\xi)K(X,\xi)Q_v^{(+)}(X-\xi) \tag{6}$$

The inclusive decay rate for the decay of a quarkonium $\Psi \to$ *light degrees of freedom* is then given by

$$\Gamma = \langle\Psi| \left[\int d^4X d^4\xi d^4\xi' T(X,\xi)T^\dagger(0,\xi') + \text{h.c.}\right] |\Psi\rangle \tag{7}$$

The next step is to perform an Operator Product Expansion (OPE) for the non-local product of the quark field operators. This expansion will yield four-quark operators of increasing dimension starting with dim-6 operators. The increasing dimension of these operators will be compensated by inverse powers of the heavy quark mass, so generically the rate takes the form

$$\Gamma = m_Q \sum_{n,i} \left(\frac{1}{m_Q}\right)^{n-2} C(\mathcal{O}_i^{(n)},\mu)\langle\Psi|\mathcal{O}_i^{(n)}|\Psi\rangle|_\mu \tag{8}$$

where $n = 6, 7, \cdots$ is the dimension of the operator and i labels different operators with the same dimension. The coefficients $C_n^{(i)}(\mu)$ are related to the short distance annihilation process and hence may be calculated in perturbation theory in terms of quarks and gluons. Once QCD radiative corrections are included, the $C_n^{(i)}(\mu)$ acquire a dependence on the renormalization scale μ which is governed by the renormalization group of the effective theory. The rate Γ is independent of μ and hence the μ dependence of $C_n^{(i)}(\mu)$ has to be compensated by a corresponding dependence of the matrix elements.

The non-perturbative contributions are encoded in the matrix elements of the local four-quark operators, and the mass dependence of these operators is also expanded in powers of $1/m_Q$ and thus the remaining m_Q dependence of the matrix elements is only due to the states. In terms of the m_Q independent static fields

$$Q_v^{(+)}(x) = h^{(+)}(x) + \mathcal{O}(1/m_Q) \quad Q_v^{(-)}(x) = h^{(-)}(x) + \mathcal{O}(1/m_Q)$$

one has in total four dim-6 operators

$$A_1^{(C)} = \bar{h}^{(+)}\gamma_5 C h^{(-)}\,\bar{h}^{(-)}\gamma_5 C h^{(+)} \text{ and } A_2^{(C)} = \bar{h}^{(+)}\gamma_\mu C h^{(-)}\,\bar{h}^{(-)}\gamma^\mu C h^{(+)}$$

where C is a color matrix, where one has the two possibilities $C \otimes C = 1 \otimes 1$ or $C \otimes C = T^a \otimes T^a$. These operators do not mix under renormalization, all anomalous dimensions vanish.

There are no dim-7 operators, since these are all proprotional to (ivD) and can be rewritten in terms of dim-8 operators by the equations of motion. At dim-8 one finds 30 local operators

$$\begin{aligned}
B_1^{(C)} &= [(iD_\mu)\bar{h}^{(+)}\gamma_5 C h^{(-)}]\,[(iD^\mu)\bar{h}^{(-)}\gamma_5 C h^{(+)}] \\
B_2^{(C)} &= [(iD_\mu)\bar{h}^{(+)}\gamma_\lambda C h^{(-)}]\,[(iD^\mu)\bar{h}^{(-)}\gamma^\lambda C h^{(+)}] \\
B_3^{(C)} &= [(iD_\mu)\bar{h}^{(+)}\gamma_\lambda C h^{(-)}]\,[(iD^\lambda)\bar{h}^{(-)}\gamma^\mu C h^{(+)}] \\
\\
C_1^{(C)} &= [(iD^\mu)\bar{h}^{(+)}\gamma_5 C h^{(-)}]\,[\bar{h}^{(-)}\gamma_5 C(i\overset{\leftrightarrow}{D}_\mu)h^{(+)}] + \text{h.c.} \\
C_2^{(C)} &= [(iD^\mu)\bar{h}^{(+)}\gamma_\mu C h^{(-)}]\,[\bar{h}^{(-)} C(i\overset{\leftrightarrow}{\not{D}})h^{(+)}] + \text{h.c.} \\
C_3^{(C)} &= [(iD^\lambda)\bar{h}^{(+)}\gamma^\mu C h^{(-)}]\,[\bar{h}^{(-)}\gamma_\mu C(i\overset{\leftrightarrow}{D})_\lambda h^{(+)}] + \text{h.c.} \\
C_4^{(C)} &= [(iD^\lambda)\bar{h}^{(+)}\gamma^\mu C h^{(-)}]\,[\bar{h}^{(-)}\gamma_\lambda C(i\overset{\leftrightarrow}{D})_\mu h^{(+)}] + \text{h.c.} \\
\\
D_1^{(C)} &= [\bar{h}^{(+)}\gamma_5 C(i\overset{\leftrightarrow}{D})_\mu h^{(-)}]\,[\bar{h}^{(-)}\gamma_5 C(i\overset{\leftrightarrow}{D})^\mu h^{(+)}] \\
D_2^{(C)} &= [\bar{h}^{(+)} C(i\overset{\leftrightarrow}{\not{D}})h^{(-)}]\,[\bar{h}^{(-)} C(i\overset{\leftrightarrow}{\not{D}})h^{(+)}] \\
D_3^{(C)} &= [\bar{h}^{(+)}\gamma_\lambda C(i\overset{\leftrightarrow}{D})_\mu h^{(-)}]\,[\bar{h}^{(-)}\gamma^\lambda C(i\overset{\leftrightarrow}{D})^\mu h^{(+)}] \\
D_4^{(C)} &= [\bar{h}^{(+)}\gamma_\mu C(i\overset{\leftrightarrow}{D})_\lambda h^{(-)}]\,[\bar{h}^{(-)}\gamma^\lambda C(i\overset{\leftrightarrow}{D})^\mu h^{(+)}] \\
\\
E_1^{(C)} &= [\bar{h}^{(+)}\gamma_5 C h^{(-)}]\,[\bar{h}^{(-)}\gamma_5 C(i\overset{\leftrightarrow}{D})^2 h^{(+)}] + \text{h.c.} \\
E_2^{(C)} &= [\bar{h}^{(+)}\gamma^\mu C h^{(-)}]\,[\bar{h}^{(-)} C(i\overset{\leftrightarrow}{\not{D}})(i\overset{\leftrightarrow}{D})_\mu h^{(+)}] + \text{h.c.} \\
E_3^{(C)} &= [\bar{h}^{(+)}\gamma^\mu C h^{(-)}]\,[\bar{h}^{(-)} C(i\overset{\leftrightarrow}{D})_\mu(i\overset{\leftrightarrow}{\not{D}})h^{(+)}] + \text{h.c.} \\
E_4^{(C)} &= [\bar{h}^{(+)}\gamma^\mu C h^{(-)}]\,[\bar{h}^{(-)}\gamma_\mu C(i\overset{\leftrightarrow}{D})^2 h^{(+)}] + \text{h.c.}
\end{aligned}$$

In addition to these contributions one has also non-local terms originating from single insertions of the Lagrangian of order $1/m_Q^2$ and from double insertions of the Lagrangian of order $1/m_Q$. Under renormalization the local dim-8 operators do not mix; only the double insertion of the kinetic energy operator of order $1/m_Q$ mixes into some of the above operators. Denoting this contribution as $T_i^{(C)}$

$$T_i^{(C)} = \frac{(-i)^2}{2} \int d^4x d^4y T[A_i^{(C)} K_1(x) K_1(y)] \tag{9}$$

one obtains in one-loop renormalization group improved perturbation theory two sets of equations for the coefficients of the operators with the spin structure $\gamma_5 \otimes \gamma_5$

$$\begin{aligned}
C(D_1^{(1)}, \mu) &= C(D_1^{(1)}, m_Q) + \frac{32}{9}\frac{1}{33-2n_f} C(T_1^{(8)}, m_Q) \ln \eta \\
C(E_1^{(1)}, \mu) &= C(E_1^{(1)}, m_Q) - \frac{8}{33-2n_f} C(T_1^{(1)}, m_Q) \ln \eta \\
C(B_1^{(8)}, \mu) &= C(B_1^{(8)}, m_Q) - \frac{24}{33-2n_f} C(T_1^{(8)}, m_Q) \ln \eta \\
C(D_1^{(8)}, \mu) &= C(D_1^{(8)}, m_Q) - \frac{16}{33-2n_f} C(T_1^{(1)}, m_Q) \ln \eta + \frac{20}{3}\frac{1}{33-2n_f} C(T_1^{(1)}, m_Q) \ln \eta \\
C(E_1^{(8)}, \mu) &= C(E_1^{(8)}, m_Q) - \frac{14}{3}\frac{1}{33-2n_f} C(T_1^{(1)}, m_Q) \ln \eta
\end{aligned}$$

where $\eta = (\alpha_s(\mu)/\alpha_s(m_Q))$. Furthermore, the coefficients $C(T_i^{(C)}, m_Q)$ are the same as the ones for the dim-6 operators $C(A_i^{(C)}, m_Q)$ since the kinetic energy operator is not renormalized.

The second set of equations is for the operators with spin structure $\gamma_\mu \otimes \gamma^\mu$ and due to heavy quarkonia spin symmetry one obtains the same equations; all other renormalization group equations are trivial.

A calculation of an annihilation decay then involves to calculate the $C(\mathcal{O}_n^{(i)}, \mu)$ at the scale $\mu = m_Q$ by matching the effective theory to full QCD. Once this is done, one may run down to some small scale μ of the order of the "binding energy" of the heavy quarkonium, thereby resumming the well known logarithms of the form $\ln(m_Q/\mu)$ that appear in the calculations of decay rates of heavy p-wave quarkonia. As an example, in [5] the decay $\eta_Q \to \gamma +$ *light hadrons* is studied in HQQET.

The matrix elements of these operators are non-perturbative quantities, which are constrained by heavy quarkonia spin symmetry. In order to exploit this symmetry, one may use the usual representation matrices for the spin singlet and spin triplet quarkonia

$$H_1(v) = \sqrt{M} P_+ \gamma_5 \text{ for } S = 0\,, \quad H_3(v) = \sqrt{M} P_+ \epsilon \text{ for } S = 1 \tag{10}$$

where $M \approx 2m_Q$ is the mass of the heavy quarkonium and $P_+ = (1 + \not{v})/2$. Using this one finds for the matrix elements of the dim-6 operators

$$\langle \Psi | \bar{h}^{(+)} \Gamma C h^{(-)} \, \bar{h}^{(-)} \Gamma' C h^{(+)} | \Psi \rangle = a^{(C)}(n, \ell) G\,, \text{ with } G = \mathrm{Tr}(\overline{H}_{2s+1} \Gamma) \mathrm{Tr}(\Gamma' H_{2s+1}) \tag{11}$$

Thus for each n and ℓ and for each color combination one finds a single parameter for both the spin singlet and spin triplet quarkonium.

Correspondingly one finds for the dim-8 operators

$$\begin{aligned}
\langle\Psi|(iD_\mu)[\bar{h}^{(+)}\Gamma C h^{(-)}]\,(iD_\nu)[\bar{h}^{(-)}\Gamma' C h^{(+)}]|\Psi\rangle &= b^{(C)}(n,\ell)(g_{\mu\nu}-v_\mu v_\nu)G \\
\langle\Psi|[\bar{h}^{(+)}\Gamma C(i\overleftrightarrow{D}_\mu)h^{(-)}]\,(iD_\nu)[\bar{h}^{(-)}\Gamma' C h^{(+)}]+\text{h.c.}|\Psi\rangle &= c^{(C)}(n,\ell)(g_{\mu\nu}-v_\mu v_\nu)G \\
\langle\Psi|[\bar{h}^{(+)}\Gamma C(i\overleftrightarrow{D}_\mu)h^{(-)}]\,[\bar{h}^{(-)}\Gamma' C(i\overleftrightarrow{D}_\nu)h^{(+)}]|\Psi\rangle &= d^{(C)}(n,\ell)(g_{\mu\nu}-v_\mu v_\nu)G \\
\langle\Psi|[\bar{h}^{(+)}\Gamma C(i\overleftrightarrow{D}_\mu)(i\overleftrightarrow{D}_\nu)h^{(-)}]\,[\bar{h}^{(-)}\Gamma' C h^{(+)}]+\text{h.c.}|\Psi\rangle &= e^{(C)}(n,\ell)(g_{\mu\nu}-v_\mu v_\nu)G
\end{aligned}$$

For fixed values of n and ℓ one finds that eight parameters are needed to describe the matrix elements of the dim-8 operators.

These matrix elements are non-perturbative, but from vacuum insertion one suspects

$$\begin{aligned}
a^{(1)}(n,0) \sim |R_{n0}(0)|^2 \gg a^{(1)}(n,\ell) \text{ for } l\neq 0\,, &\quad a^{(1)}(n,0) \gg a^{(8)}(n,\ell) \text{ for all } n,l \\
d^{(1)}(n,1) \sim |R'_{n1}(0)|^2 \gg d^{(1)}(n,\ell) \text{ for } \ell\neq 1\,, &\quad d^{(1)}(n,1) \gg d^{(8)}(n,\ell) \text{ for all } n,\ell \\
e^{(1)}(n,0) \sim \mathrm{Re}\,[R''_{n0}(0)R^*_{n0}(0)] \gg e^{(1)}(n,\ell) \text{ for } \ell\neq 0\,, &\quad e^{(1)}(n,1) \gg e^{(8)}(n,\ell) \text{ for all } n,\ell
\end{aligned}$$

where $R_{nl}(r)$ is the radial wave function of the quarkonium. The same reasoning yields the expectation that $b^{(C)}(n,\ell)$ and $c^{(C)}(n,\ell)$ are small compared to the coefficients that are non-vanishing in vacuum insertion.

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